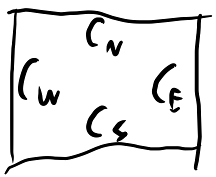
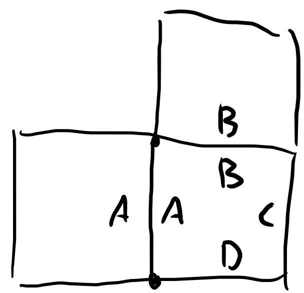
Definitions:

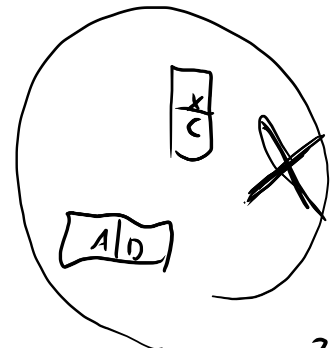
Wang tile over color set Σ: $(c_E, c_N, c_W, c_S) \in \Sigma^4$



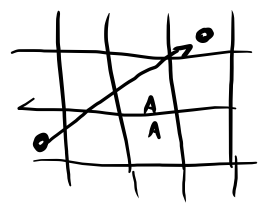
Tile set: $T \subseteq \Sigma^4$

Adjacency rules:





Tiling over T: ...

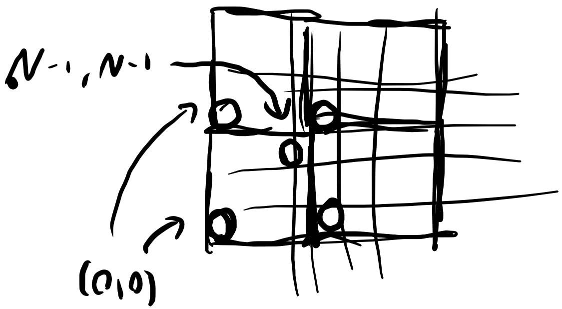


... $= x \in \underline{X_T} \subseteq \underline{T^{\mathbb{Z}^2}}$

Period of x: vector v st. $x_{w+v} = x_w \ \forall w$

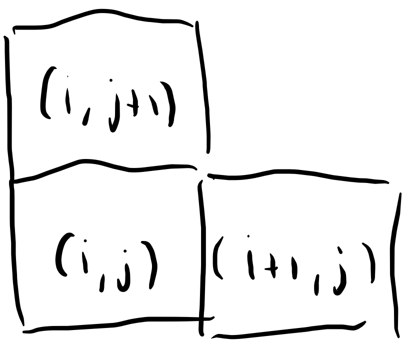
Shift space (of finite type) or subshift

Fixed point const. for tile sets



How to simulate any given tile set S:

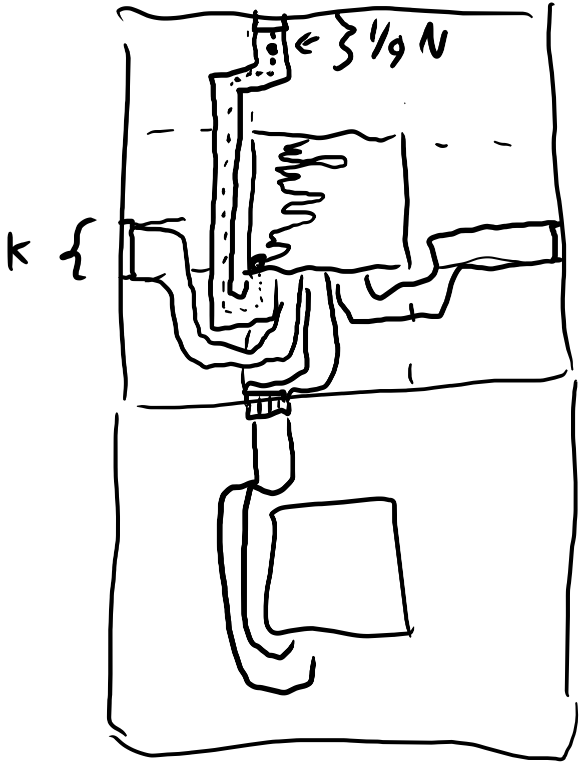
- Tiles of T have address $\in [0, N-1]^2$



mod N

Role of tile depends on its address!

Computation zone: addresses $[\frac{N}{3}, \frac{2N}{3}]^2$, simulate Turing machine M_S



Border bits: encode colors of S, route to M_S, it checks they form a tile of S

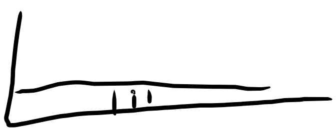
$\Rightarrow$ Simulation for large enough N

Want: $T = S$

Encode colors of T as bit vectors of length $\underbrace{2\log N}_{\text{address}} + \underbrace{m}_{\substack{\text{other}\\\text{stuff}\\\text{(constant)}}}$, replace M_s by a universal TM M_u and give it a program $\underline{P_T} \in \{0,1\}^*$ on a separate tape.

length $\log N + \underbrace{m'}_{\text{constant}}$

P_T is hard-coded: address $\frac{N}{3}+i$ has bit $(P_T)_i$



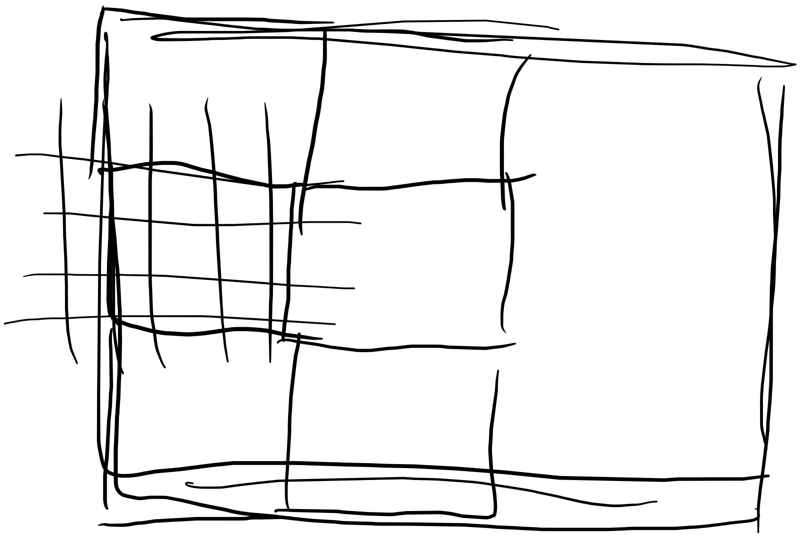
M_u can enforce this in the simulated tiles, since it can read P_T

Since length of encoding of T is $2\log N + m \ll N$

and $|P_T| = \log N + m' \ll N$

and time and space req. of M_u = poly$(\log N) \ll N$,

for large N, T simulates itself.

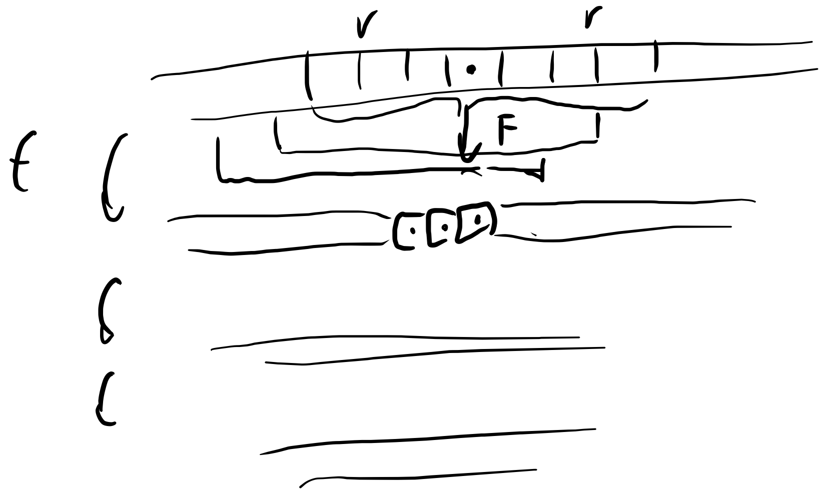


Variable zoom factor:

a macrotile can know its level and simulate tiling defined by $P_T(L)$ computable from $P_T(L-1)$

Fixed point CA:

CA on $A^{\mathbb{Z}}$ has radius $r \geq 0$ and local rule $F: A^{2r+1} \to A$, global rule $f: A^{\mathbb{Z}} \to A^{\mathbb{Z}}$



address $\in [0, Q-1]$, age $\in [0, U-1]$