Cantor Equicontinuous Factors of the Coven Cellular Automaton of Three Neighbours

Saliha Djenaoui

LMAM, Guelma university, Algeria

AUTOMATA'2021

CIRM, Luminy, Marseille, France

Plan

A (topological) dynamical system (DS) is (X, F) : X is a compact metric space and $F : X \to X$ is continuous.
If X is the Cantor space, (X, F) is a Cantor system.

A (topological) dynamical system (DS) is (X, F) : X is a compact metric space and $F : X \to X$ is continuous.
If X is the Cantor space, (X, F) is a Cantor system.

$(A^{\mathbb{Z}}, F)$ is a cellular automaton (CA) if there exist two integers $r_- \leq r_+$ and a local rule $f : A^{\mathbf{r}_+ - \mathbf{r}_- + 1} \to A$ such that

$$\forall x \in A^{\mathbb{Z}}, \forall i \in \mathbb{Z}, F(x)_i = f(x_{i+\mathbf{r}_-}, \ldots, x_{i+\mathbf{r}_+}).$$

- $A^{\mathbb{Z}}$ is the configuration space.
- $d(x, y) = 2^{-min\{\,|i| \in \mathbb{Z} \mid x_i \neq y_i\}}$ is the Cantor metric.

A (topological) dynamical system (DS) is (X, F) : X is a compact metric space and $F : X \to X$ is continuous.
If X is the Cantor space, (X, F) is a Cantor system.

$(A^{\mathbb{Z}}, F)$ is a cellular automaton (CA) if there exist two integers $r_- \leq r_+$ and a local rule $f : A^{\mathbf{r}_+ - \mathbf{r}_- + 1} \to A$ such that

$$\forall x \in A^{\mathbb{Z}}, \forall i \in \mathbb{Z}, F(x)_i = f(x_{i+\mathbf{r}_-}, \ldots, x_{i+\mathbf{r}_+}).$$

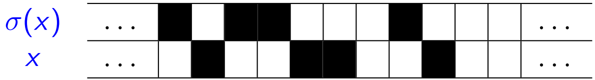
- $u \in A^n$, a cylinder of u is $[u]_j = \left\{ x \in A^{\mathbb{Z}} \middle| x_{[\![j, j+n[\![} = u \right\}$.

A (topological) dynamical system (DS) is (X, F) : X is a compact metric space and $F : X \to X$ is continuous.
If X is the Cantor space, (X, F) is a Cantor system.

$(A^{\mathbb{Z}}, F)$ is a cellular automaton (CA) if there exist two integers $r_- \leq r_+$ and a local rule $f : A^{\mathbf{r}_+ - \mathbf{r}_- + 1} \to A$ such that

$$\forall x \in A^{\mathbb{Z}}, \forall i \in \mathbb{Z}, F(x)_i = f(x_{i+\mathbf{r}_-}, \dots, x_{i+\mathbf{r}_+}).$$

- $u \in A^n$, a cylinder of u is $[u]_j = \left\{ x \in A^{\mathbb{Z}} \middle| x_{[\![j, j+n [\![} = u \right\}$.
- $(A^{\mathbb{Z}}, \sigma)$ is the shift if $\forall x \in A^{\mathbb{Z}}, \forall i \in \mathbb{Z}, \sigma(x)_i = x_{i+1}$.



A (topological) dynamical system (DS) is (X, F) : X is a compact metric space and $F : X \to X$ is continuous.
If X is the Cantor space, (X, F) is a Cantor system.

$(A^{\mathbb{Z}}, F)$ is a cellular automaton (CA) if there exist two integers $r_- \leq r_+$ and a local rule $f : A^{\mathbf{r}_+ - \mathbf{r}_- + 1} \to A$ such that

$$\forall x \in A^{\mathbb{Z}}, \forall i \in \mathbb{Z}, F(x)_i = f(x_{i+\mathbf{r}_-}, \ldots, x_{i+\mathbf{r}_+}).$$

- $u \in A^n$, a cylinder of u is $[u]_j = \left\{ x \in A^{\mathbb{Z}} \middle| x_{[\![j,j+n[\![} = u \right\}$.
- $(A^{\mathbb{Z}}, \sigma)$ is the shift if $\forall x \in A^{\mathbb{Z}}, \forall i \in \mathbb{Z}, \sigma(x)_i = x_{i+1}$.
  * F is a CA on $A^{\mathbb{Z}}$ iff it is continuous and commutes with σ.

A (topological) dynamical system (DS) is (X, F) : X is a compact metric space and $F : X \to X$ is continuous.
If X is the Cantor space, (X, F) is a Cantor system.

$(A^{\mathbb{Z}}, F)$ is a cellular automaton (CA) if there exist two integers $r_- \leq r_+$ and a local rule $f : A^{\mathbf{r}_+ - \mathbf{r}_- + 1} \to A$ such that

$$\forall x \in A^{\mathbb{Z}}, \forall i \in \mathbb{Z}, F(x)_i = f(x_{i+\mathbf{r}_-}, \dots, x_{i+\mathbf{r}_+}).$$

- $u \in A^n$, a cylinder of u is $[u]_j = \left\{ x \in A^{\mathbb{Z}} \middle| x_{[\![j, j+n[\![} = u \right\}$.
- $(A^{\mathbb{Z}}, \sigma)$ is the shift if $\forall x \in A^{\mathbb{Z}}, \forall i \in \mathbb{Z}, \sigma(x)_i = x_{i+1}$.
  * F is a CA on $A^{\mathbb{Z}}$ iff it is continuous and commutes with σ.
- $A^{\mathbb{Z}}$ is a Cantor space : perfect and totally disconnected.

A (topological) dynamical system (DS) is (X, F) : X is a compact metric space and $F : X \to X$ is continuous.
If X is the Cantor space, (X, F) is a Cantor system.

$(A^{\mathbb{Z}}, F)$ is a cellular automaton (CA) if there exist two integers $r_- \leq r_+$ and a local rule $f : A^{\mathbf{r}_+ - \mathbf{r}_- + 1} \to A$ such that

$$\forall x \in A^{\mathbb{Z}}, \forall i \in \mathbb{Z}, F(x)_i = f(x_{i+\mathbf{r}_-}, \ldots, x_{i+\mathbf{r}_+}).$$

- $u \in A^n$, a cylinder of u is $[u]_j = \left\{ x \in A^{\mathbb{Z}} \middle| x_{[\![j,j+n[\![} = u \right\}$.
- $(A^{\mathbb{Z}}, \sigma)$ is the shift if $\forall x \in A^{\mathbb{Z}}, \forall i \in \mathbb{Z}, \sigma(x)_i = x_{i+1}$.
  * F is a CA on $A^{\mathbb{Z}}$ iff it is continuous and commutes with σ.
- $A^{\mathbb{Z}}$ is a Cantor space : perfect and totally disconnected.

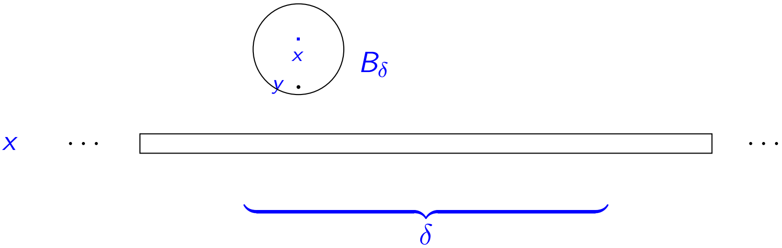
- A subshift is a closed σ-invariant subset $\Sigma \subseteq A^{\mathbb{Z}}$.

$\mathcal{E}_F \subseteq X$: the set of equicontinuous points. $x \in \mathcal{E}_F$ iff

$$\forall \varepsilon > 0, \exists \delta > 0, \forall y \in B_\delta(x), \forall t \in \mathbb{N}, d(F^t(y), F^t(x)) < \varepsilon.$$

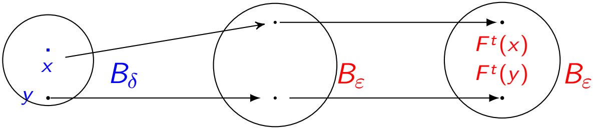
$\mathcal{E}_F \subseteq X$: the set of equicontinuous points. $x \in \mathcal{E}_F$ iff

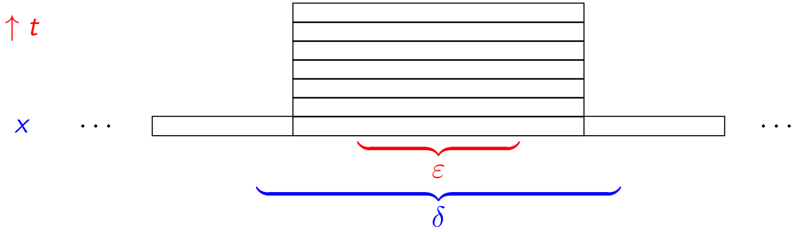
$$\forall \varepsilon > 0, \exists \delta > 0, \forall y \in B_\delta(x), \forall t \in \mathbb{N}, d(F^t(y), F^t(x)) < \varepsilon.$$



$\mathcal{E}_F \subseteq X$: the set of equicontinuous points. $x \in \mathcal{E}_F$ iff

$$\forall \varepsilon > 0, \exists \delta > 0, \forall y \in B_\delta(x), \forall t \in \mathbb{N}, d(F^t(y), F^t(x)) < \varepsilon.$$





$\mathcal{E}_F \subseteq X$: the set of equicontinuous points. $x \in \mathcal{E}_F$ iff

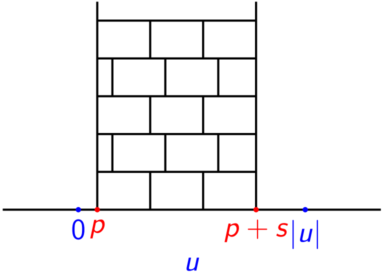
$$\forall \varepsilon > 0, \exists \delta > 0, \forall y \in B_\delta(x), \forall t \in \mathbb{N}, d(F^t(y), F^t(x)) < \varepsilon.$$

F is equicontinuous if $\mathcal{E}_F = X$.

$\mathcal{E}_F \subseteq X$: the set of equicontinuous points. $x \in \mathcal{E}_F$ iff

$$\forall \varepsilon > 0, \exists \delta > 0, \forall y \in B_\delta(x), \forall t \in \mathbb{N}, d(F^t(y), F^t(x)) < \varepsilon.$$

Let $s > 0$. u is a s-blocking word of CA F if $\exists p \in [\![0, |u| - s]\!]$ such that $\forall x, y \in [u]_0, \forall t \geq 0, F^t(x)_{[\![p,p+s[\![} = F^t(y)_{[\![p,p+s[\![}$.



$\mathcal{E}_F \subseteq X$: the set of equicontinuous points. $x \in \mathcal{E}_F$ iff

$$\forall \varepsilon > 0, \exists \delta > 0, \forall y \in B_\delta(x), \forall t \in \mathbb{N}, d(F^t(y), F^t(x)) < \varepsilon.$$

Let $s > 0$. u is a s-blocking word of CA F if $\exists p \in [\![0, |u| - s]\!]$ such that $\forall x, y \in [u]_0, \forall t \geq 0, F^t(x)_{[\![p,p+s[\![} = F^t(y)_{[\![p,p+s[\![}$.

* A Cantor system is equicontinuous iff all of its trace subshifts are finite.

$\mathcal{E}_F \subseteq X$: the set of equicontinuous points. $x \in \mathcal{E}_F$ iff

$$\forall \varepsilon > 0, \exists \delta > 0, \forall y \in B_\delta(x), \forall t \in \mathbb{N}, d(F^t(y), F^t(x)) < \varepsilon.$$

Let $s > 0$. u is a s-blocking word of CA F if $\exists p \in [\![0, |u| - s]\!]$ such that $\forall x, y \in [u]_0, \forall t \geq 0, F^t(x)_{[\![p,p+s[\![} = F^t(y)_{[\![p,p+s[\![}$.

* A Cantor system is equicontinuous iff all of its trace subshifts are finite.

The trace of a Cantor system $(A^{\mathbb{Z}}, F)$ is

$$T_F^{[\![-n,n[\![} : \quad A^{\mathbb{Z}} \to (A^{2n+1})^{\mathbb{N}}$$

$$x \to (F^t(x)_{[\![-n,n[\![})_{t\in\mathbb{N}}$$

(Y, G) is a factor of a DS (X, F) by a map $\Phi : X \to Y$ if Φ is continuous, surjective and

$$\Phi \circ F = G \circ \Phi.$$

* Every DS admits a maximal equicontinuous factor.

(Y, G) is a factor of a DS (X, F) by a map $\Phi : X \to Y$ if Φ is continuous, surjective and

$$\Phi \circ F = G \circ \Phi.$$

* Every DS admits a maximal equicontinuous factor.

A DS (X, F) is weakly mixing, if for any nonempty open sets $U, V, U', V' \subseteq X$, $\exists t \in \mathbb{N}, F^t(U) \cap U' \neq \emptyset$ and $F^t(V) \cap V' \neq \emptyset$.

* A weakly mixing DS has no nontrivial equicontinuous factor.

(Y, G) is a factor of a DS (X, F) by a map $\Phi : X \to Y$ if Φ is continuous, surjective and

$$\Phi \circ F = G \circ \Phi.$$

> Proposition 1 : A DS F admits a nontrivial Cantor equicontinuous factor if and only if F admits a nontrivial finite factor.

Proof:

(Y, G) is a factor of a DS (X, F) by a map $\Phi : X \to Y$ if Φ is continuous, surjective and

$$\Phi \circ F = G \circ \Phi.$$

Proposition 1 : A DS F admits a nontrivial Cantor equicontinuous factor if and only if F admits a nontrivial finite factor.

Proof:

G is a Cantor EF of F

$\downarrow$

Trace subshifts of G are finite + G is nontrivial

$\downarrow$

G has a nontrivial finite trace subshift

$\downarrow$

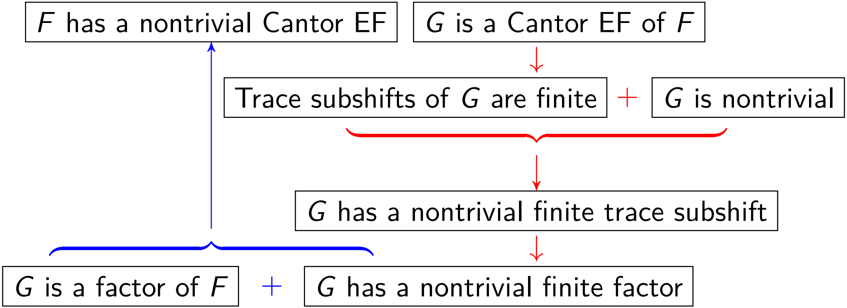
G has a nontrivial finite factor

(Y, G) is a factor of a DS (X, F) by a map $\Phi : X \to Y$ if Φ is continuous, surjective and

$$\Phi \circ F = G \circ \Phi.$$

> Proposition 1 : A DS F admits a nontrivial Cantor equicontinuous factor if and only if F admits a nontrivial finite factor.

Proof:



Plan

Proposition 2 : (Main Property) Let F be a surjective DS. If there exists a weakly mixing subsystem that intersects every nonempty strongly F-invariant clopen set, then F admits no nontrivial Cantor equicontinuous factor.

Proof :

Proposition 2 : (Main Property) Let F be a surjective DS. If there exists a weakly mixing subsystem that intersects every nonempty strongly F-invariant clopen set, then F admits no nontrivial Cantor equicontinuous factor.

Proof :

$F_{|W}$ is w. mixing + G is a finite factor

$\downarrow$

$G_{|\Phi(W)}$ is a singleton

Proposition 2 : (Main Property) Let F be a surjective DS. If there exists a weakly mixing subsystem that intersects every nonempty strongly F-invariant clopen set, then F admits no nontrivial Cantor equicontinuous factor.

Proof :

$\Phi^{-1}(\bigcup_n G^n \Phi(W)^C) \subseteq W^C$ + $F_{|W}$ is w. mixing + G is a finite factor

$\bigcup_{n\in\mathbb{Z}} G^n \subseteq \Phi(W)$

$G_{|\Phi(W)}$ is a singleton

Proposition 2 : (Main Property) Let F be a surjective DS. If there exists a weakly mixing subsystem that intersects every nonempty strongly F-invariant clopen set, then F admits no nontrivial Cantor equicontinuous factor.

Proof :

$\Phi^{-1}(\bigcup_n G^n\Phi(W)^C) \subseteq W^C$ + $F_{|W}$ is w. mixing + G is a finite factor

↓

G is surjective + $\bigcup_{n\in\mathbb{Z}} G^n \subseteq \Phi(W)$ + $G_{|\Phi(W)}$ is a singleton

↓

G is the identity over a singleton ; G is trivial.

Plan

The Coven CA of three neighbours is

$F : \{0,1\}^{\mathbb{Z}} \to \{0,1\}^{\mathbb{Z}}$ defined by $f : \{0,1\}^3 \to \{0,1\}$ such that

$$F(x)_i = f(x_i\ x_{i+1}\ x_{i+2}) = \begin{cases} x_i + 1 \bmod 2 & \textit{if} \quad x_{i+1} = 1 \textit{ and } x_{i+2} = 0 \\ x_i & \textit{otherwise} \end{cases} .$$

t+1	0	0	0	1	1	1	1	0
t	0 00	0 01	0 11	1 00	1 01	1 11	0 10	1 10

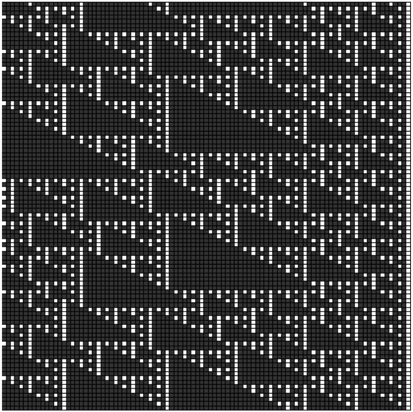


Figure: Space-time diagram of the Coven CA of three neighbours ↑ *Time*. 0s (resp. 1s) are represented by white squares (resp. black squares).

The Coven CA of three neighbours is $F : \{0,1\}^{\mathbb{Z}} \to \{0,1\}^{\mathbb{Z}}$ defined by $f : \{0,1\}^3 \to \{0,1\}$ such that

$$F(x)_i = f(x_i\ x_{i+1}\ x_{i+2}) = \begin{cases} x_i + 1 \bmod 2 & \textit{if} \quad x_{i+1} = 1 \textit{ and } x_{i+2} = 0 \\ x_i & \textit{otherwise} \end{cases}.$$

t+1	0	0	0	1	1	1	1	0
t	0 00	0 01	0 11	1 00	1 01	1 11	0 10	1 10

${}^{\infty}1^{\infty}$ is a fixed point, so

$(\{{}^{\infty}1^{\infty}\}, F)$ is a weakly mixing subsystem of this CA.

The Coven CA of three neighbours is $F : \{0,1\}^{\mathbb{Z}} \to \{0,1\}^{\mathbb{Z}}$ defined by $f : \{0,1\}^3 \to \{0,1\}$ such that

$$F(x)_i = f(x_i\ x_{i+1}\ x_{i+2}) = \begin{cases} x_i + 1 \bmod 2 & \textit{if} \quad x_{i+1} = 1 \textit{ and } x_{i+2} = 0 \\ x_i & \textit{otherwise} \end{cases} .$$

${}^{\infty}1^{\infty}$ is a fixed point, so

$(\{{}^{\infty}1^{\infty}\}, F)$ is a weakly mixing subsystem of this CA.

* A chain-transitive DS with a fixed point is chain-mixing.

The Coven CA of three neighbours is $F : \{0,1\}^{\mathbb{Z}} \to \{0,1\}^{\mathbb{Z}}$ defined by $f : \{0,1\}^3 \to \{0,1\}$ such that

$$F(x)_i = f(x_i\ x_{i+1}\ x_{i+2}) = \begin{cases} x_i + 1 \bmod 2 & \textit{if} \quad x_{i+1} = 1 \textit{ and } x_{i+2} = 0 \\ x_i & \textit{otherwise} \end{cases}.$$

${}^{\infty}1^{\infty}$ is a fixed point, so

$(\{{}^{\infty}1^{\infty}\}, F)$ is a weakly mixing subsystem of this CA.

* A chain-transitive DS with a fixed point is chain-mixing.
* A chain-mixing DS has no nontrivial (finite) periodic factor.

$\Sigma_0 \cup \Sigma_1$ is the subshift Σ_F, $F = \{01^{2k}0, k \in \mathbb{N}\}$. $x \in \Sigma_0 \cup \Sigma_1$ iff in x, between each 2 successive zeros, there isan odd number of 1.

$\Sigma_0 \cup \Sigma_1$ is the subshift Σ_F, $F = \{01^{2k}0, k \in \mathbb{N}\}$. $x \in \Sigma_0 \cup \Sigma_1$ iff in x, between each 2 successive zeros, there isan odd number of 1.

Proposition 3 : Let $k \in \mathbb{N}$. Then,

- $01^{2k}0$ is a minimal 1-blocking word with offset 0.

Proof : By Induction.

- In this CA, 00 is a minimal 1-blocking word.

$$F^k([01^{2k}0]) \subseteq [00].$$

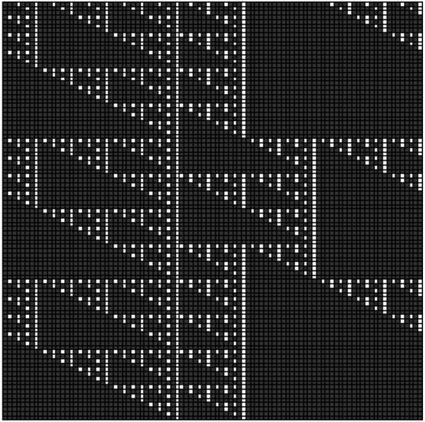


Figure: Diagram with the blocking word $01^{14}0$ ↑ *Time*.
0s (resp. 1s) are represented by white squares (resp. black squares)

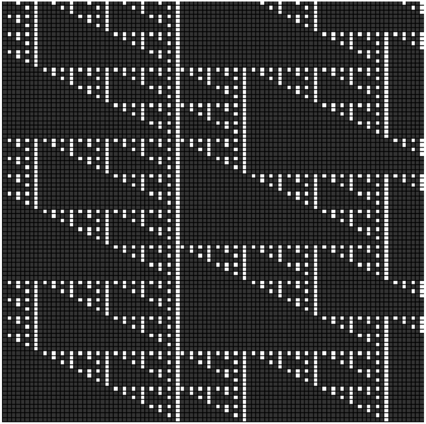


Figure: Diagram with the blocking word $01^{14}0$ $\uparrow$ *Time*.
0s (resp. 1s) are represented by white squares (resp. black squares).

$\Sigma_0 \cup \Sigma_1$ is the subshift Σ_F, $F = \{01^{2k}0, k \in \mathbb{N}\}$. $x \in \Sigma_0 \cup \Sigma_1$ iff in x, between each 2 successive zeros, there isan odd number of 1.

Proposition 3 : Let $k \in \mathbb{N}$. Then,

- $01^{2k}0$ is a minimal 1-blocking word with offset 0.
- The minimal blocking words are all of the form $01^{2k}0$.

$\Sigma_0 \cup \Sigma_1$ is the subshift Σ_F, $F = \{01^{2k}0, k \in \mathbb{N}\}$. $x \in \Sigma_0 \cup \Sigma_1$ iff in x, between each 2 successive zeros, there isan odd number of 1.

Proposition 3 : Let $k \in \mathbb{N}$. Then,

- $01^{2k}0$ is a minimal 1-blocking word with offset 0.
- The minimal blocking words are all of the form $01^{2k}0$.

Proof : By Induction.

- Let $a, b \in \{0,1\}$, $w \in \mathcal{L}(\Sigma_0)$, $|w| = 2^n - 1$ and $awb \in \mathcal{L}(\Sigma_0)$.

$$F^{2^{n-1}}([awb]) \subseteq \begin{cases} [1] & \text{if} \quad a = b \\ [0] & \text{if} \quad a \neq b \end{cases}.$$

$\Sigma_0 \cup \Sigma_1$ is the subshift Σ_F, $F = \{01^{2k}0, k \in \mathbb{N}\}$. $x \in \Sigma_0 \cup \Sigma_1$ iff in x, between each 2 successive zeros, there isan odd number of 1.

Proposition 3 : Let $k \in \mathbb{N}$. Then,

- $01^{2k}0$ is a minimal 1-blocking word with offset 0.
- The minimal blocking words are all of the form $01^{2k}0$.

Proof : By Induction.

- Let $a, b \in \{0,1\}$, $w \in \mathcal{L}(\Sigma_0)$, $|w| = 2^n - 1$ and $awb \in \mathcal{L}(\Sigma_0)$.

$$F^{2^{n-1}}([awb]) \subseteq \begin{cases} [1] & \textit{if} \quad a = b \\ [0] & \textit{if} \quad a \neq b \end{cases}.$$

So, $x \in \Sigma_0 \cup \Sigma_1$ iff x is without blocking words.

Lemma 1: Let U be a strongly F-invariant clopen set.
If $U \cap (\Sigma_0 \cup \Sigma_1) \neq \emptyset$, then U contains ${}^{\infty}1^{\infty}$.

Lemma 1: Let U be a strongly F-invariant clopen set.
If $U \cap (\Sigma_0 \cup \Sigma_1) \neq \emptyset$, then U contains ${}^{\infty}1^{\infty}$.

Proof : By Induction : Base Case.

Let U be a strongly F-invariant clopen set and U contains $[u_0]_j, j \in \mathbb{Z}$, u_0 contains a single zero. Let $n > 1$ and $x \in [u_0]_j$,

$$x = 1^{\infty}\ 0\ 1^{2^n - 1}\ 0\ 1^{\infty}.$$

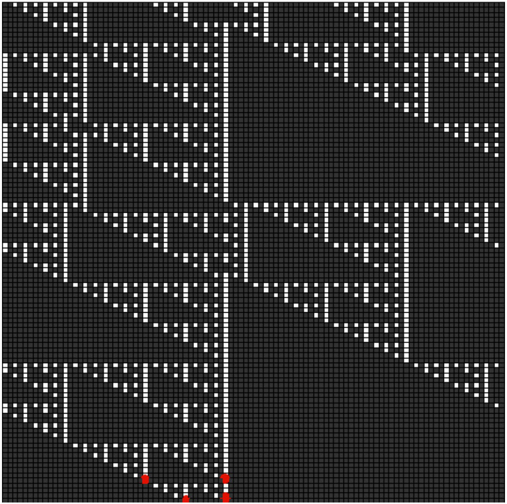


Figure: Base Case: Invariant clopen set contains a single zero ↑ *Time*.
0s (resp. 1s) are represented by white (red) squares (resp. black sq.).

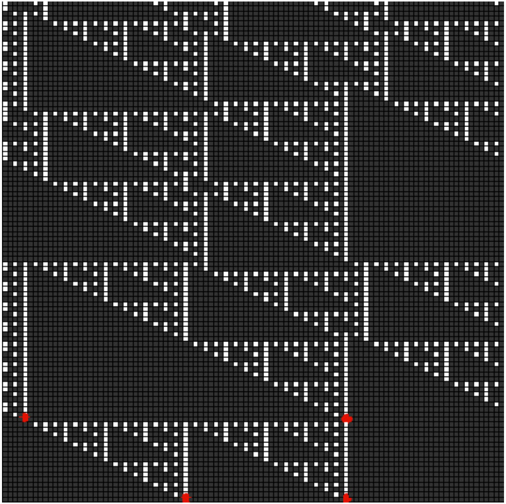


Figure: Base Case: Invariant clopen set contains a single zero ↑ *Time*.
0s (resp. 1s) are represented by white (red) squares (resp. black sq.).

Lemma 1: Let U be a strongly F-invariant clopen set.
If $U \cap (\Sigma_0 \cup \Sigma_1) \neq \emptyset$, then U contains ${}^{\infty}1^{\infty}$.

Proof : By Induction : Base Case.
Let U be a strongly F-invariant clopen set and U contains $[u_0]_j, j \in \mathbb{Z}$, u_0 contains a single zero. Let $n > 1$ and $x \in [u_0]_j$,

$$x = 1^{\infty}\ 0\ 1^{2^n-1}\ 0\ 1^{\infty}.$$

$$\Longrightarrow F^{2^{n-1}}(x) = 1^{\infty}\ 0\ 1^{2^{n+1}-1}\ 0\ 1^{\infty}.$$

Thus,

$$F^{2^{n-1}}(x) \underset{n\to\infty}{\longrightarrow} {}^{\infty}1^{\infty} \in U.$$

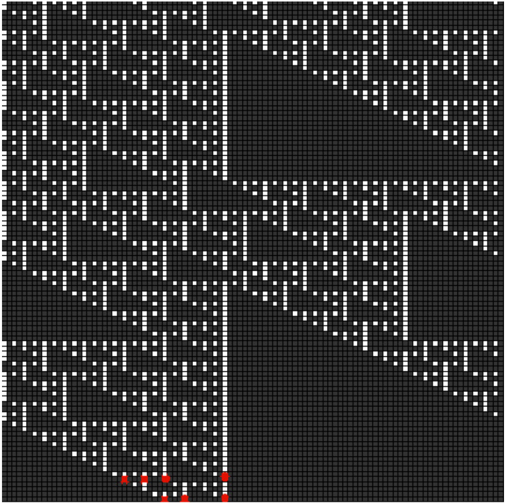


Figure: Induction Step: Invariant clopen set contains two zeros ↑ *Time*.
0s (resp. 1s) are represented by white (red) squares (resp. black sq.).

Figure: Induction Step: Invariant clopen set contains two zeros ↑ *Time*.
0s (resp. 1s) are represented by white (red) squares (resp. black sq.).

Lemma 1: Let U be a strongly F-invariant clopen set.
If $U \cap (\Sigma_0 \cup \Sigma_1) \neq \emptyset$, then U contains ${}^{\infty}1^{\infty}$.

Proof : By Induction : Inductive Step.
If U contains $[u]_j$, $u \in \mathcal{L}(\Sigma_0)$ and contains $N+1$ zeros, u is of the form $v01^{k_1}$, $k_1 \geq 0$, with v contains N zeros. Let $x \in [u]_j$.

$$x = 1^{\infty}\ v\ 0\ 1^{2^n-1}\ 0\ 1^{\infty}.$$

$$\Longrightarrow F^{2^{n-1}}(x) = 1^{\infty}\ 1^{2^n-1-|v|}\ v\ 0\ 1^{2^n-1-|v|}\ v\ 1^{2^n}\ 0\ 1^{\infty}.$$

Thus,

$$F^{2^{n-1}}(x) \underset{n\to\infty}{\longrightarrow} {}^{\infty}1^{\infty}\ v\ {}^{\infty}1^{\infty} \in U.$$

Proposition 4 : Let U be a strongly F-invariant clopen set. Then, $U \cap (\Sigma_0 \cup \Sigma_1) \neq \emptyset$.

Proof : By Induction : Base Case.

If U contains $[u]_j$, u contains a single minimal 1-blocking word.

$$\exists v \in \; 0\,1\,(\mathfrak{2}1)^{k_1'}\;0\;1^{2k_1}\;0\;1\;(\mathfrak{2}1)^{k_1''}\;0$$

where $k_1, k_1', k_1'' \geq 0$, $[v]_m \subseteq [u]_j$. Let $x \in [v]_m$.

$$x \in \quad 1^\infty\;(\mathfrak{2}1)^{2^{n-1}}\;0\;w\;0\;1^{k'}\;w\;0\;1^\infty.$$

where $w \in \mathcal{L}(\Sigma_0)$, $|w| = 2^n - 1 - k'$ and $01^{2k_1}0 \sqsubseteq 0w0$.

$$\Longrightarrow F^{2^{n-1}}(x) \in \quad 1^\infty\;(\mathfrak{2}1)^{2^n}\;0\;1^{2^n}\;w\;0\;1^\infty.$$

Thus,

$$F^{2^{n-1}}(x) \underset{n\to\infty}{\in} \quad (\mathfrak{2}1)^\infty\;0\;1^\infty \subseteq (\Sigma_0 \cup \Sigma_1).$$

Proposition 4 : Let U be a strongly F-invariant clopen set. Then, $U \cap (\Sigma_0 \cup \Sigma_1) \neq \emptyset$.

Proof : By Induction : Inductive Step.
If U contains $[u]_j$, u contains $N+1$ minimal 1-blocking words,

$$\exists\ v = \ 0\ v_1\ 0\ v_2\ 0 \dots 0\ v_N\ 0\ v_{N+1}\ 0$$

with $[v]_m \subseteq [u]_j$ and $0v_i0$ has a single minimal 1-blocking word, $\forall i$.
Let $x \in [v]_m$.

$$x \in \quad 1^\infty\ (\mathfrak{2}1)^{2^{n-1}}\ 0\ 1^{2k_1}\ 0\ v_2 0 \dots 0 v_N\ 0\ w\ 01^{k'}\ w\ 01^{2^n-1}1^\infty$$

where $w \in \mathcal{L}(\Sigma_0)$, $|w| = 2^n - 1 - k'$ and $01^{2k_{N+1}}0 \sqsubseteq 0w0$.

$$\Longrightarrow F^{2^{n-1}}(x) \in \quad 1^\infty\ (\mathfrak{2}1)^{2^n}\ 0v_1'0 \dots 0v_N'\ 0\ 1^{2^n}\ w\ 0\ 1^\infty$$

Thus,

$$F^{2^{n-1}}(x) \underset{n\to\infty}{\in} \quad (\mathfrak{2}1)^\infty\ 0v_1'0 \dots 0v_N' 0\ 1^\infty.$$

Plan

The Coven CA of three neighbours satisfies the main property, therefore, it has no nontrivial Cantor equicontinuous factors.

The Coven CA of three neighbours satisfies the main property, therefore, it has no nontrivial Cantor equicontinuous factors.

Questions :

- Does a chain-mixing system have a nontrivial equicontinuous factor, which is not a Cantor system ?

The Coven CA of three neighbours satisfies the main property, therefore, it has no nontrivial Cantor equicontinuous factors.

Questions :

- Does a chain-mixing system have a nontrivial equicontinuous factor, which is not a Cantor system ?
- Can a DS which has a nontrivial Cantor equicontinuous factor be chain-mixing ?