Regular Languages: To Finite Automata and Beyond

Succinct Descriptions and Optimal Simulations

Luca Prigioniero

AUTOMATA 2021

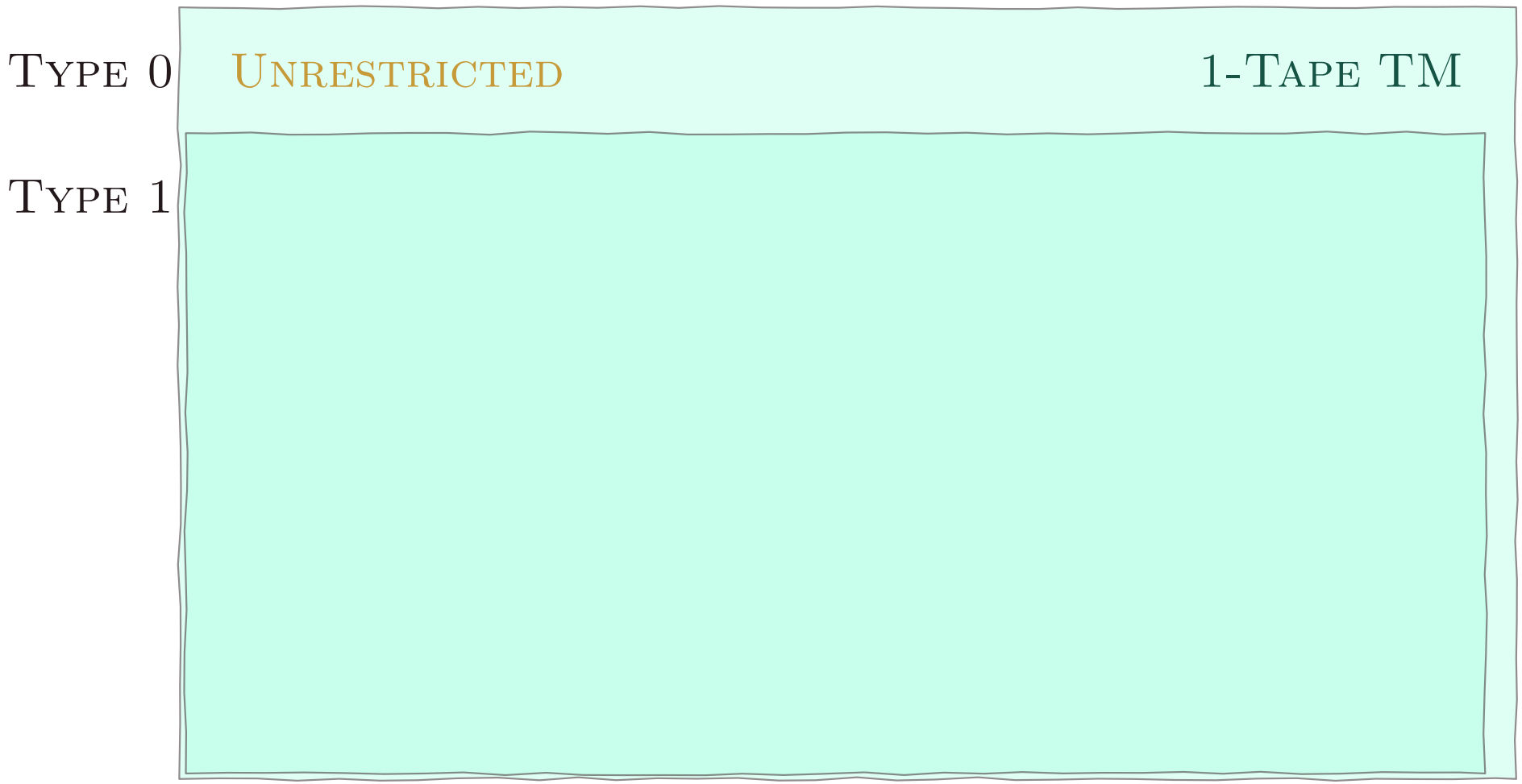
July 13th, 2021

UNIVERSITÀ DEGLI STUDI DI MILANO

DIPARTIMENTO DI INFORMATICA


Type 0


Type 0
Unrestricted
1-Tape TM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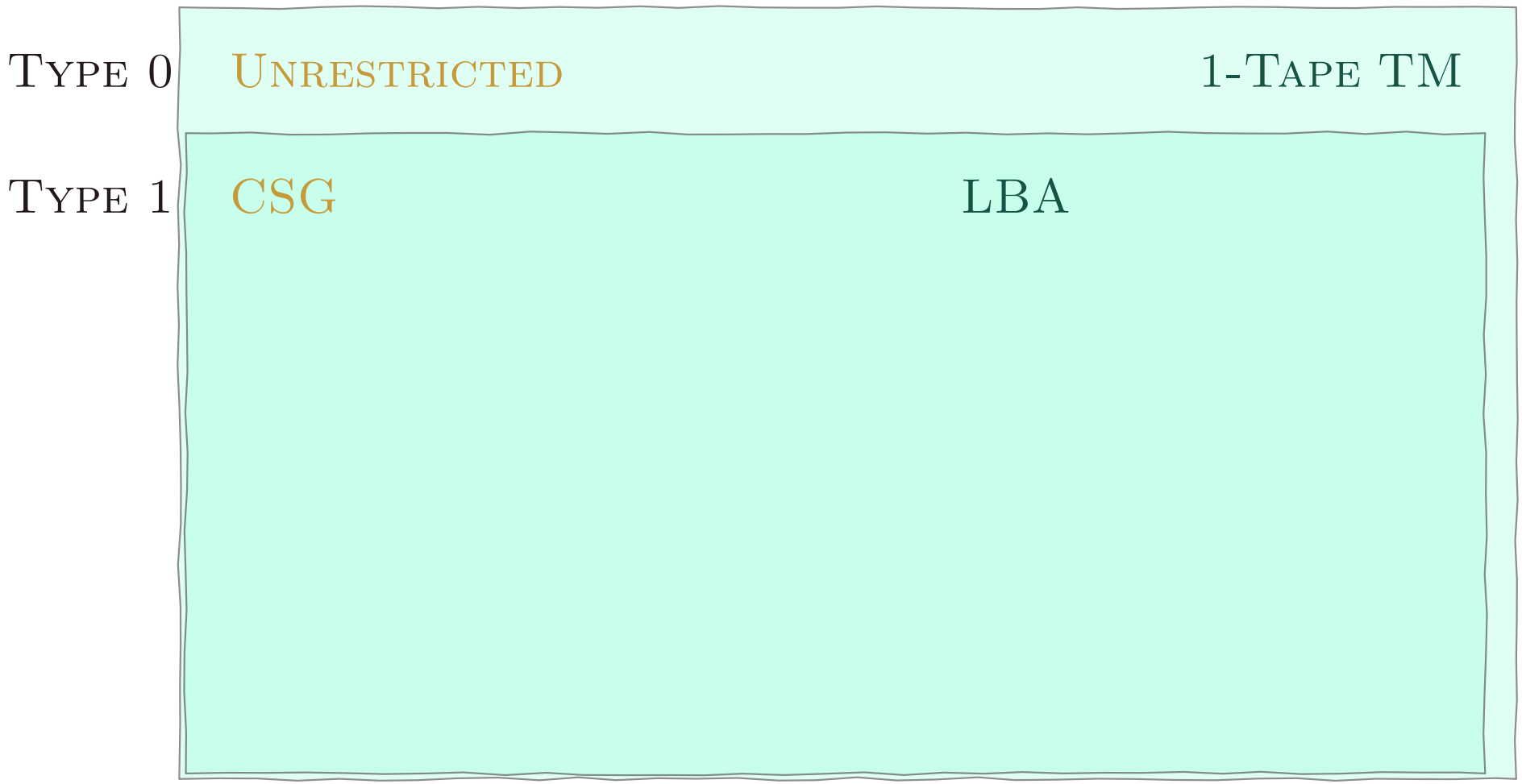
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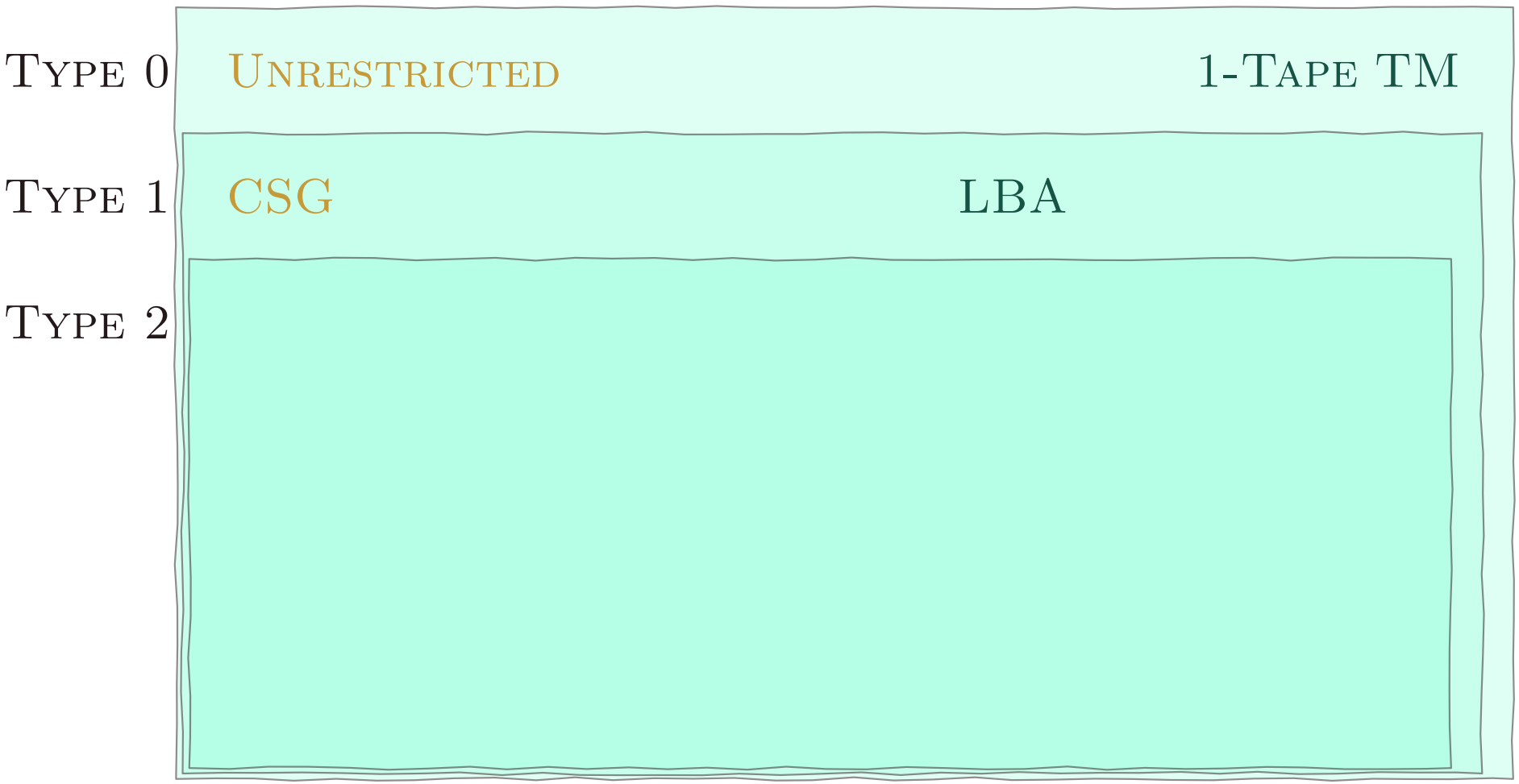
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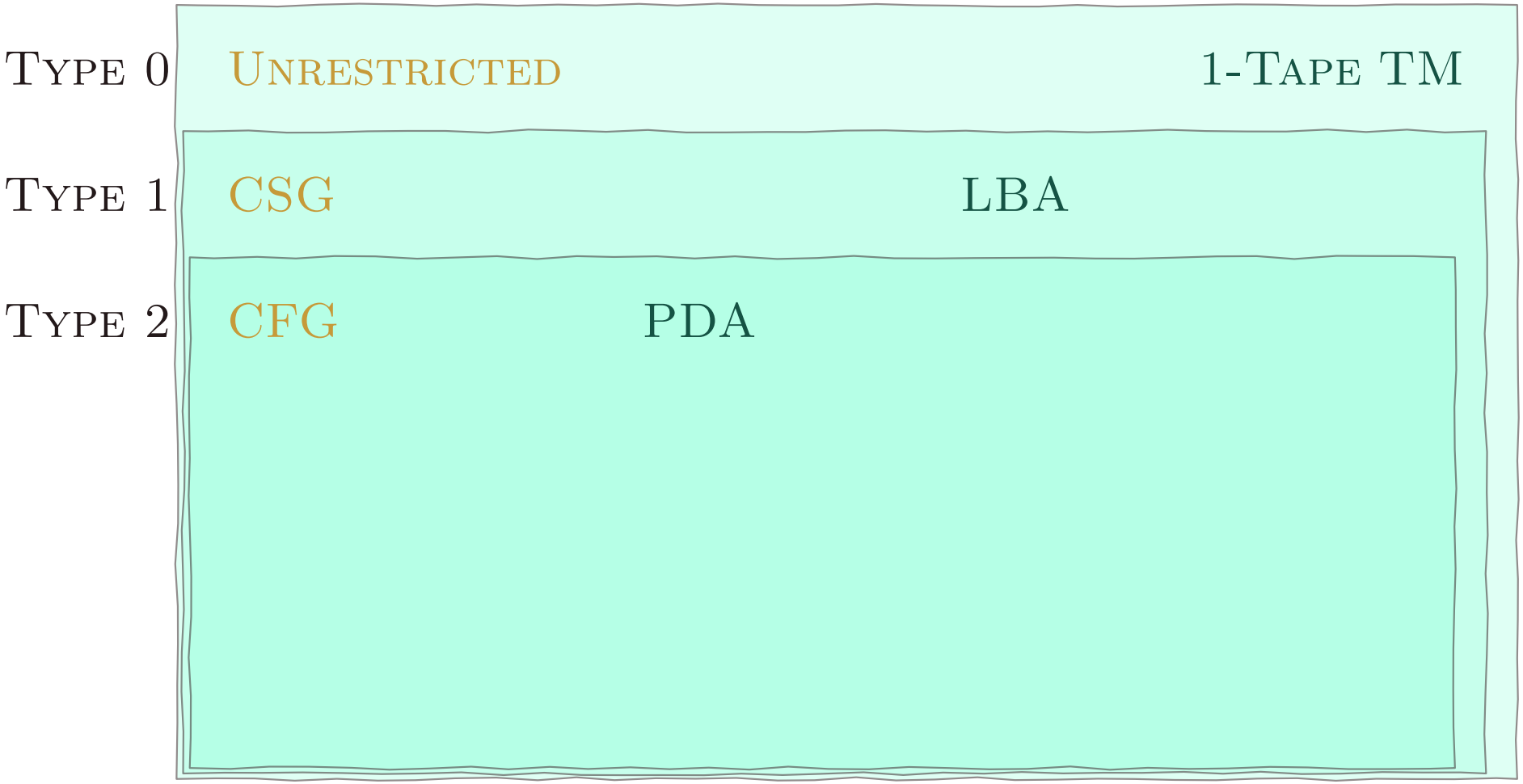
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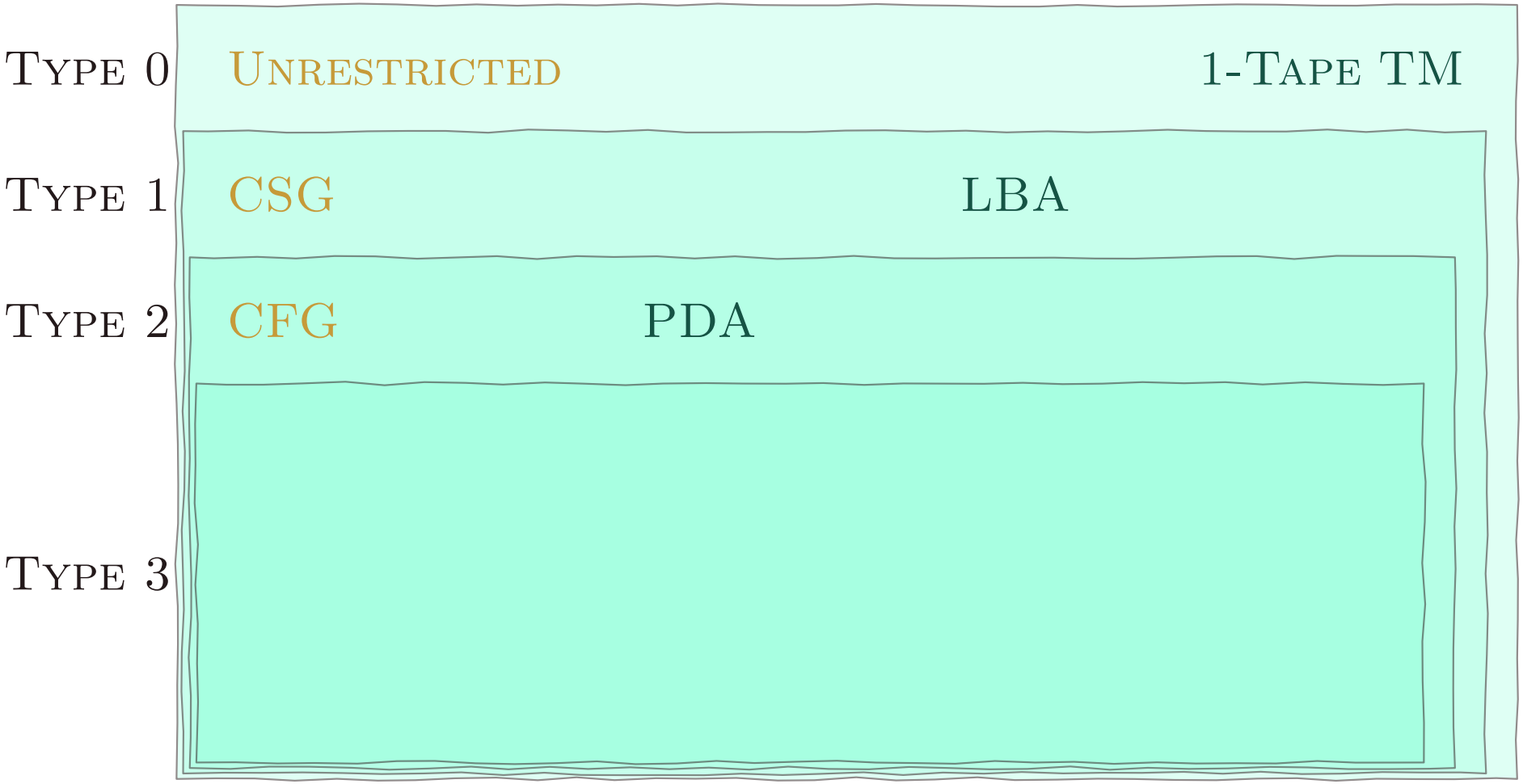
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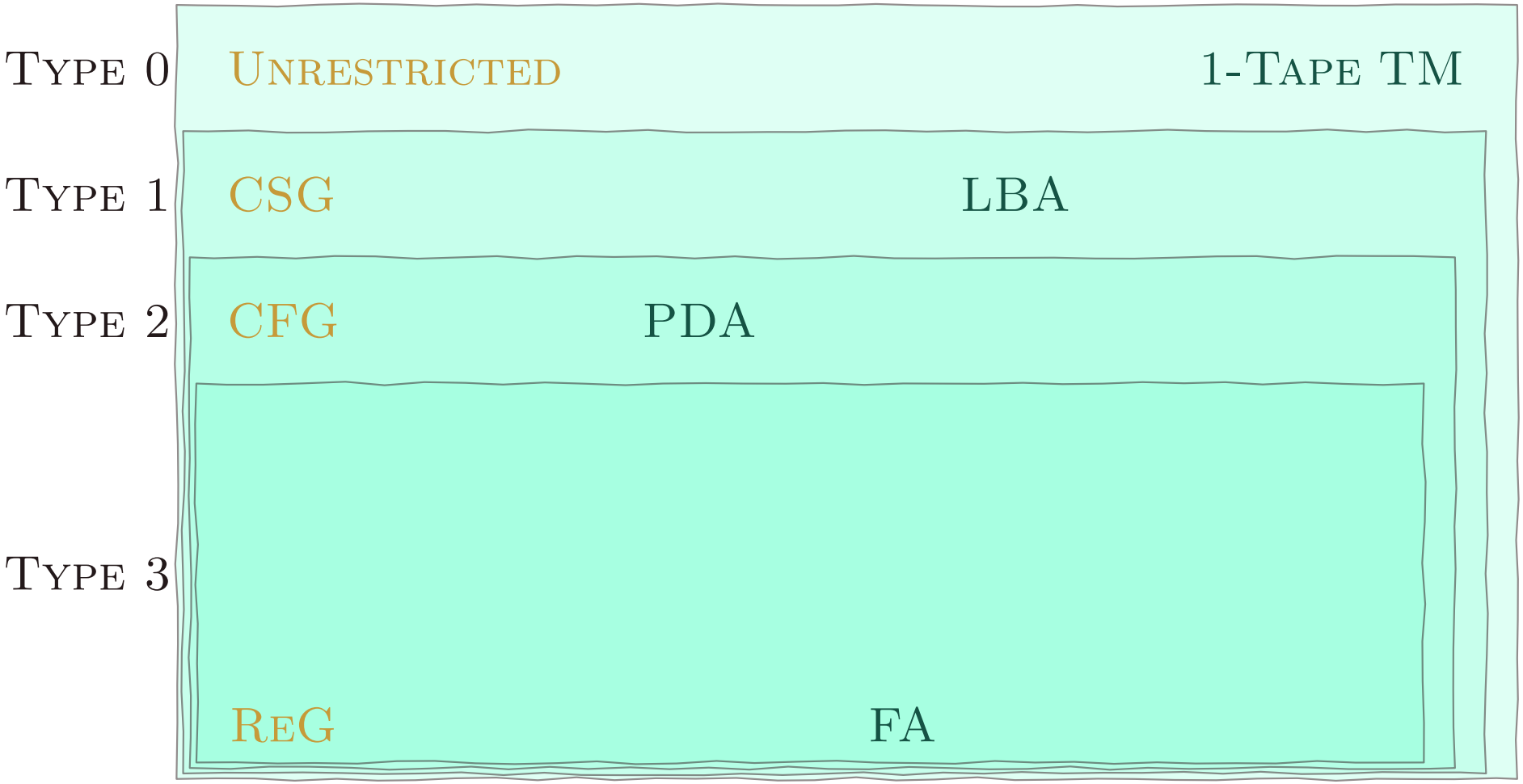
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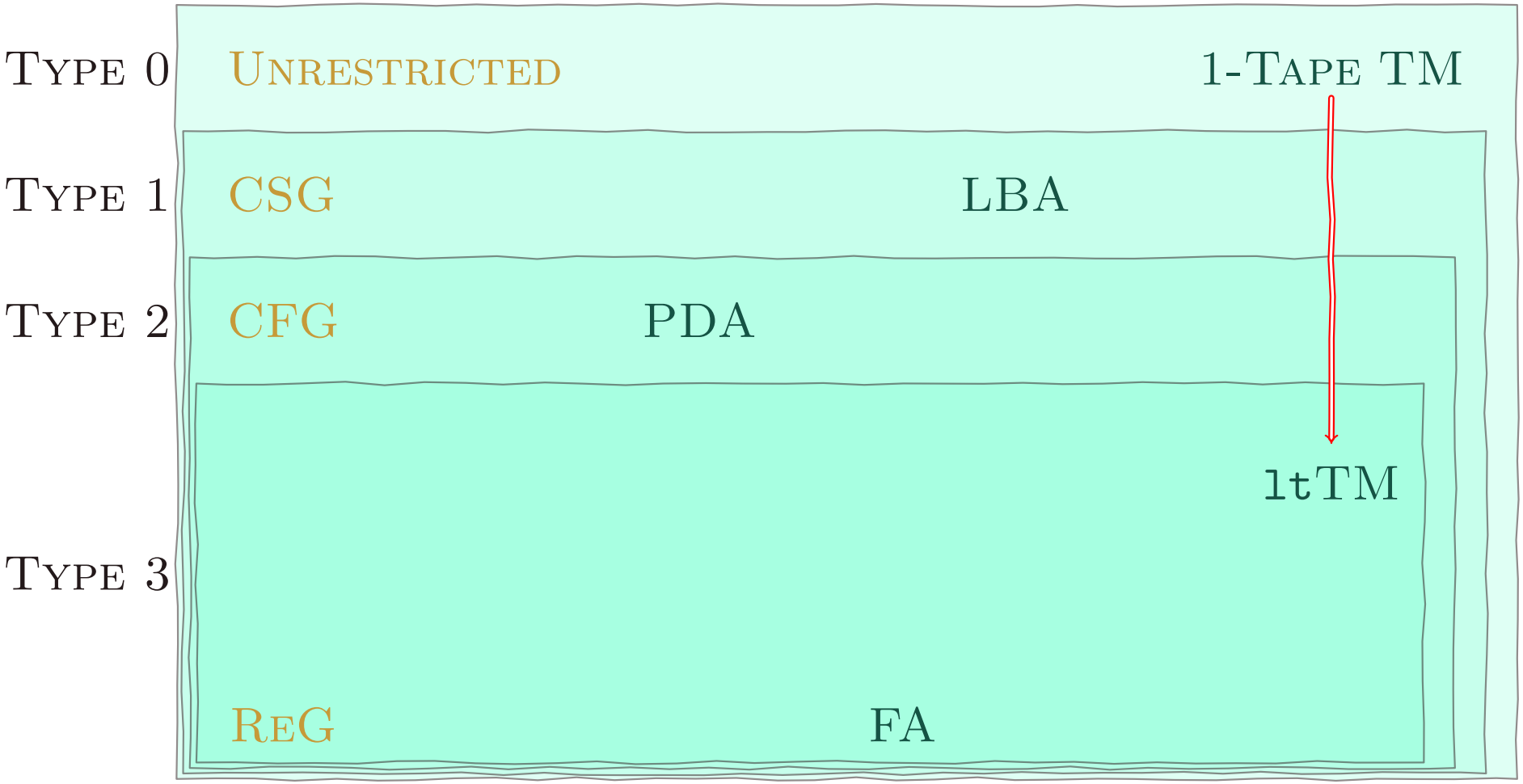
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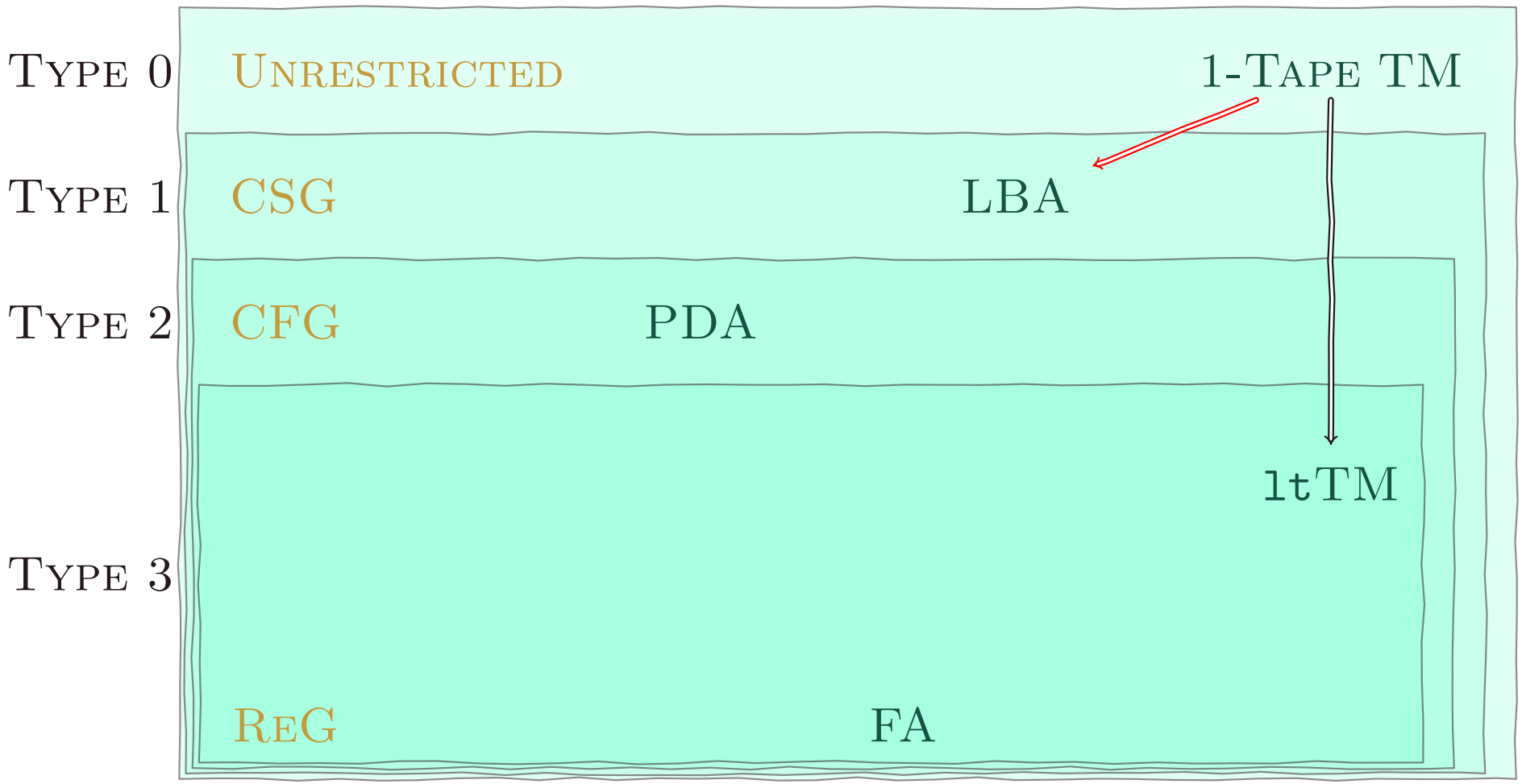
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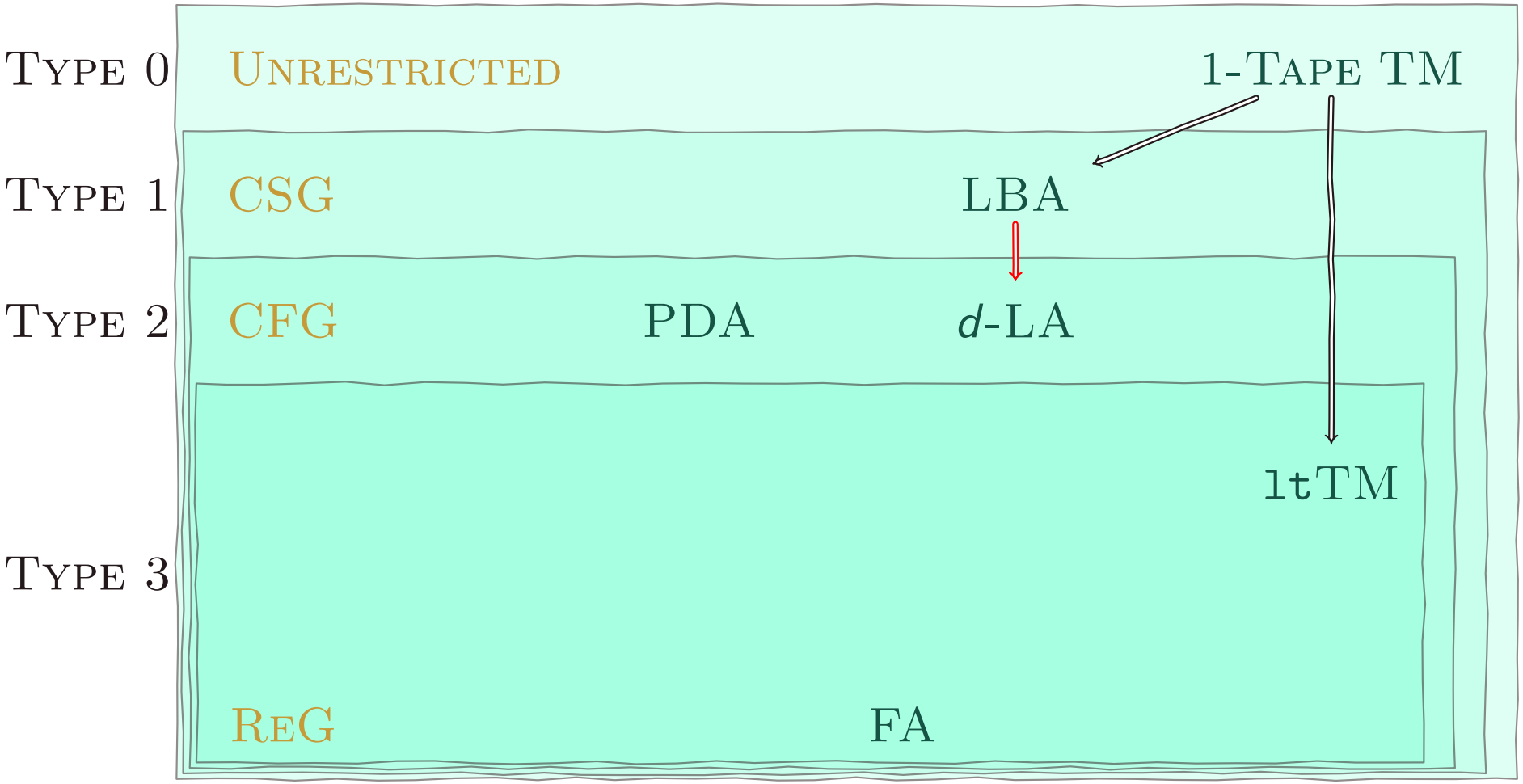
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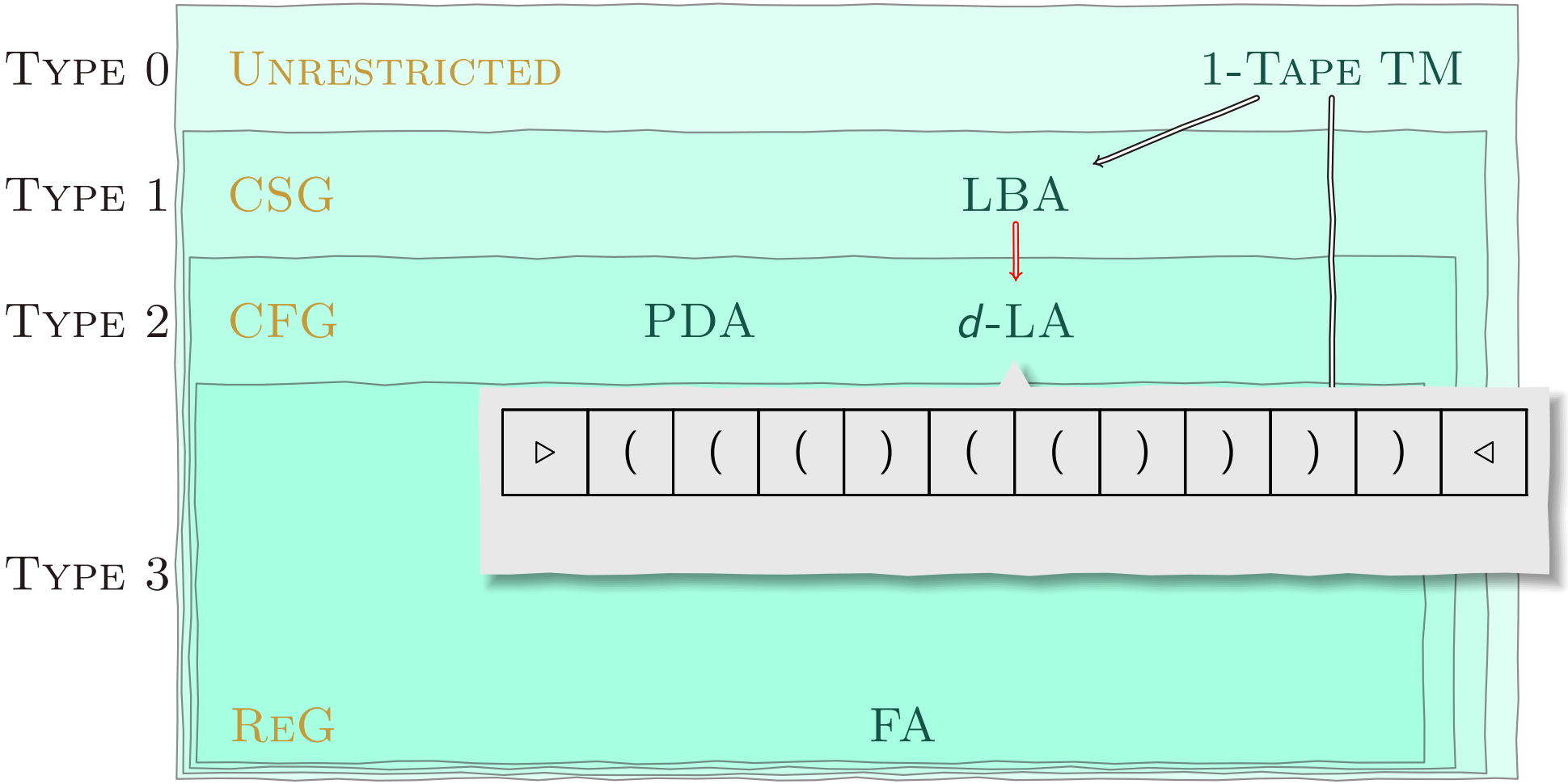
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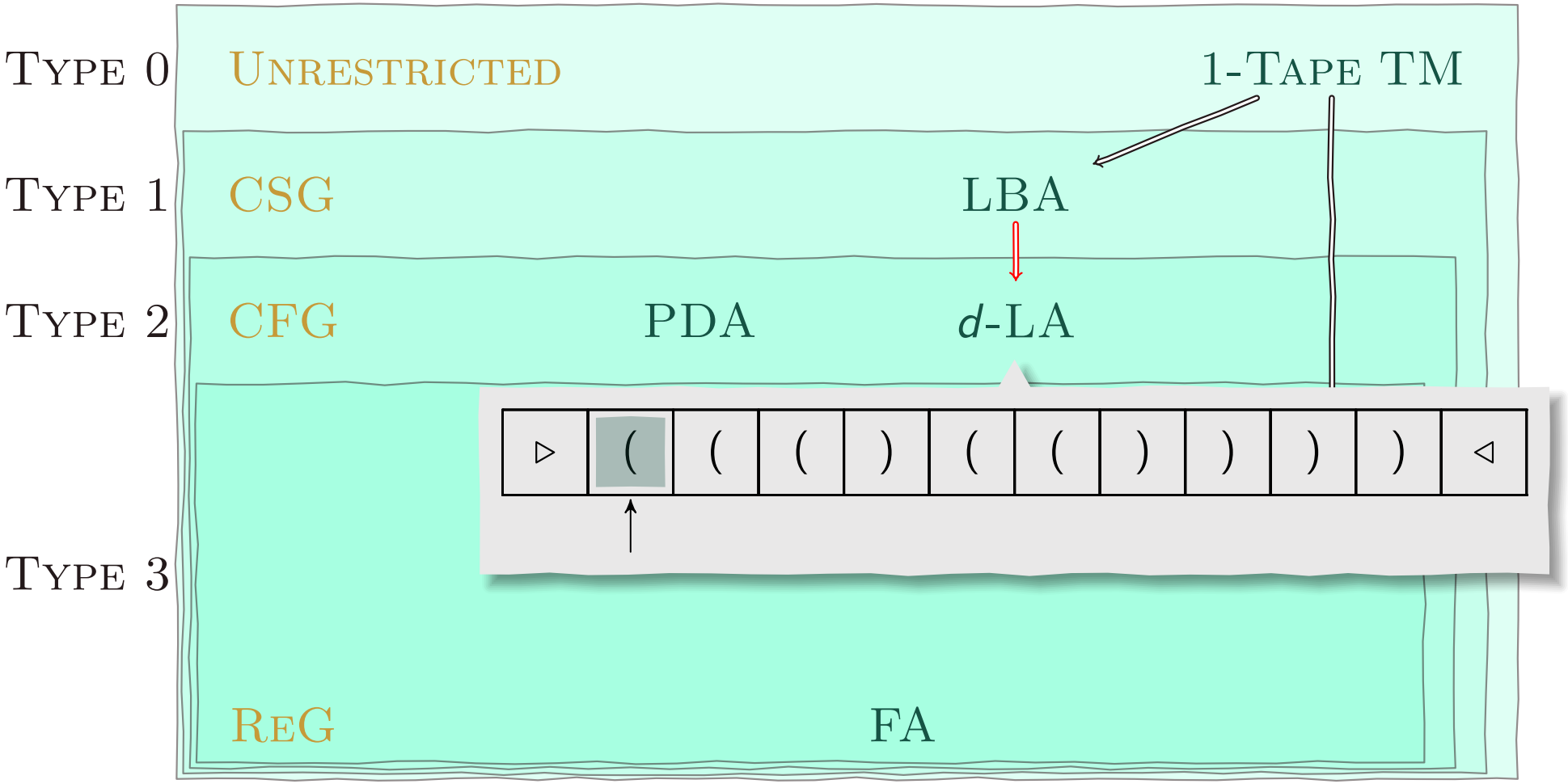
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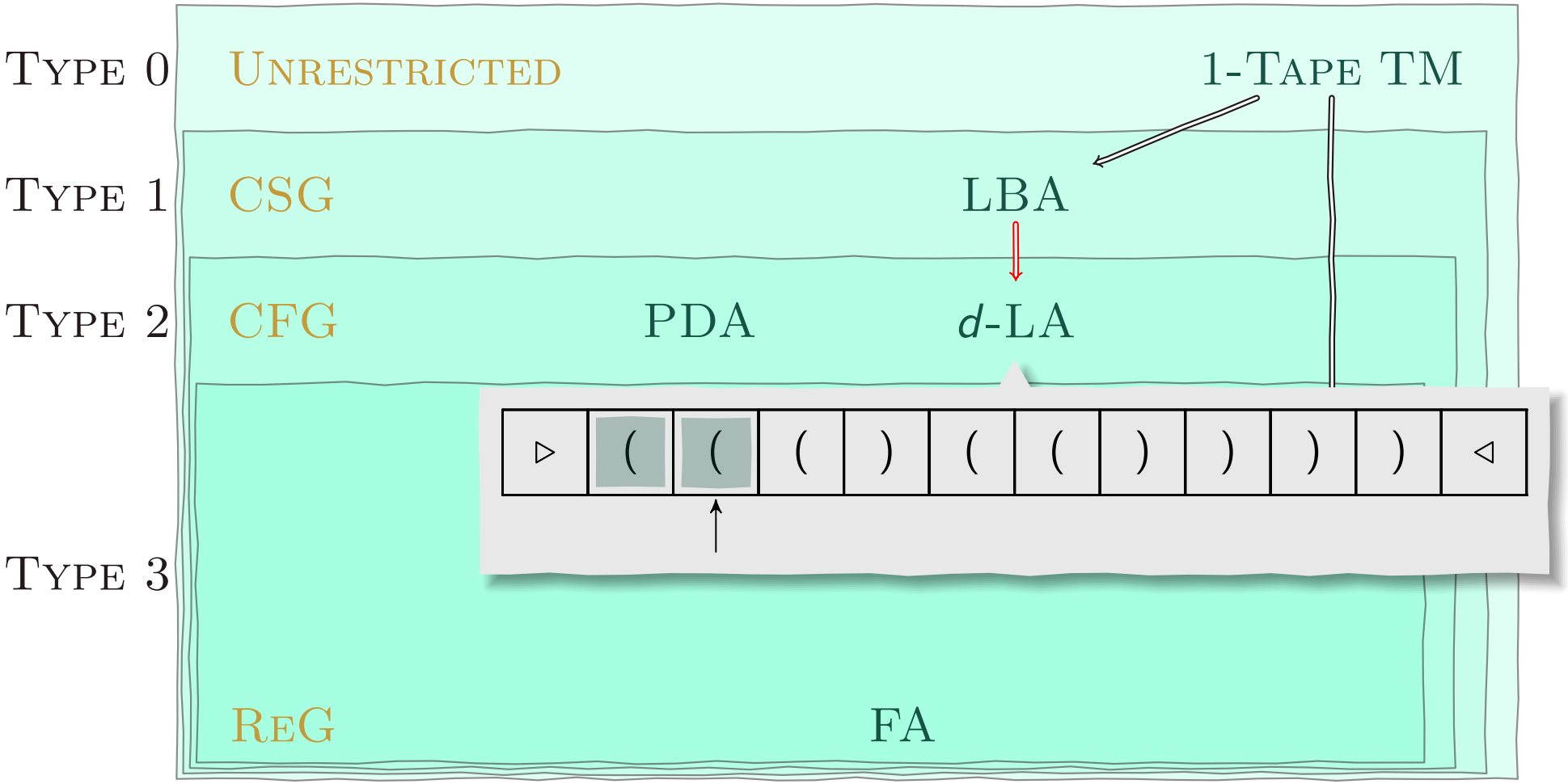
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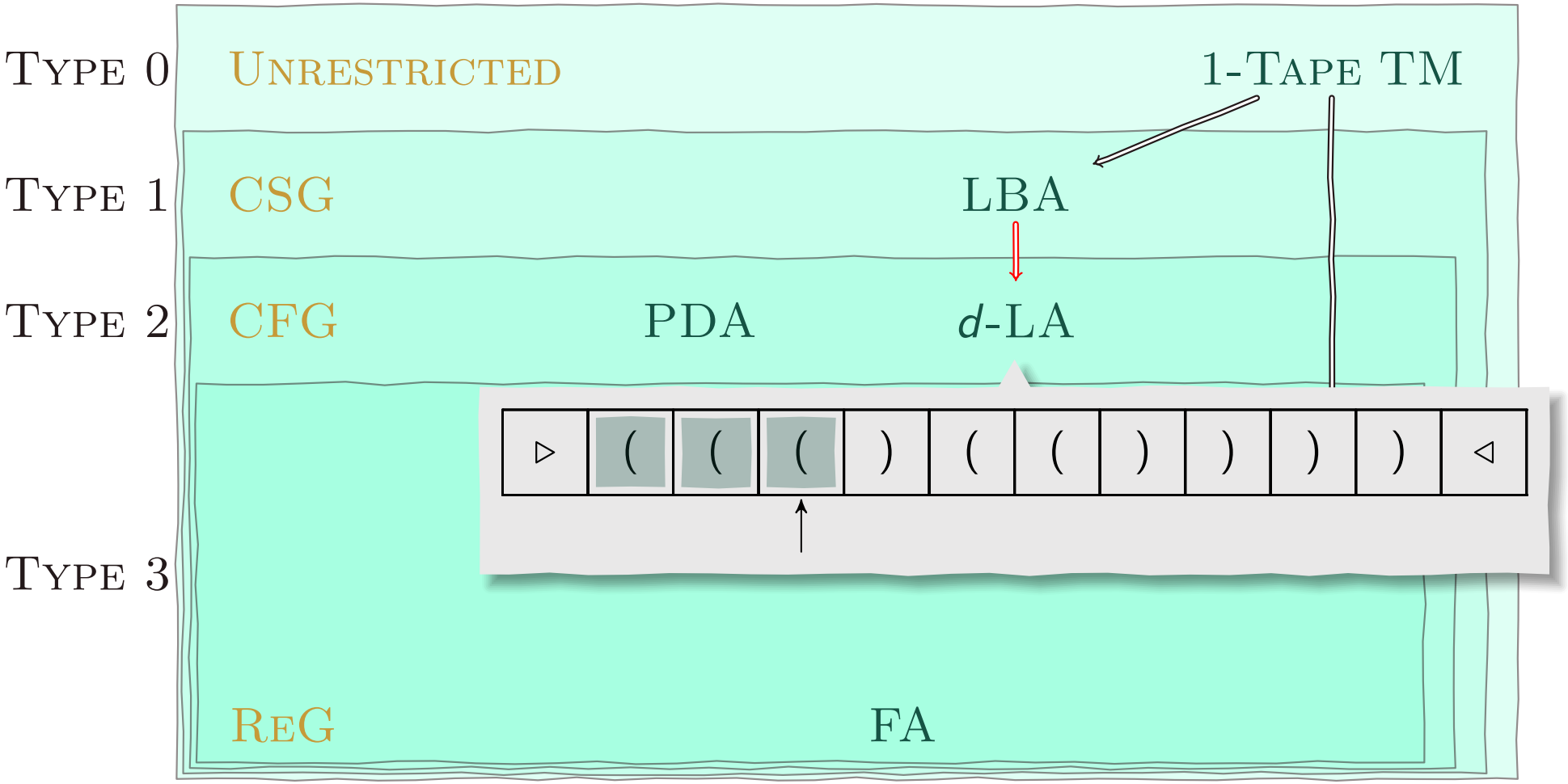
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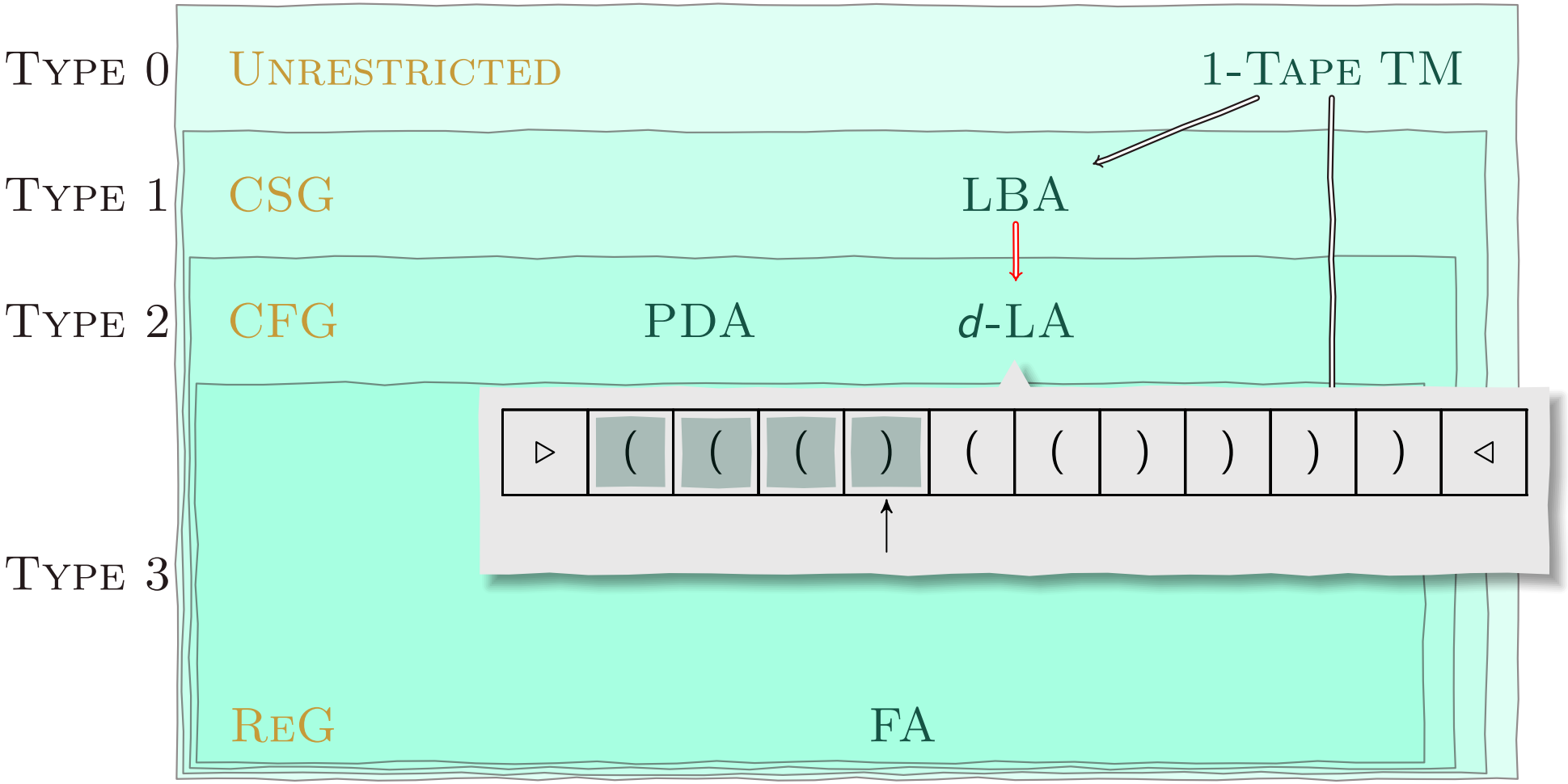
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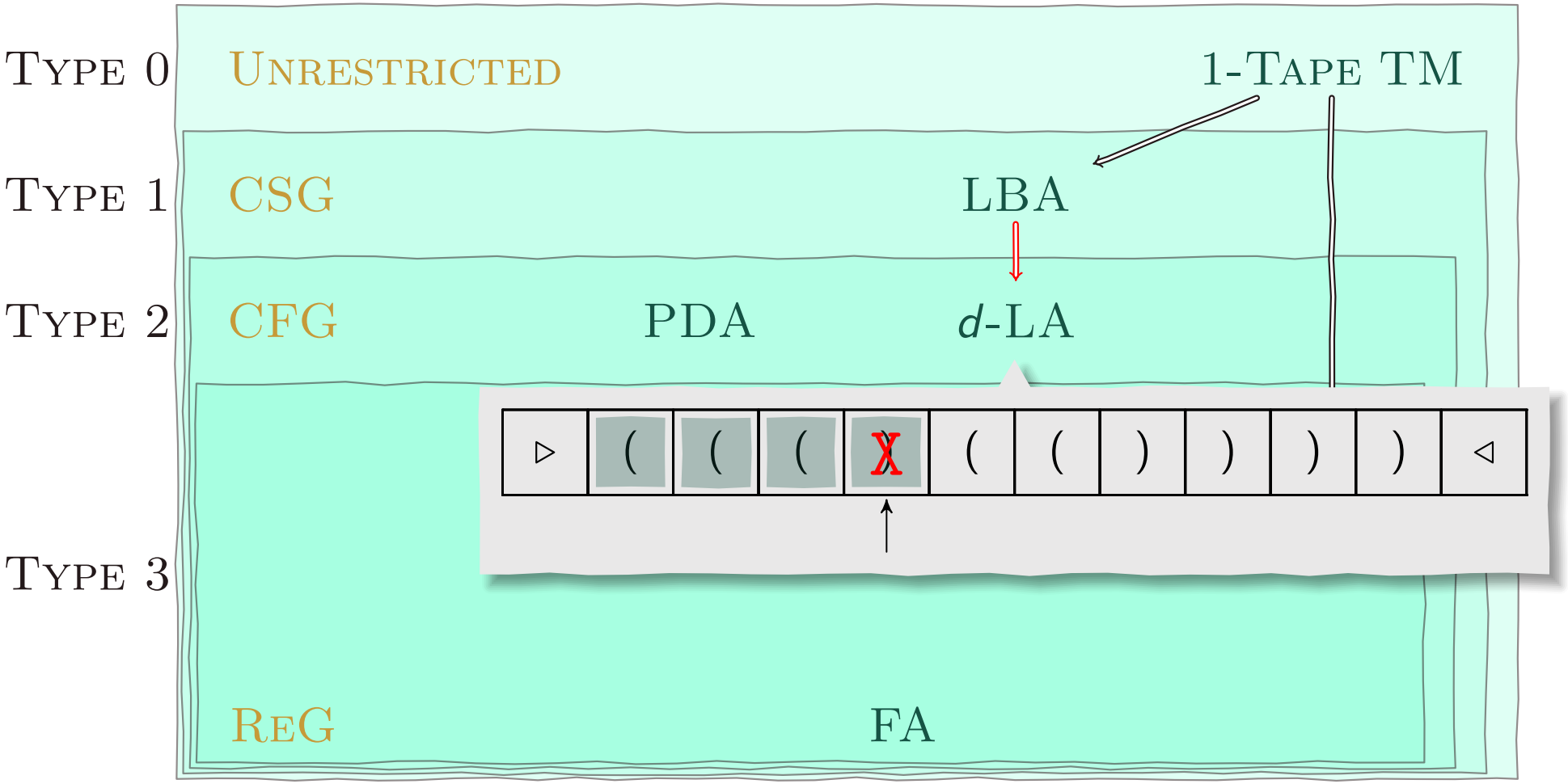
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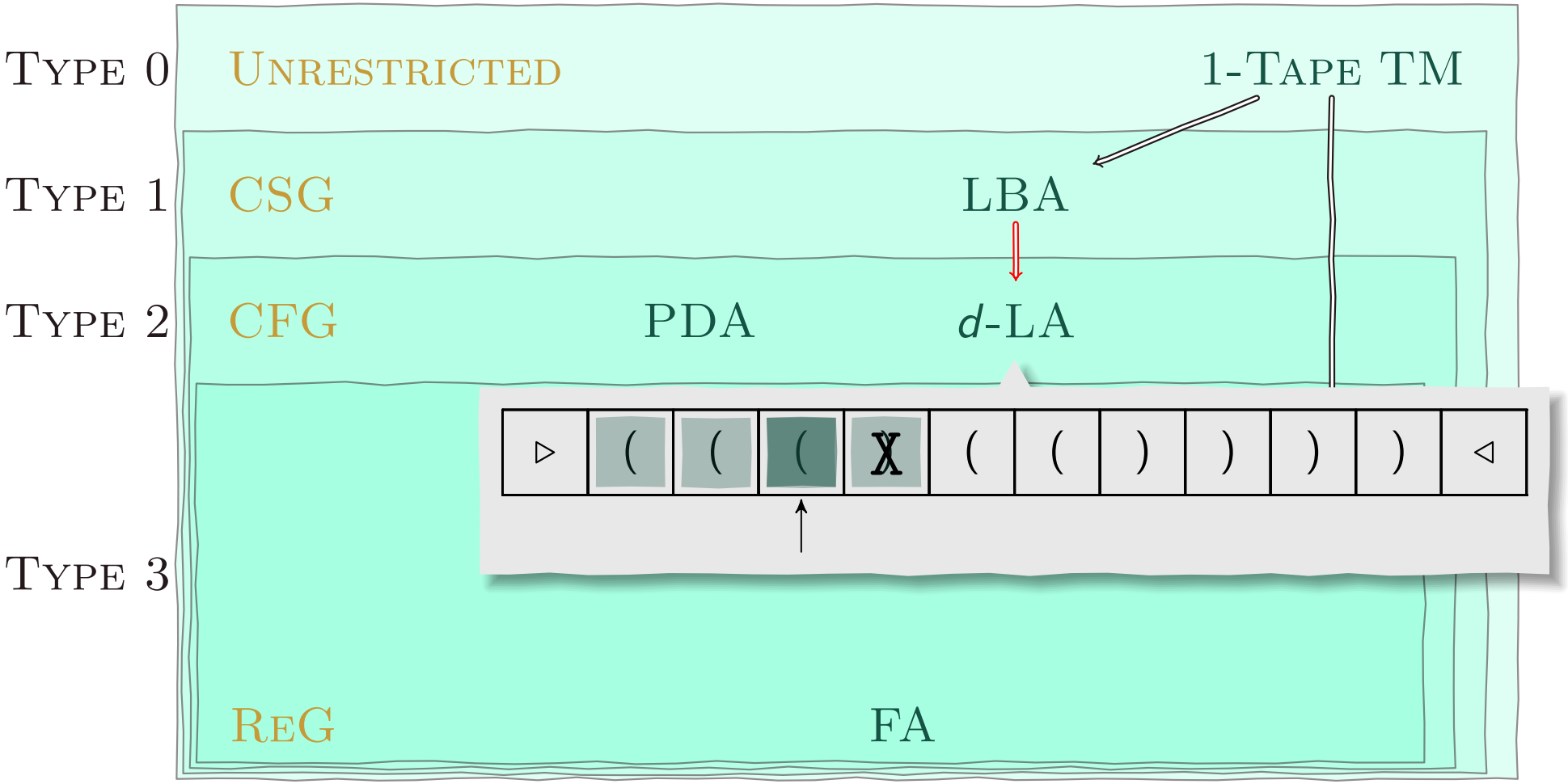
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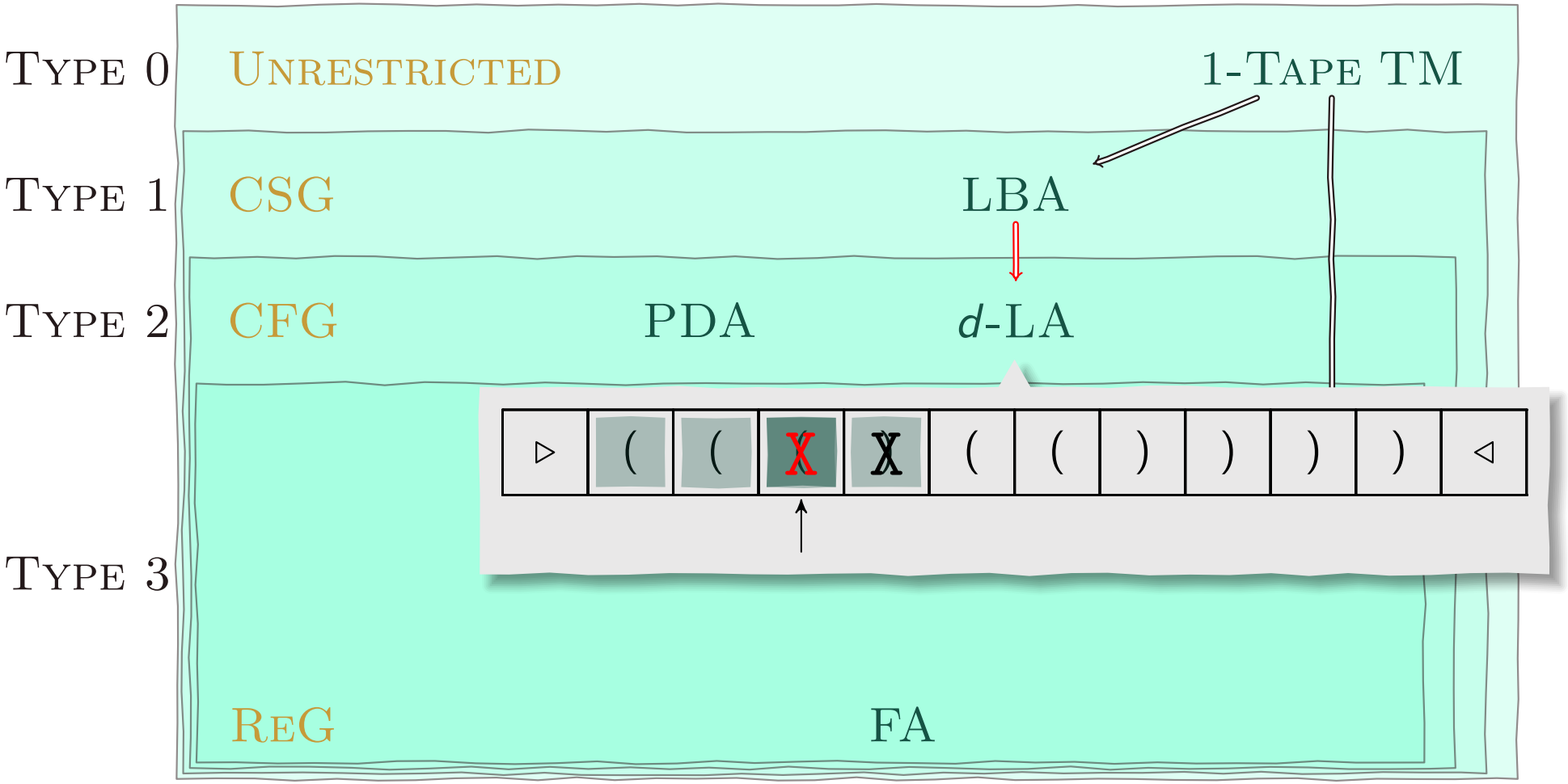
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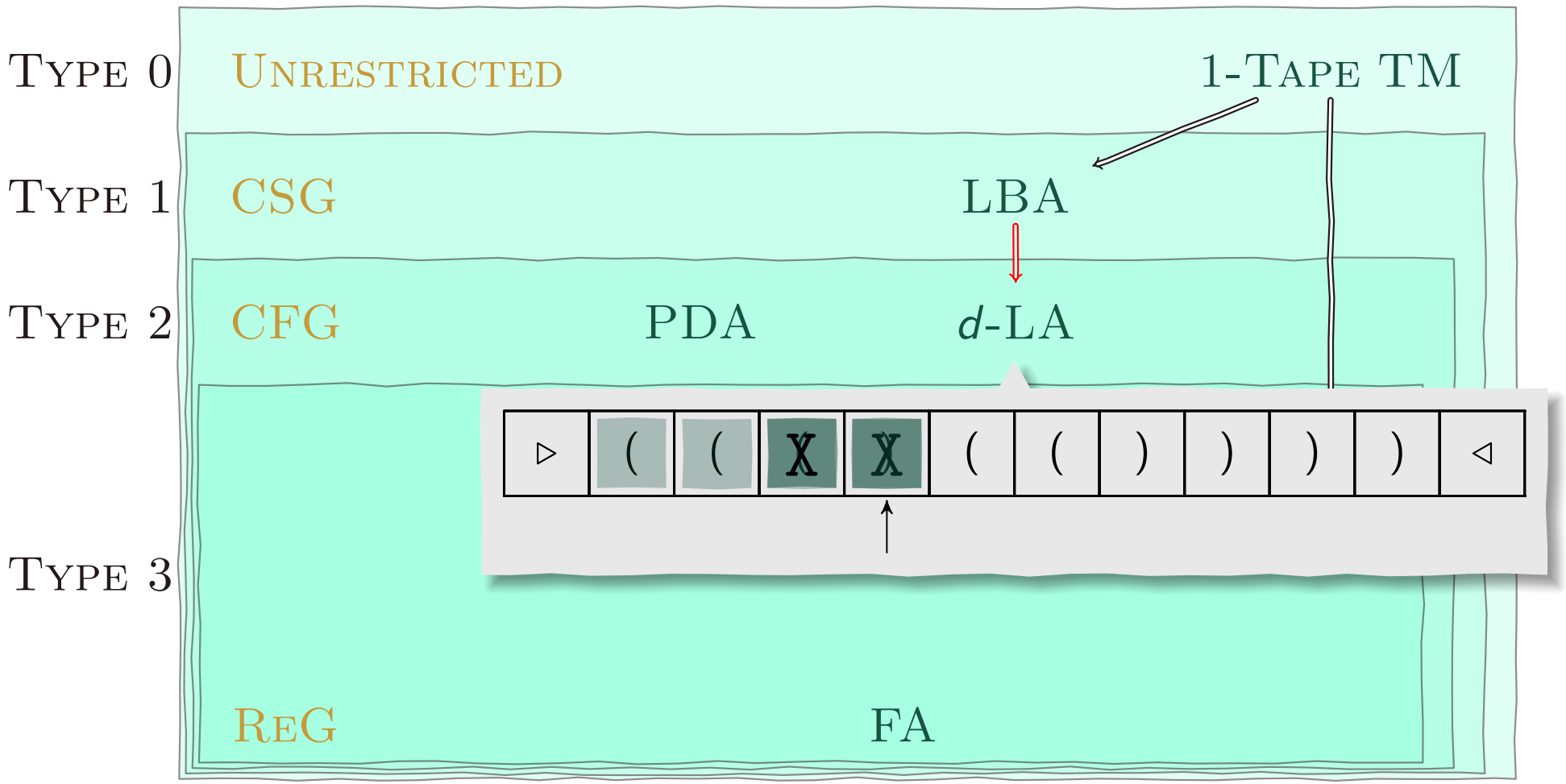
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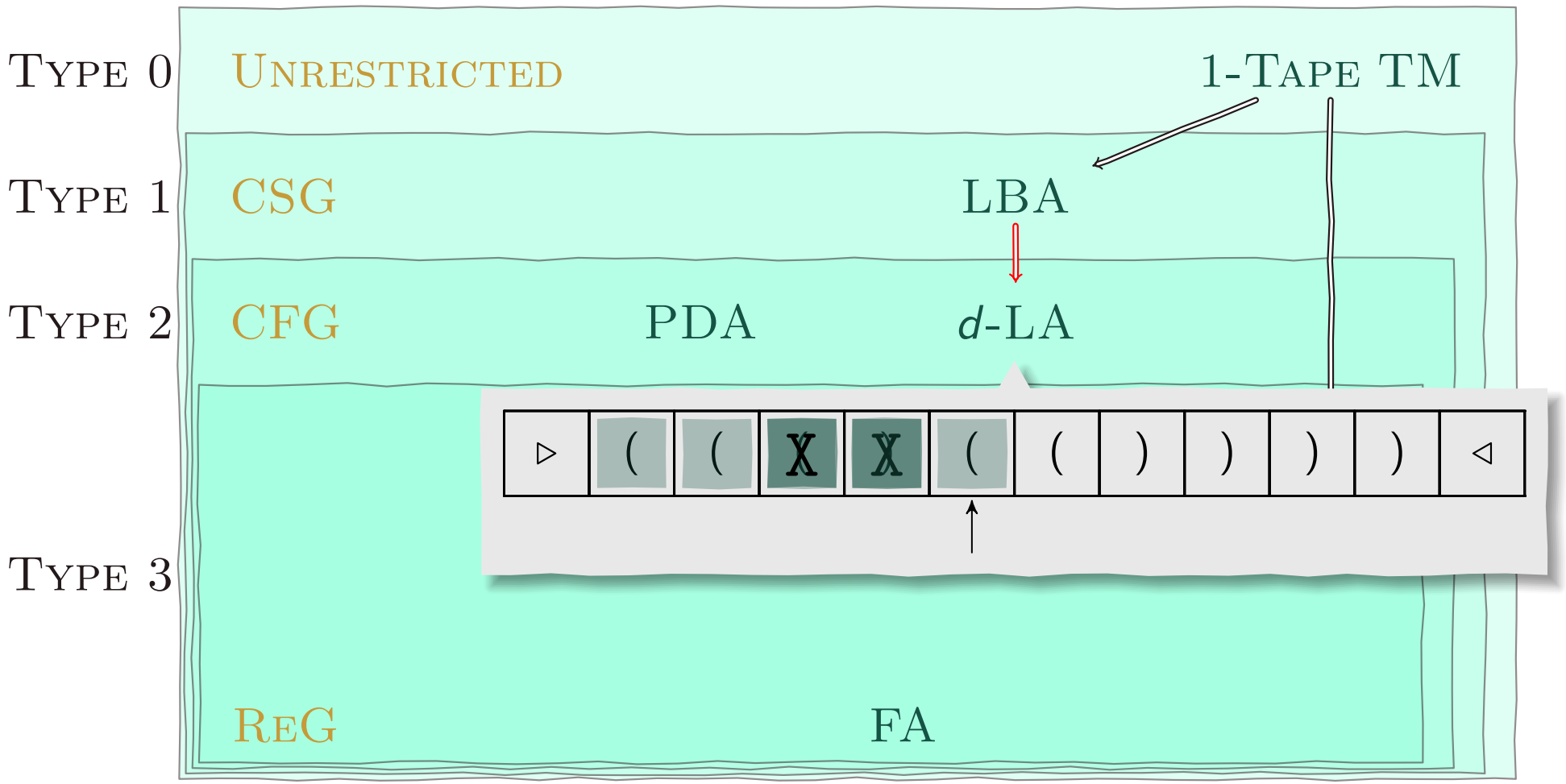
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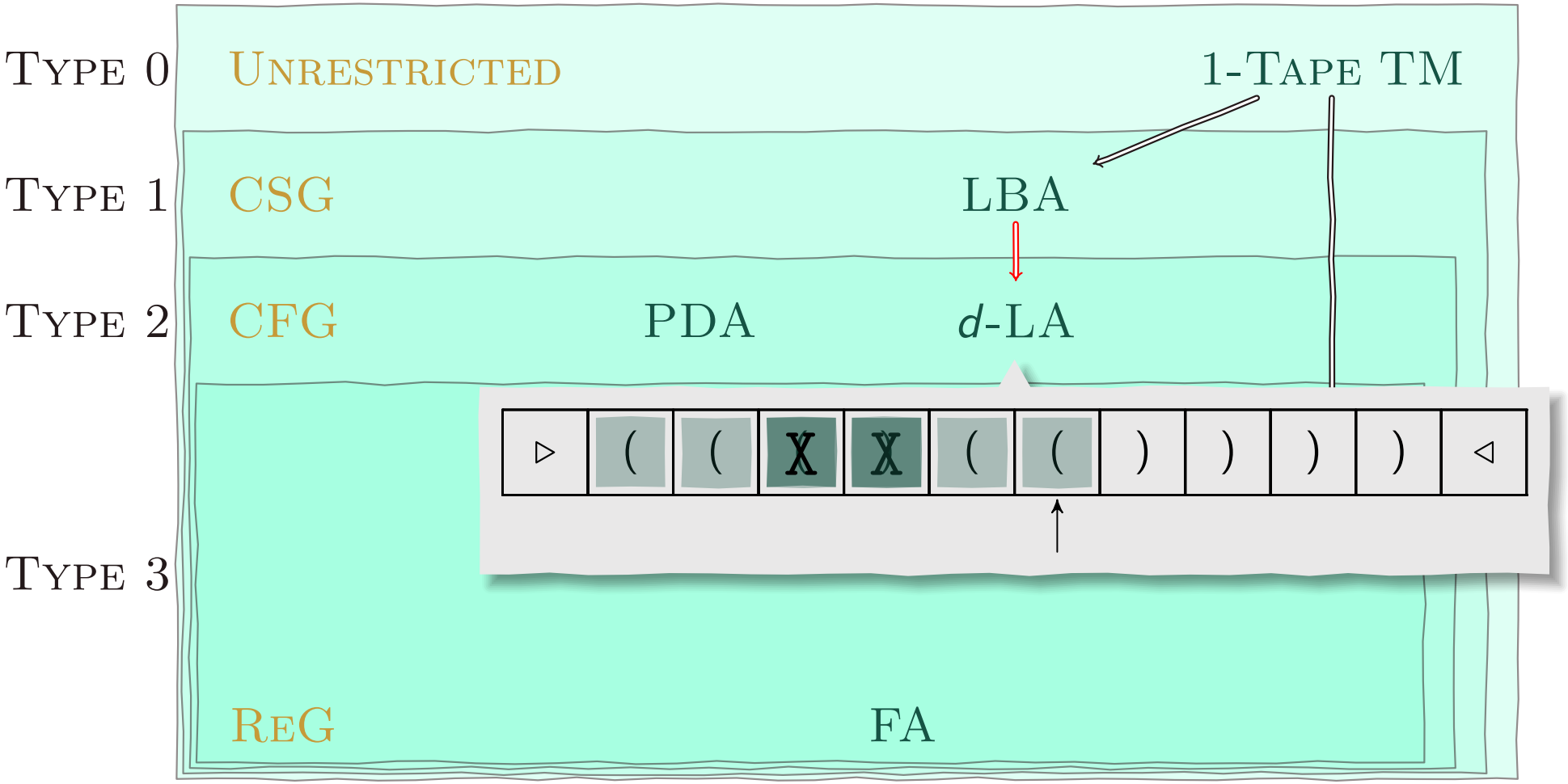
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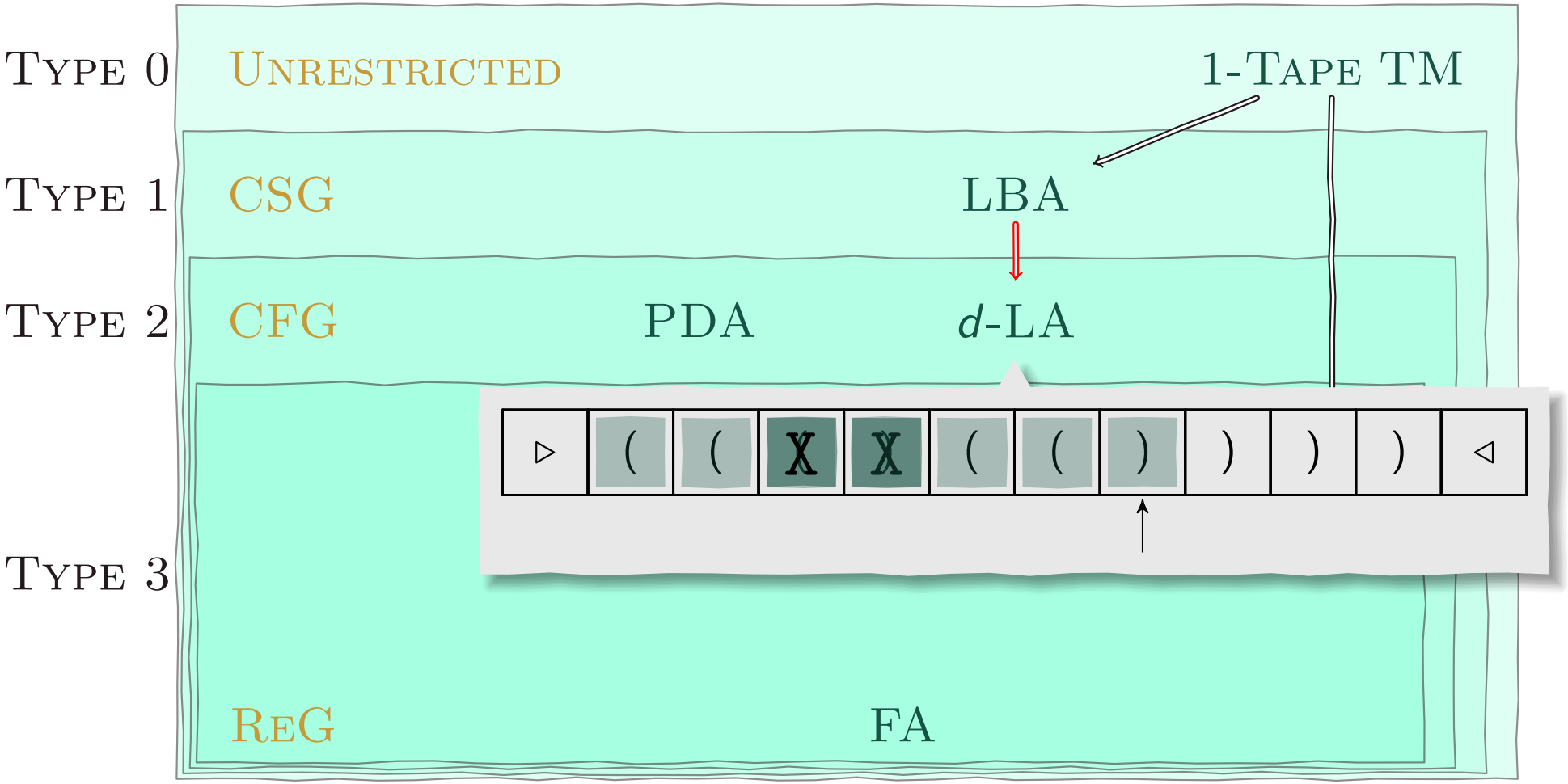
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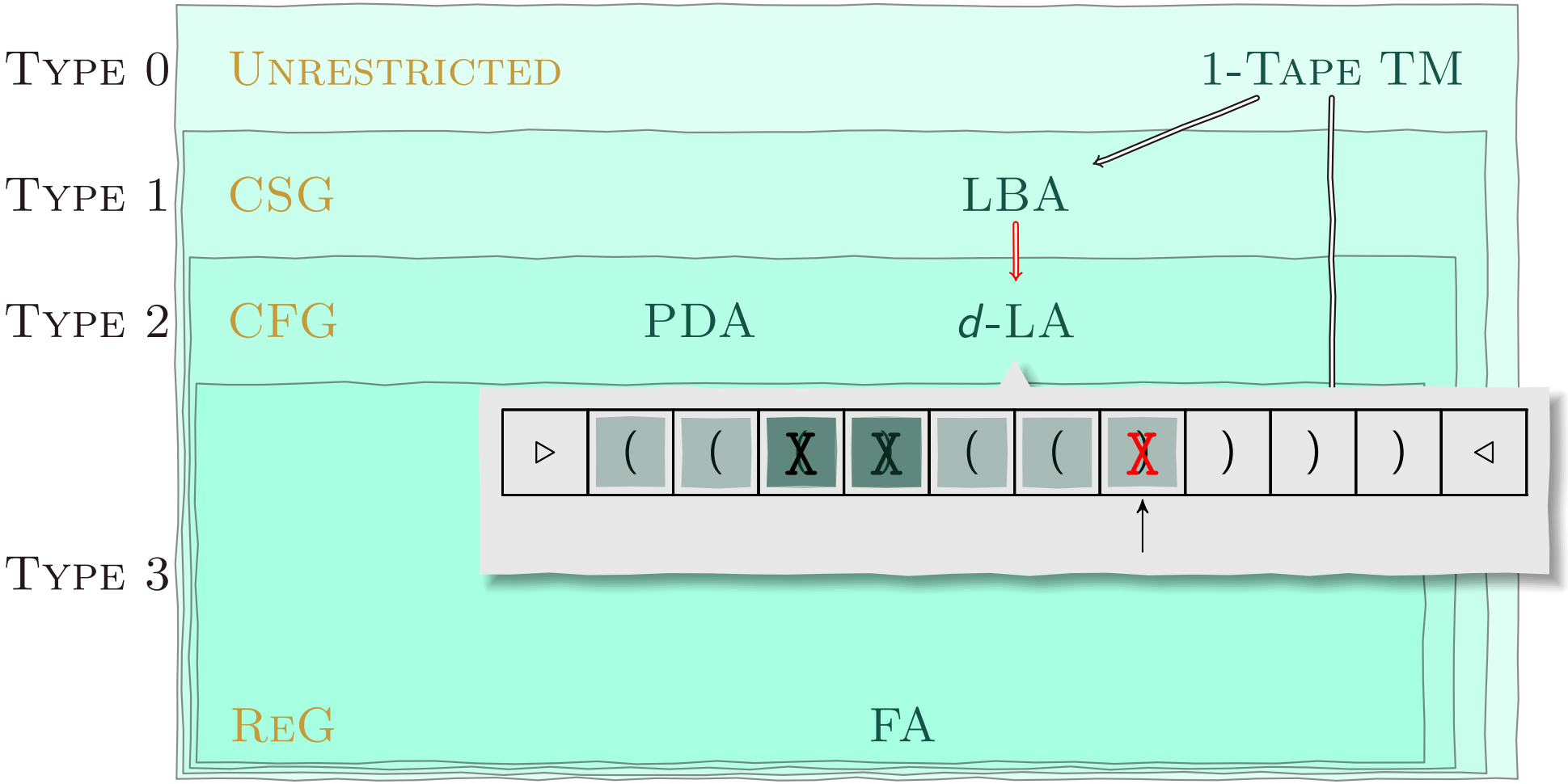
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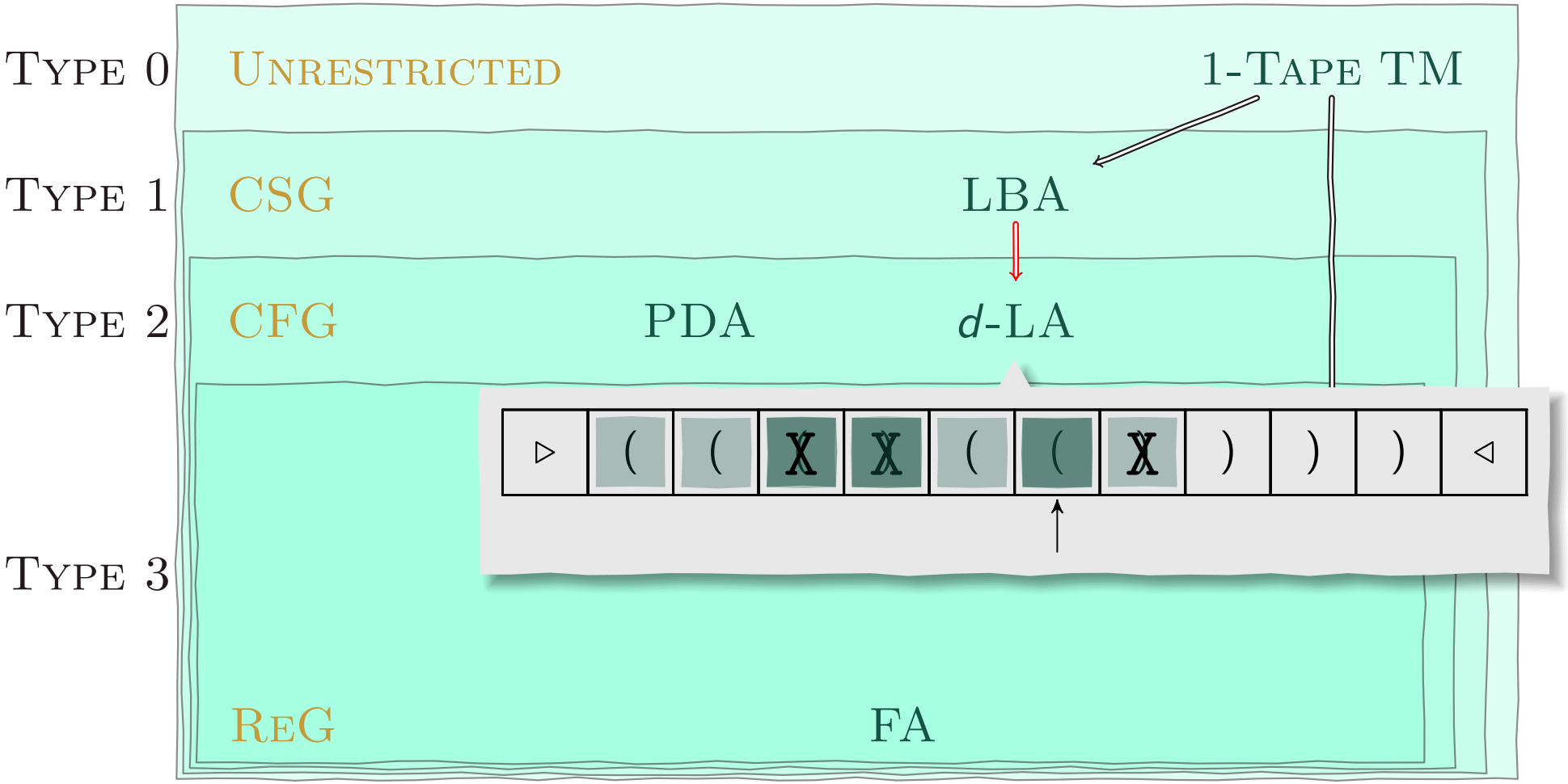
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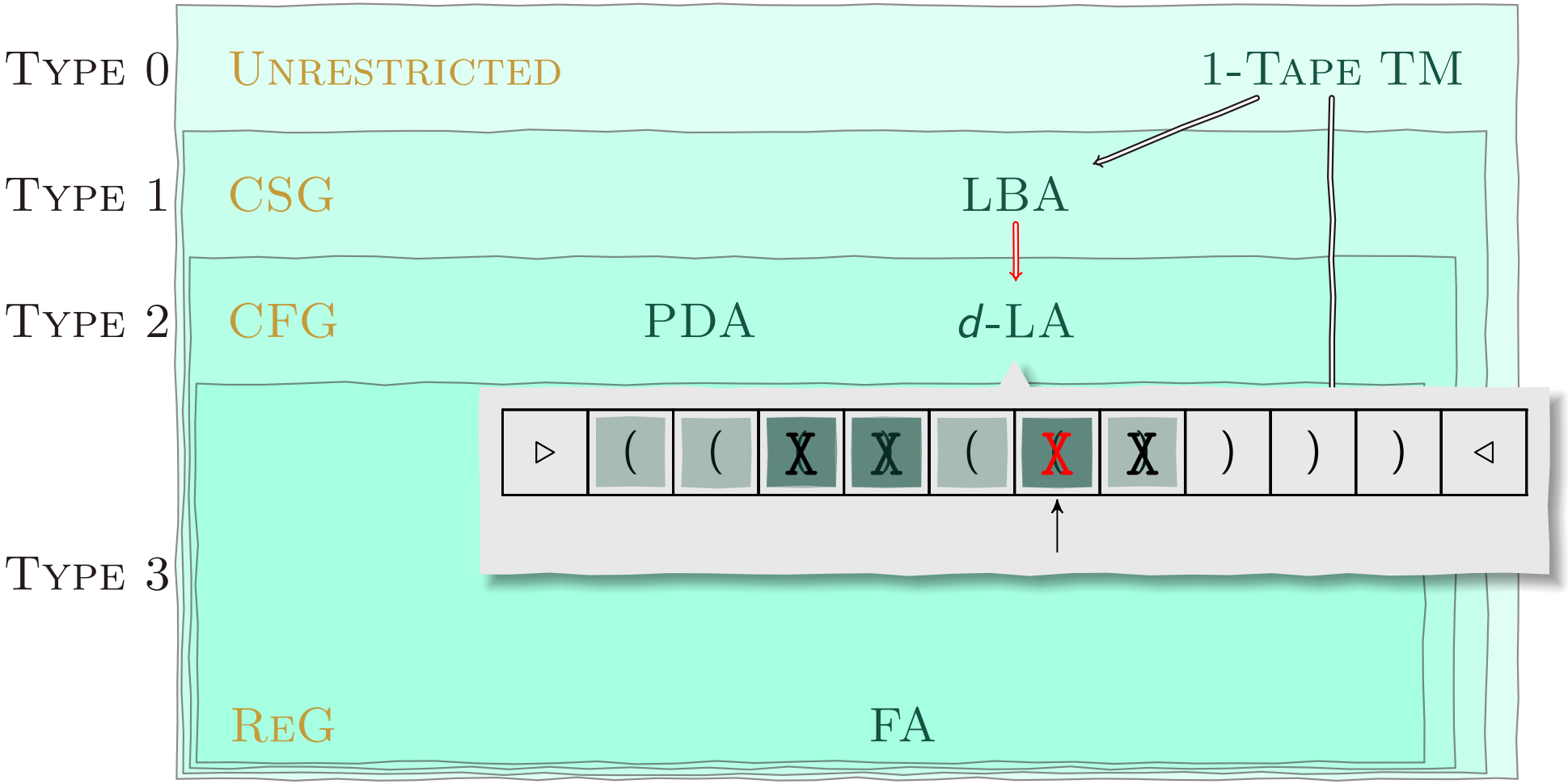
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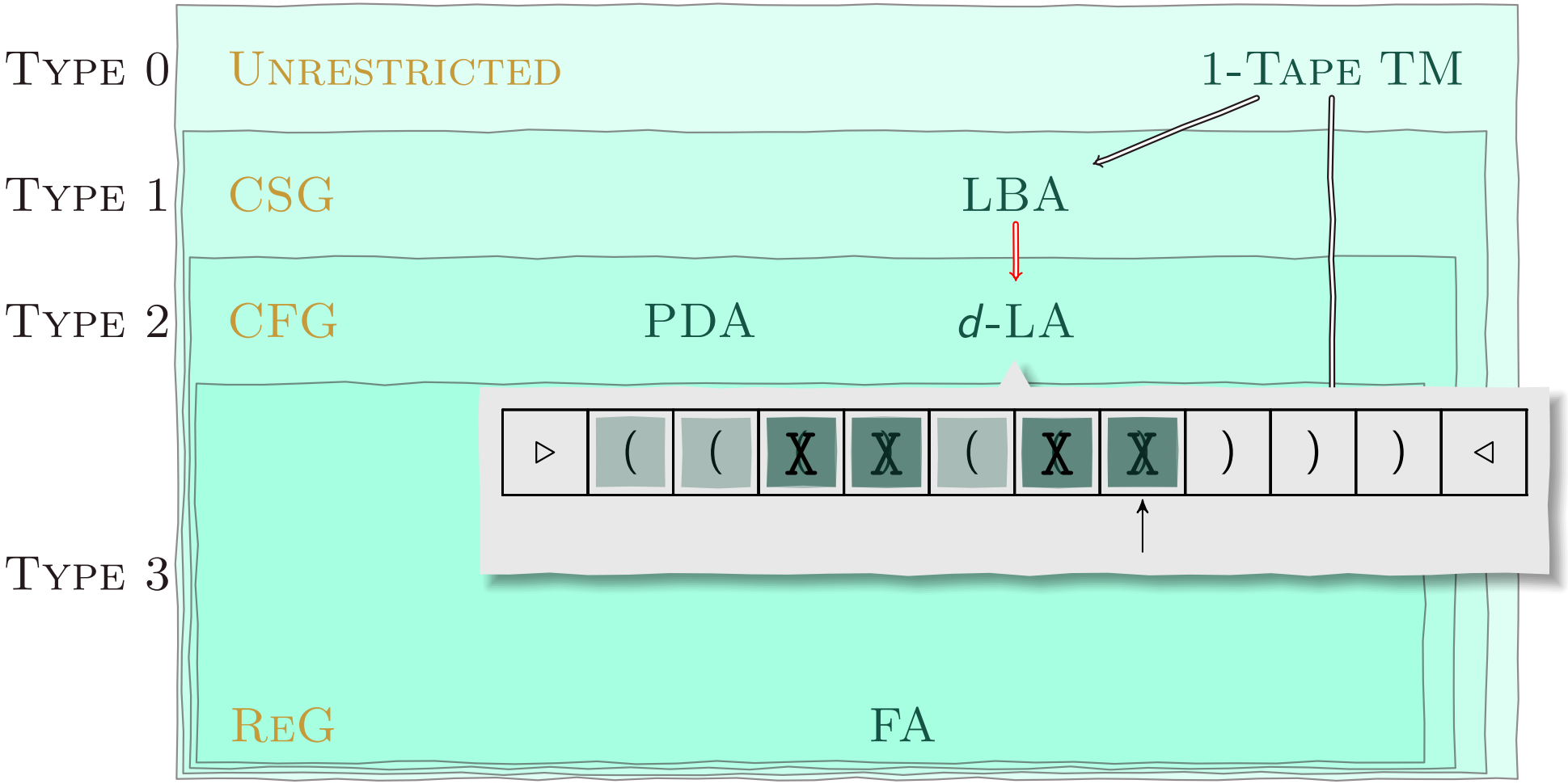
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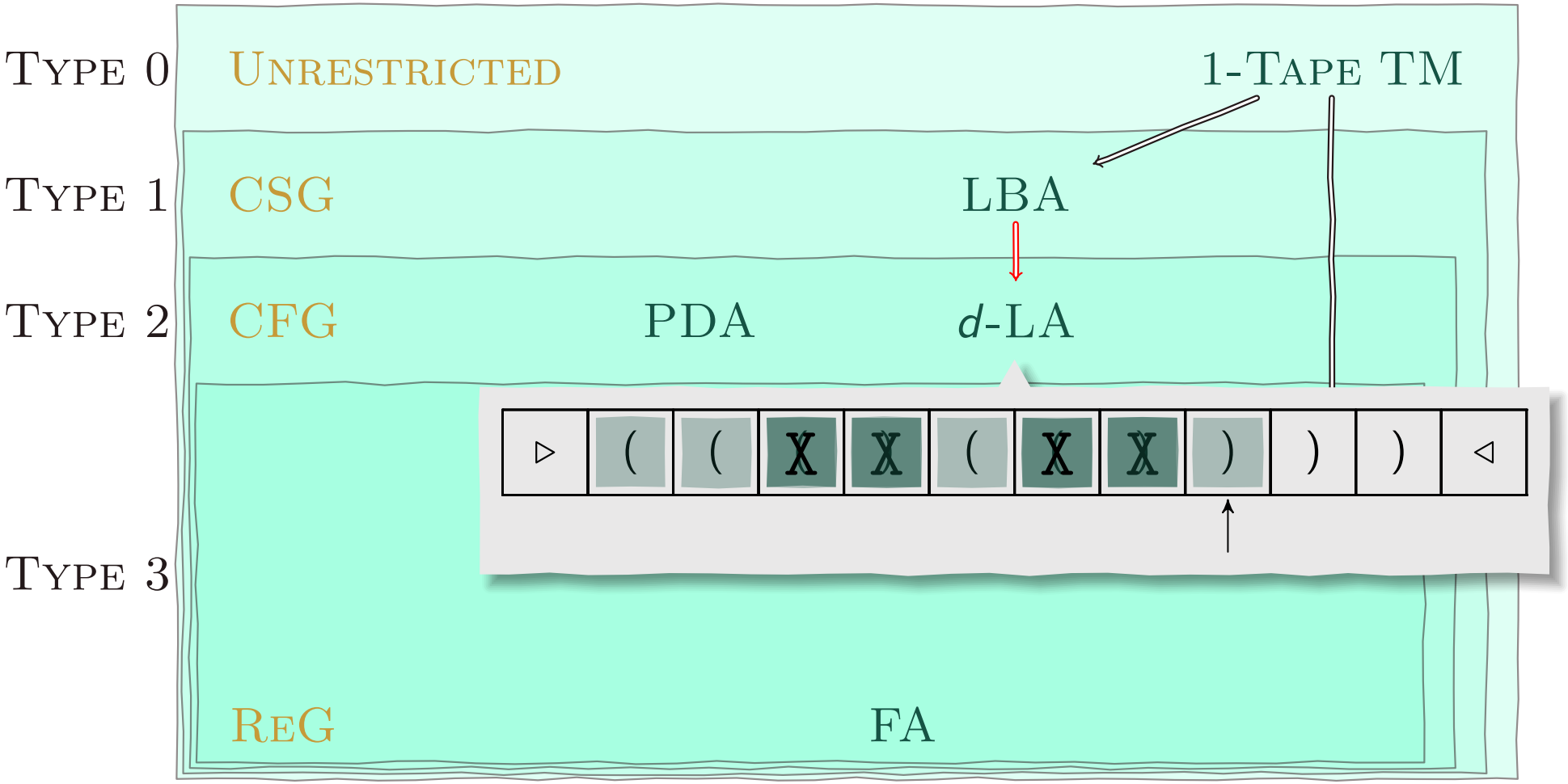
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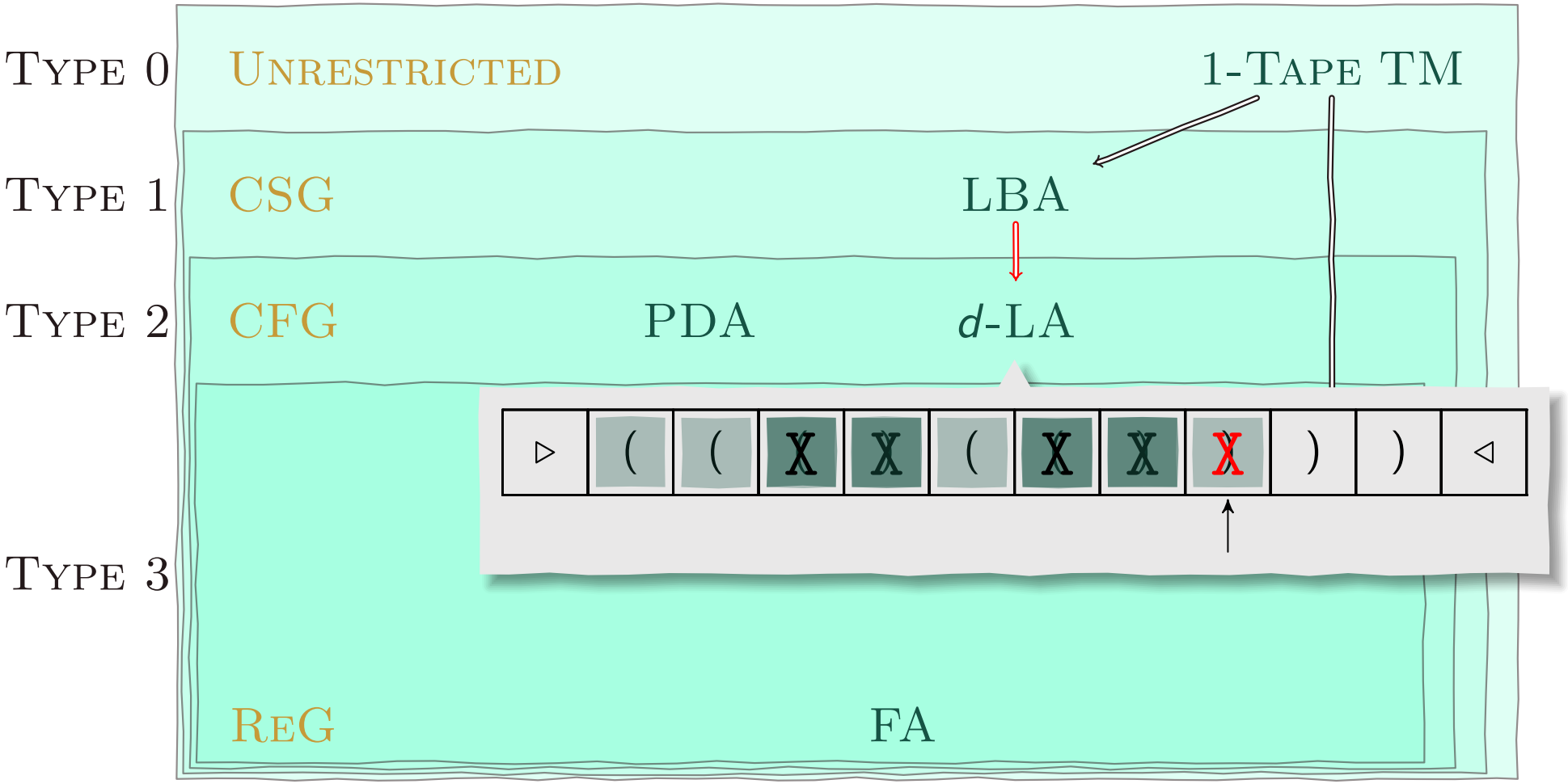
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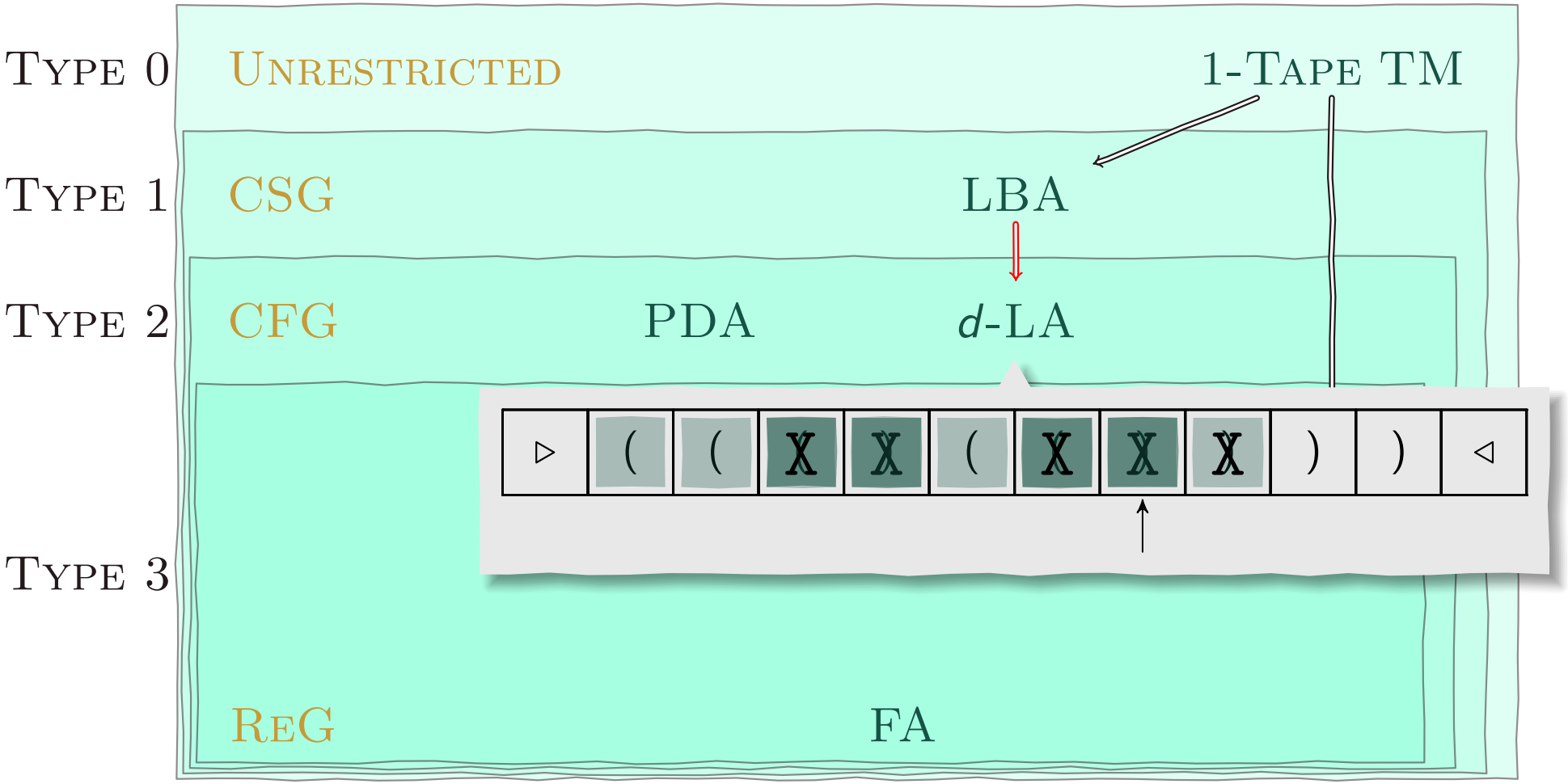
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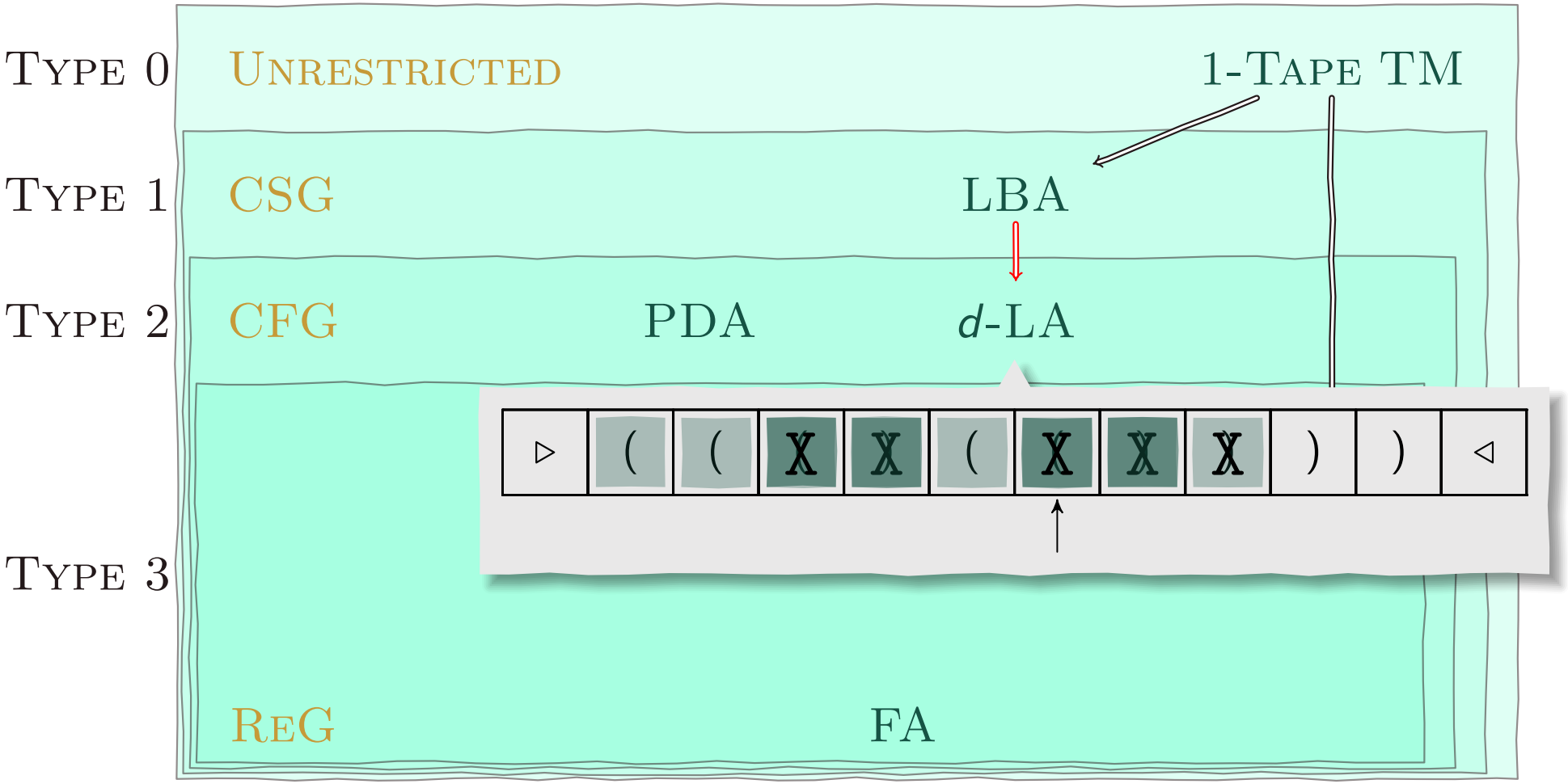
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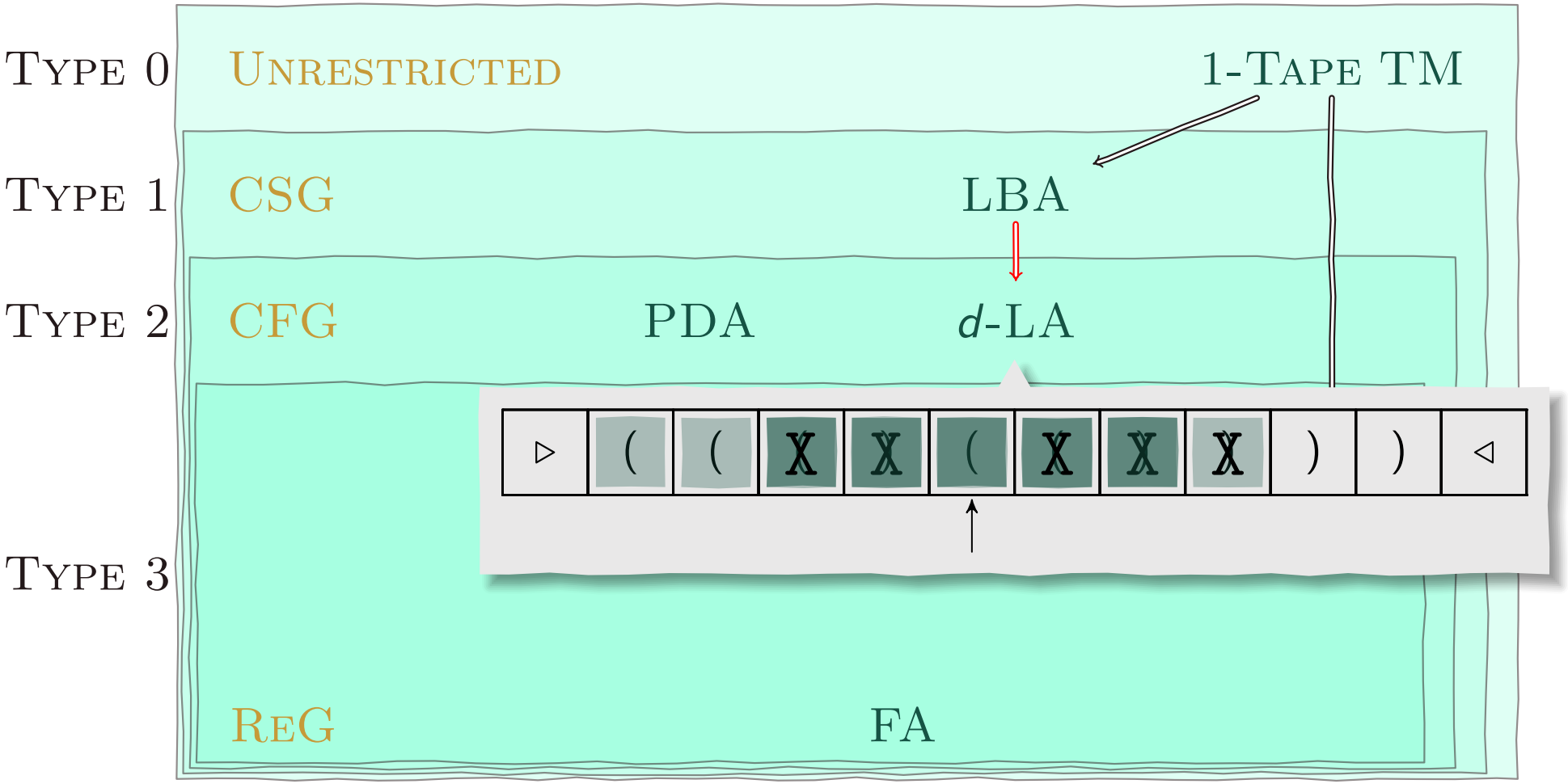
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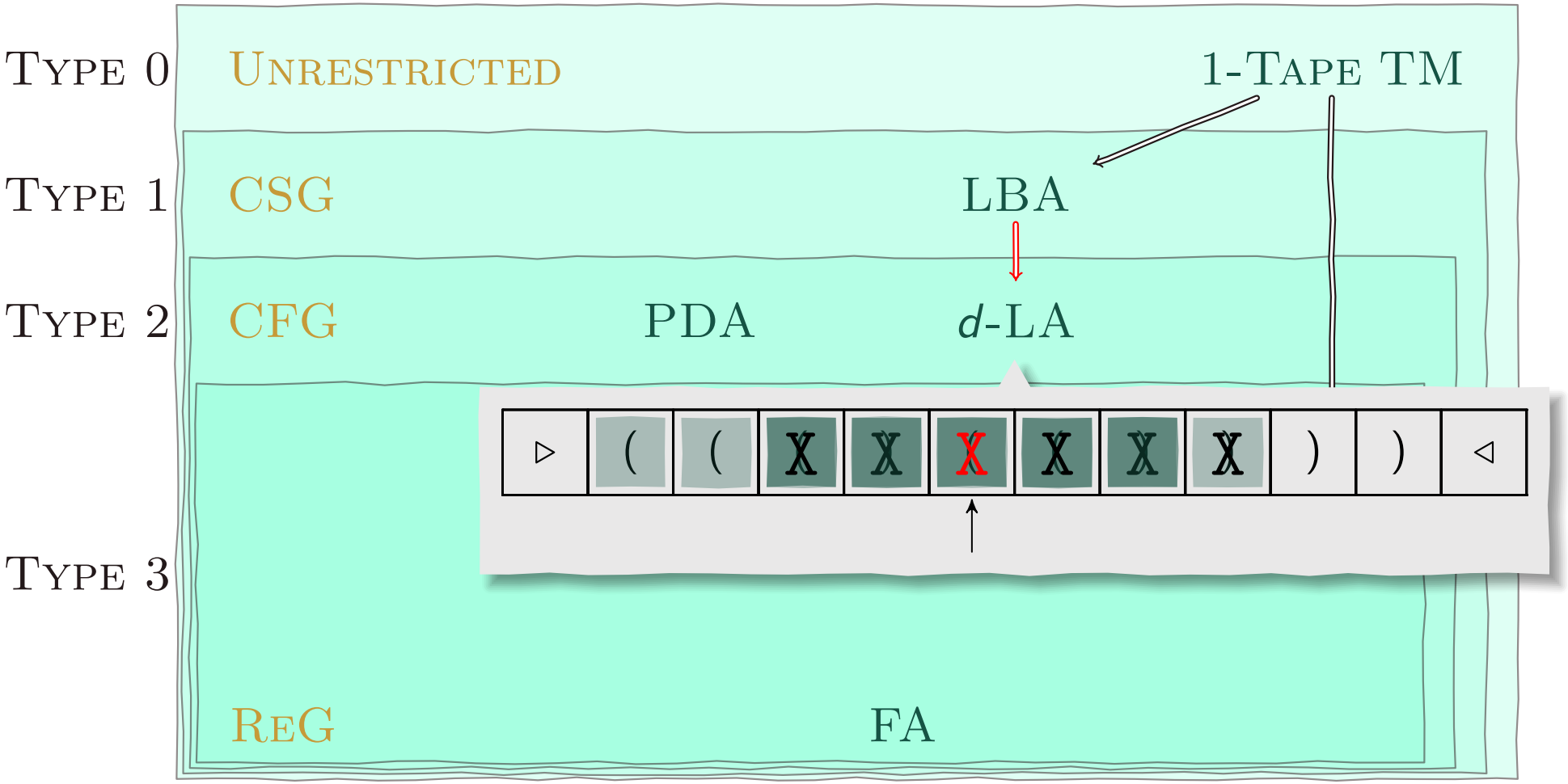
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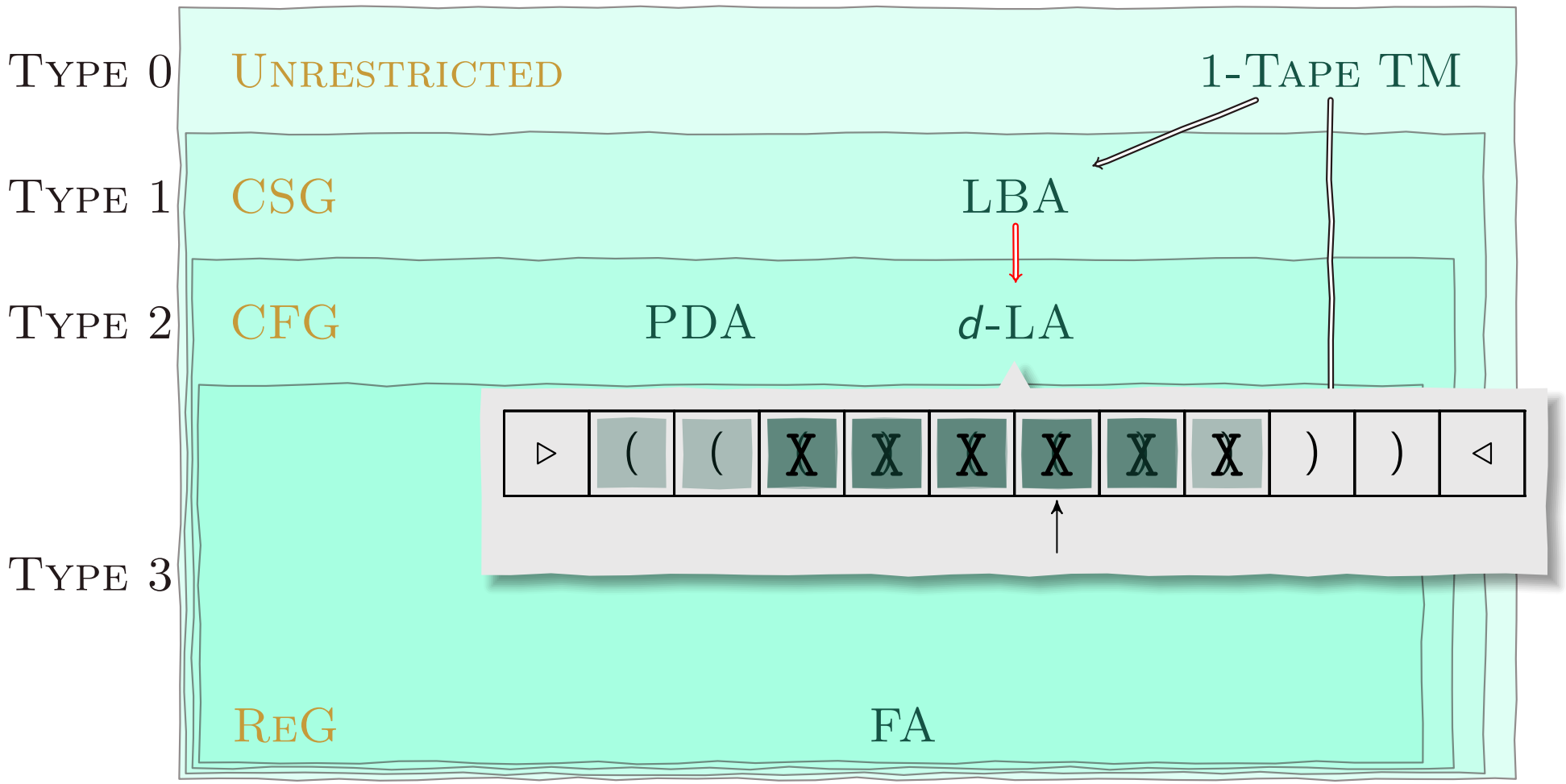
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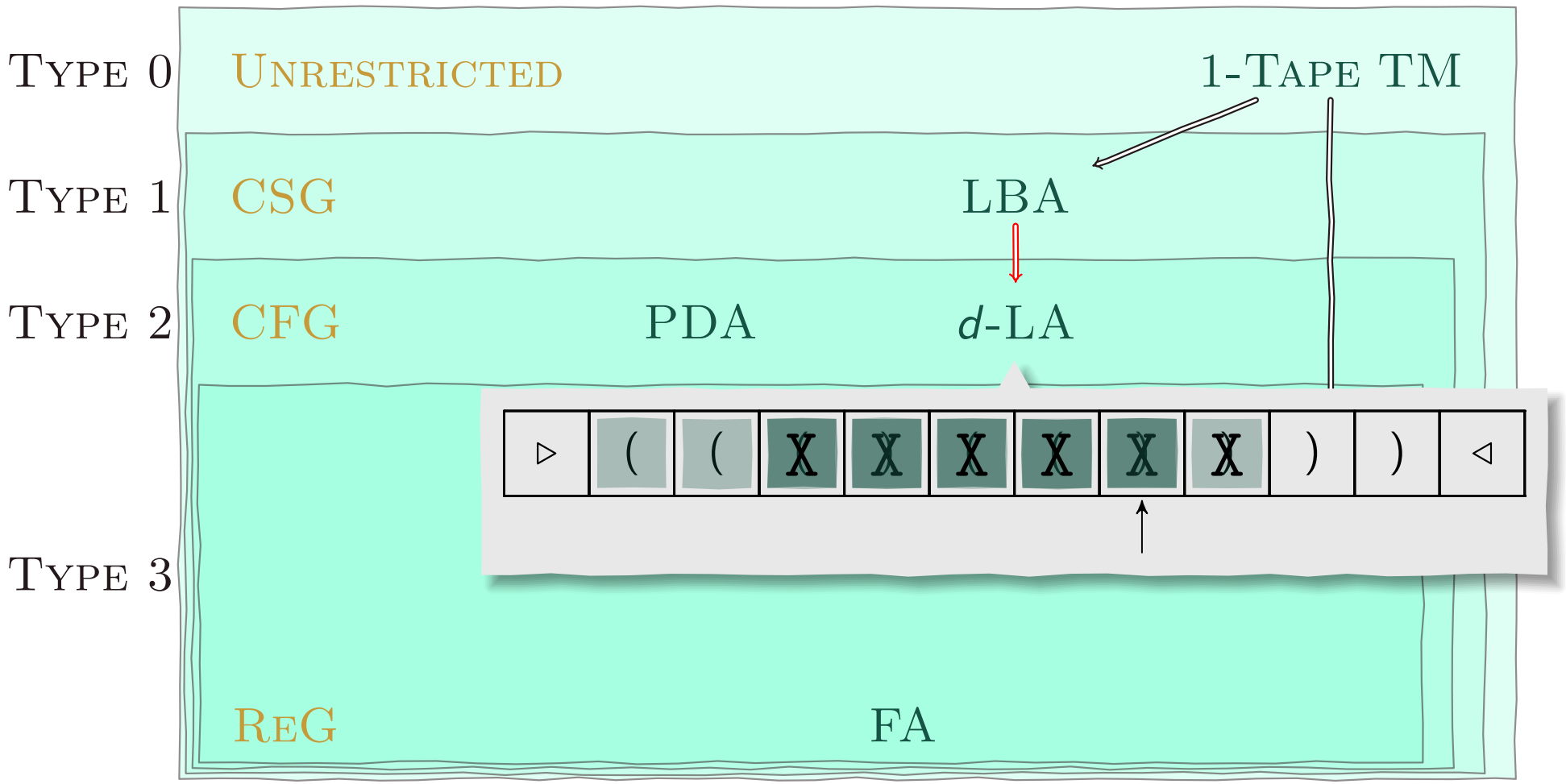
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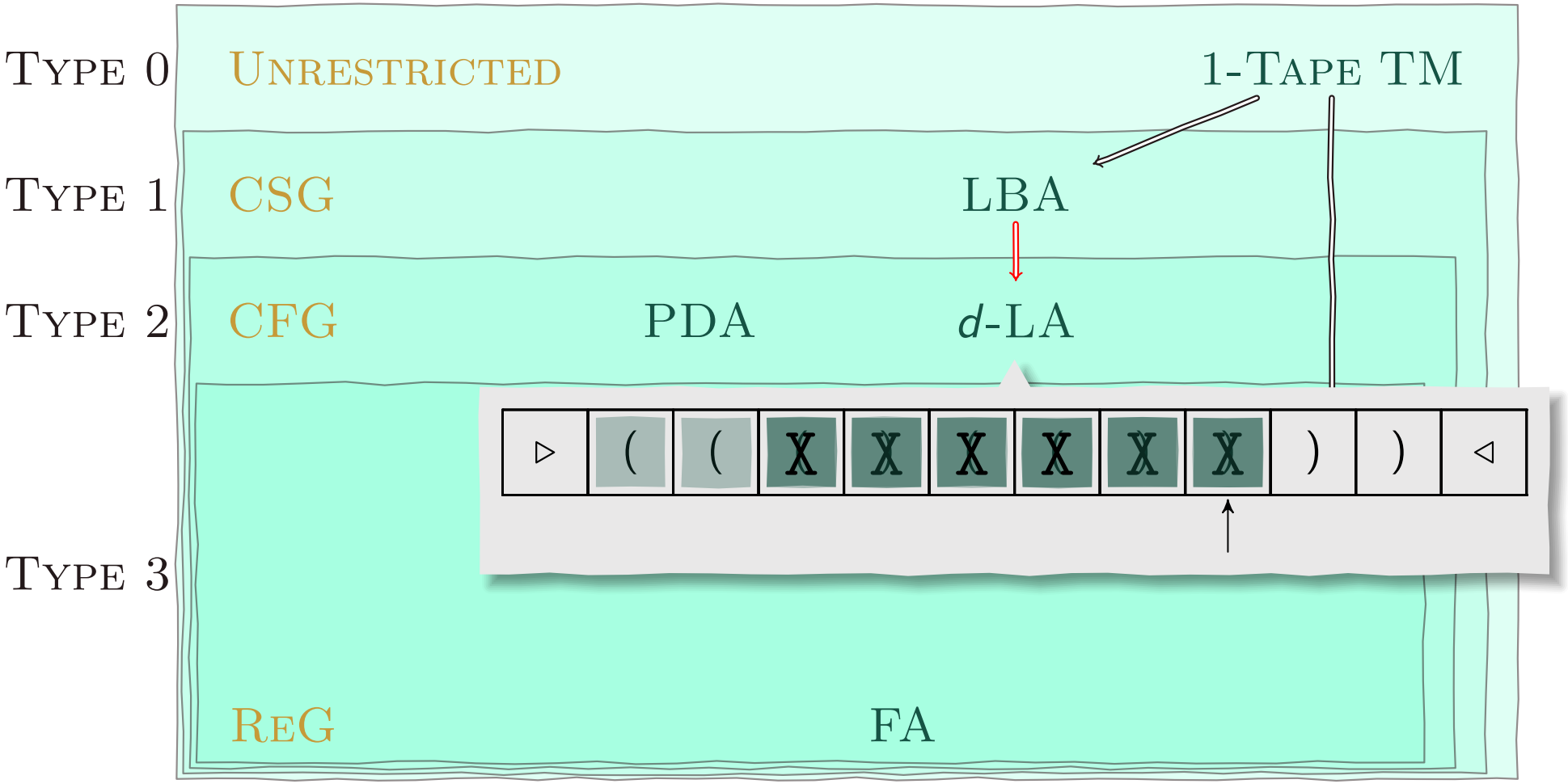
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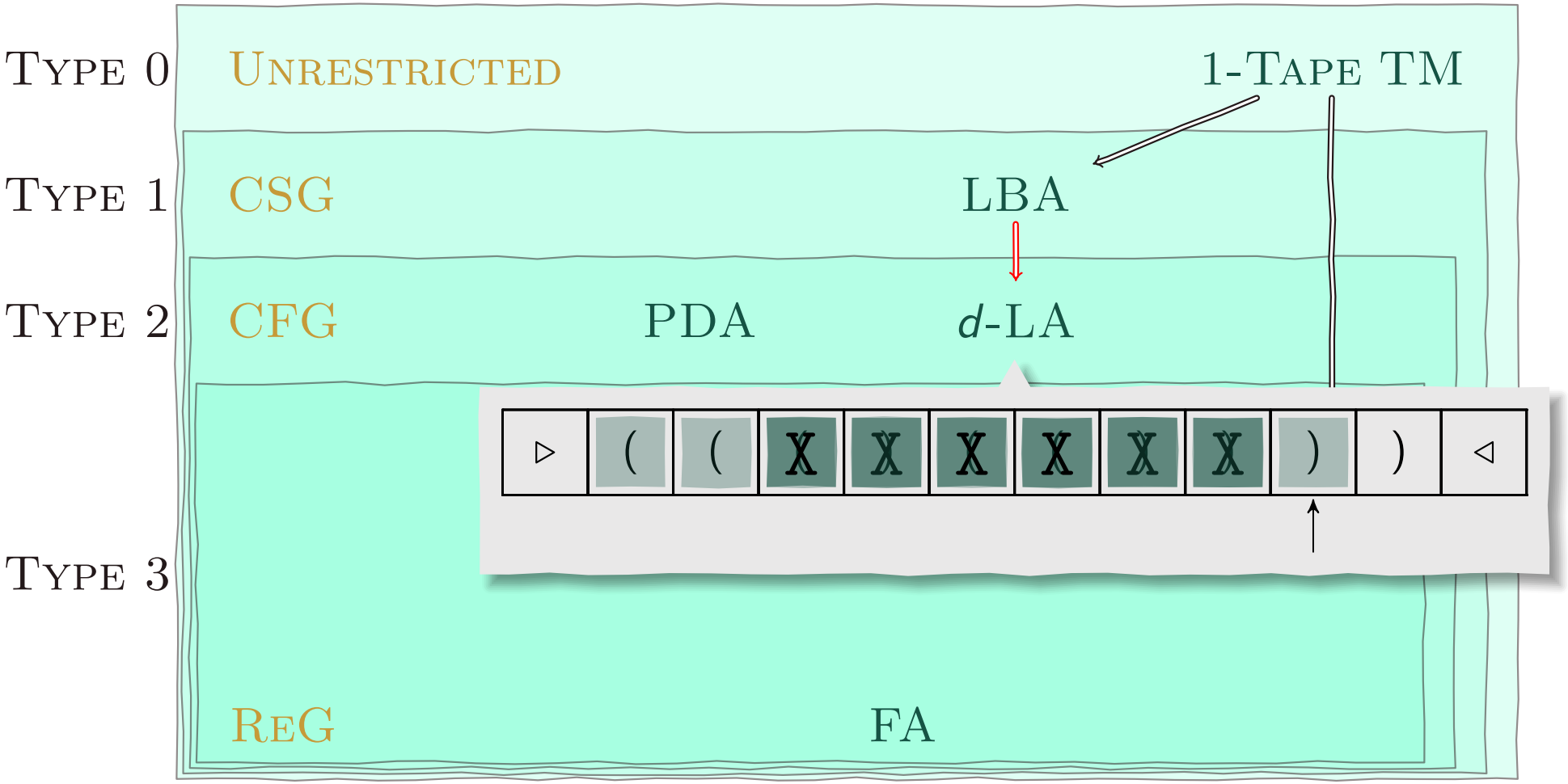
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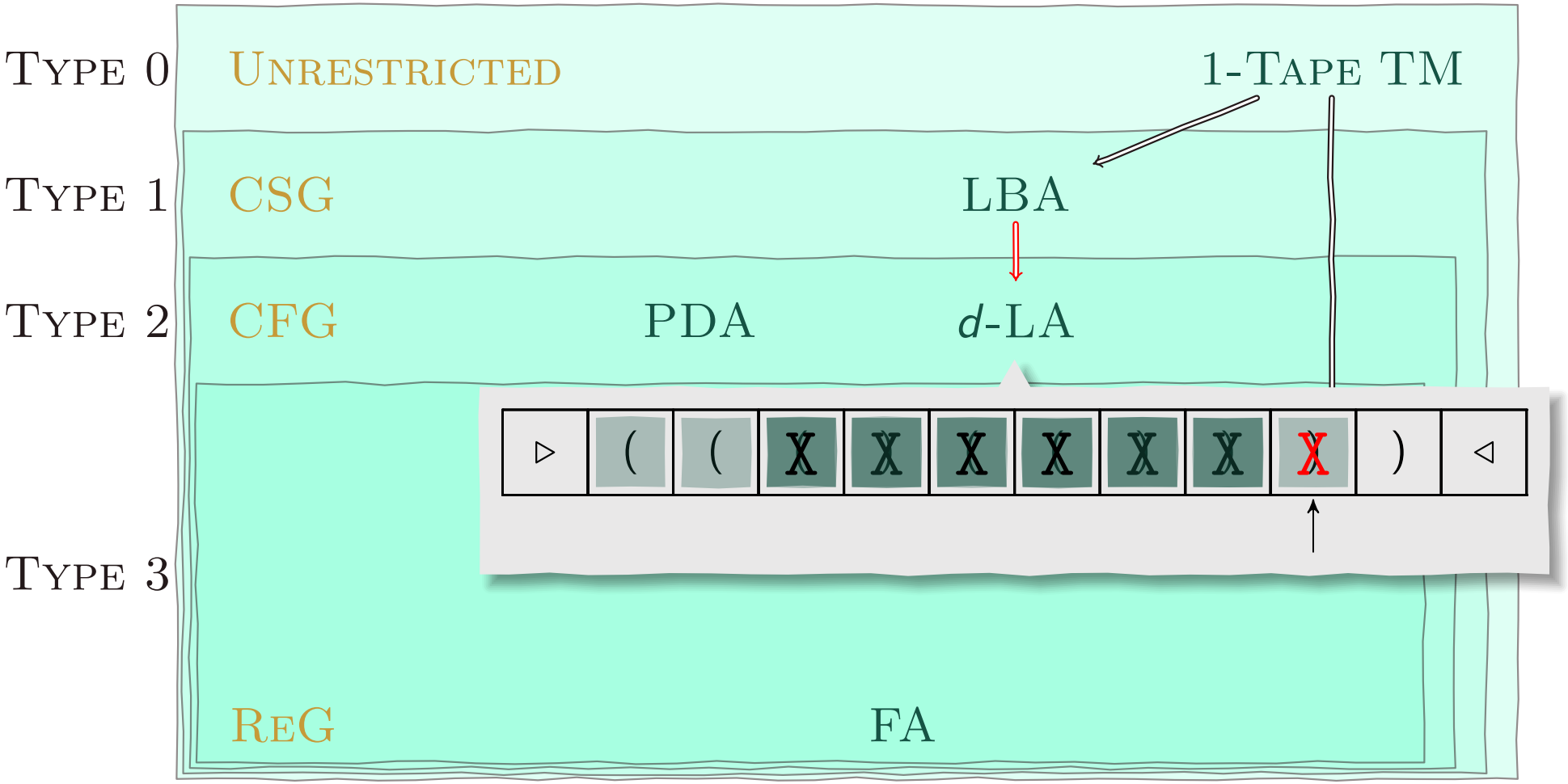
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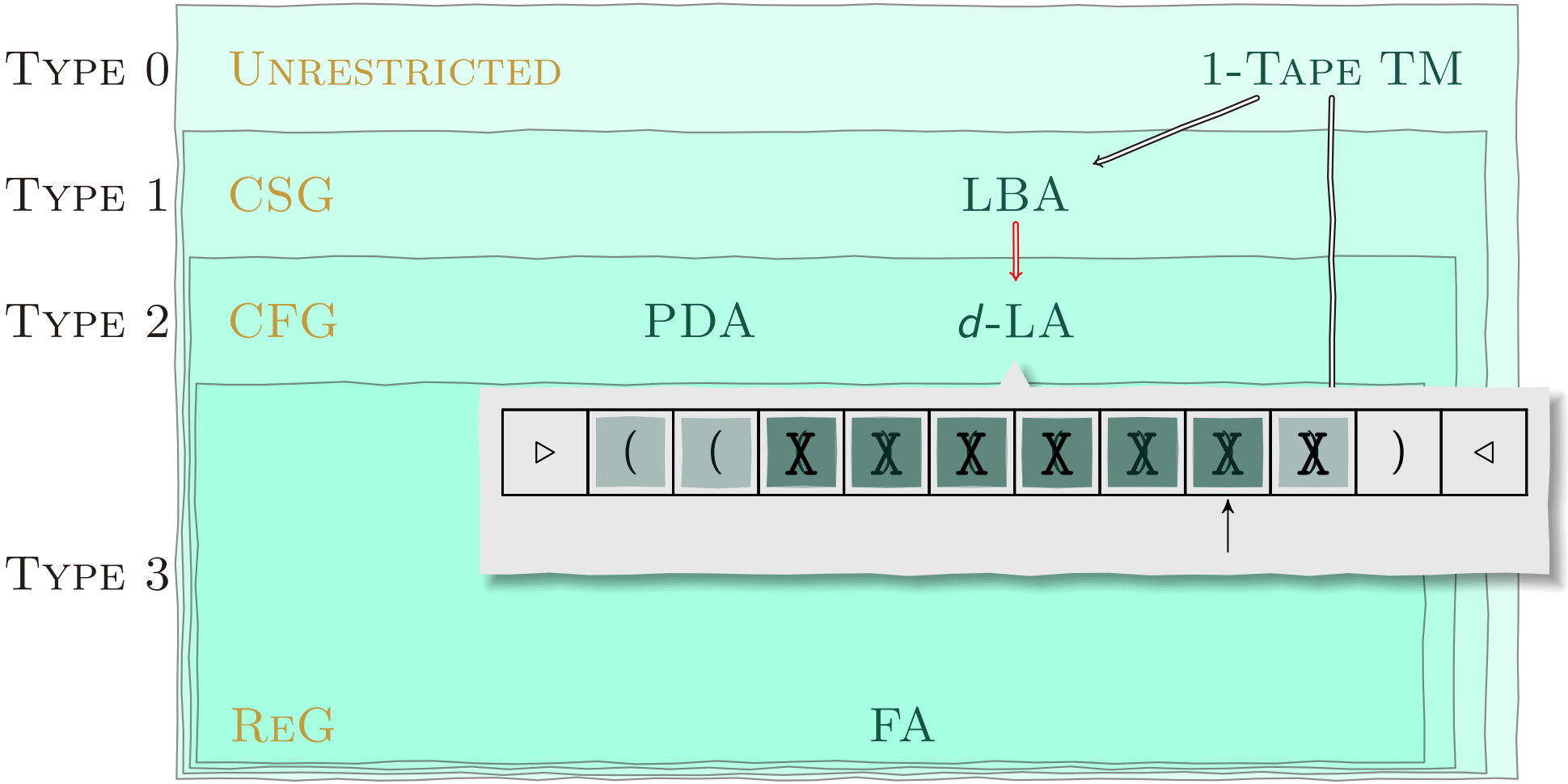
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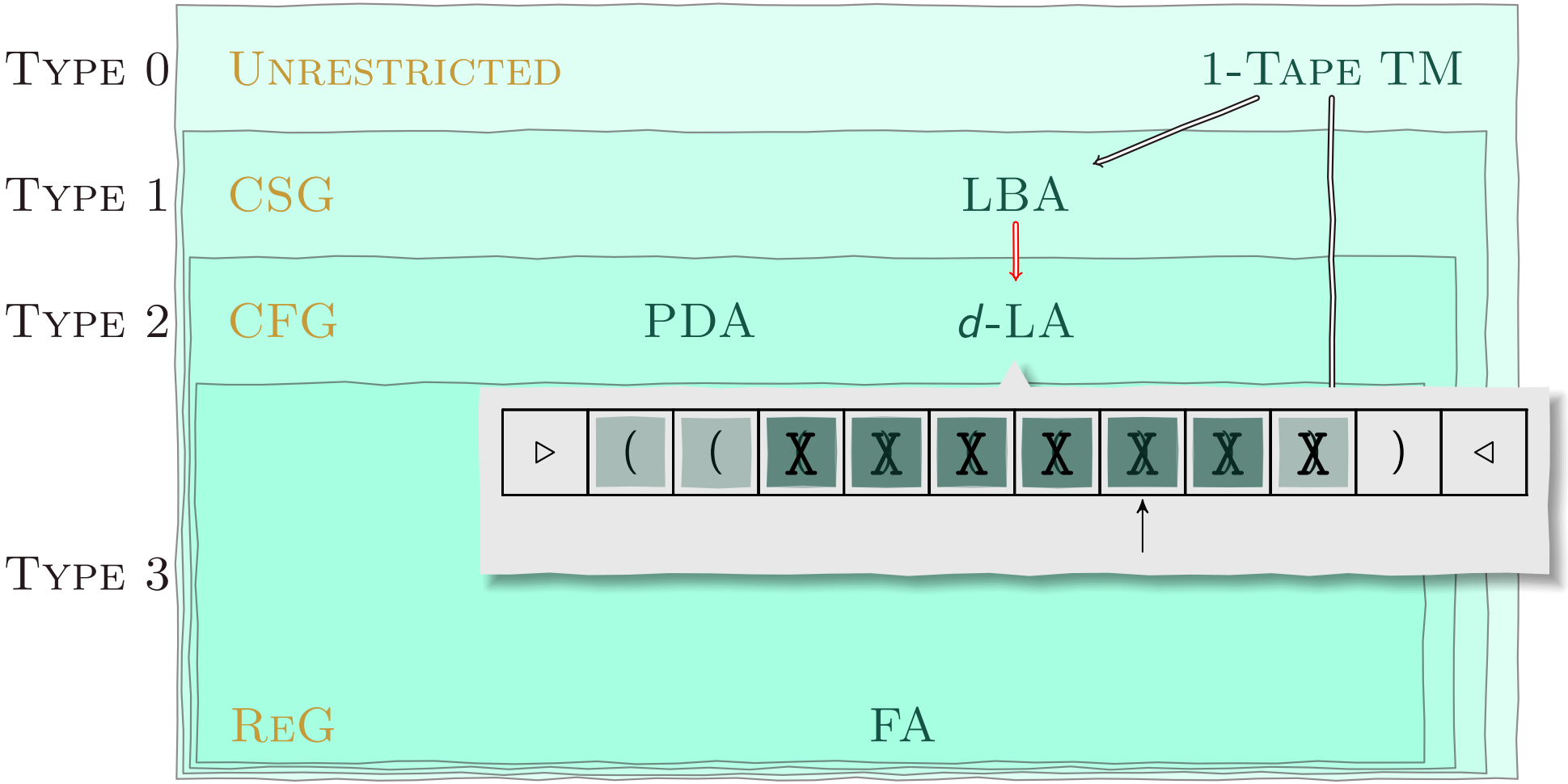
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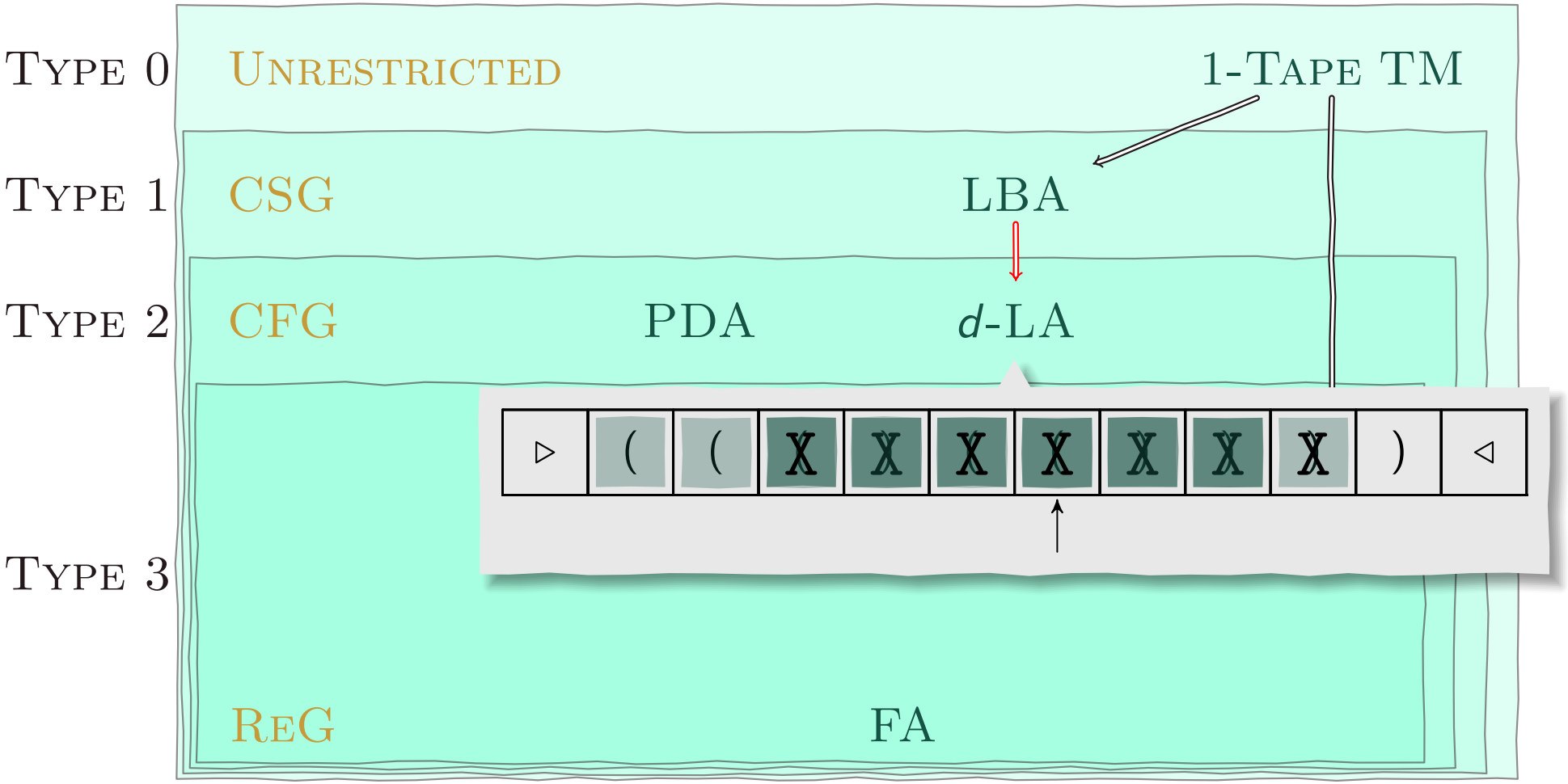
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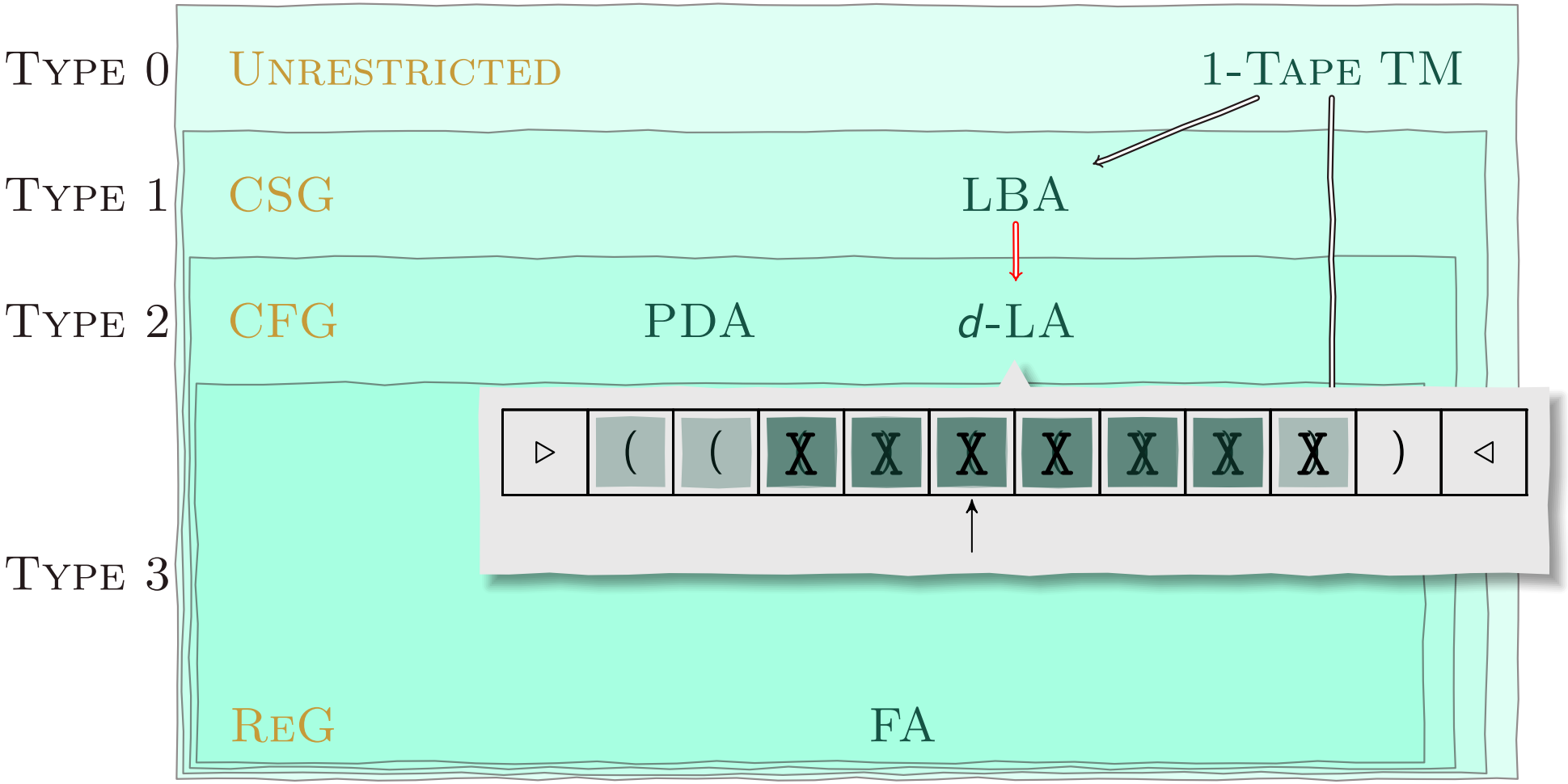
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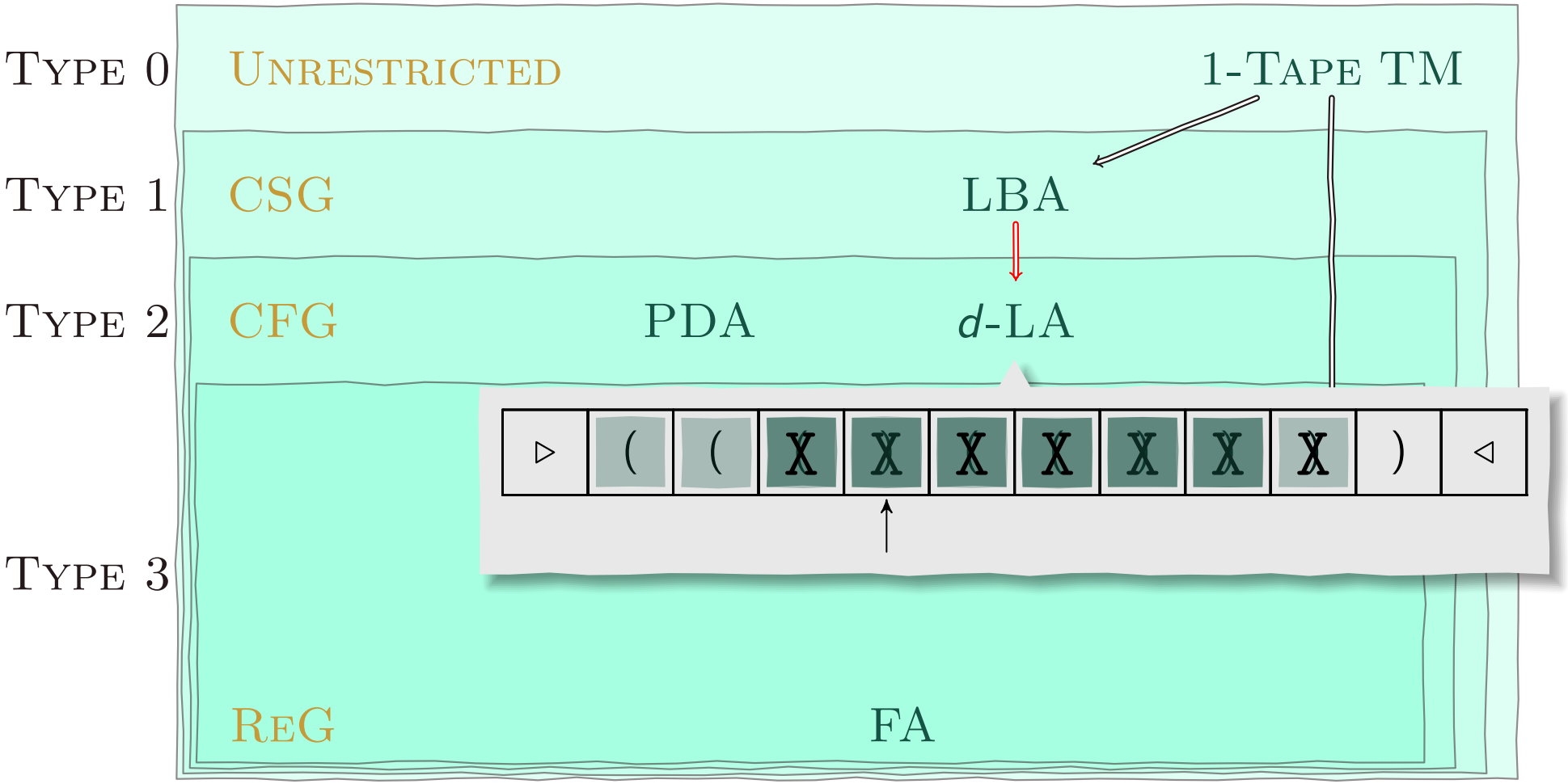
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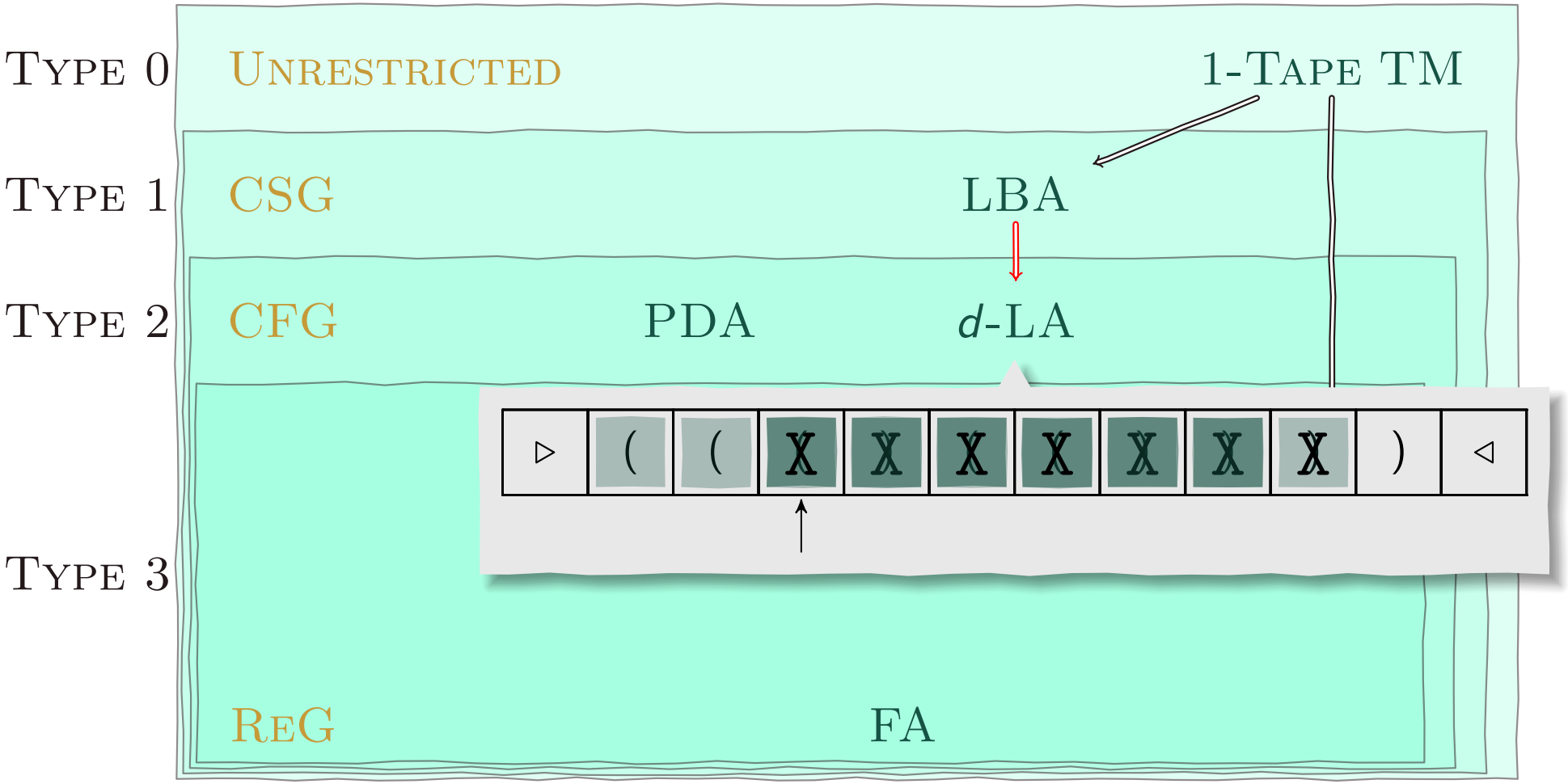
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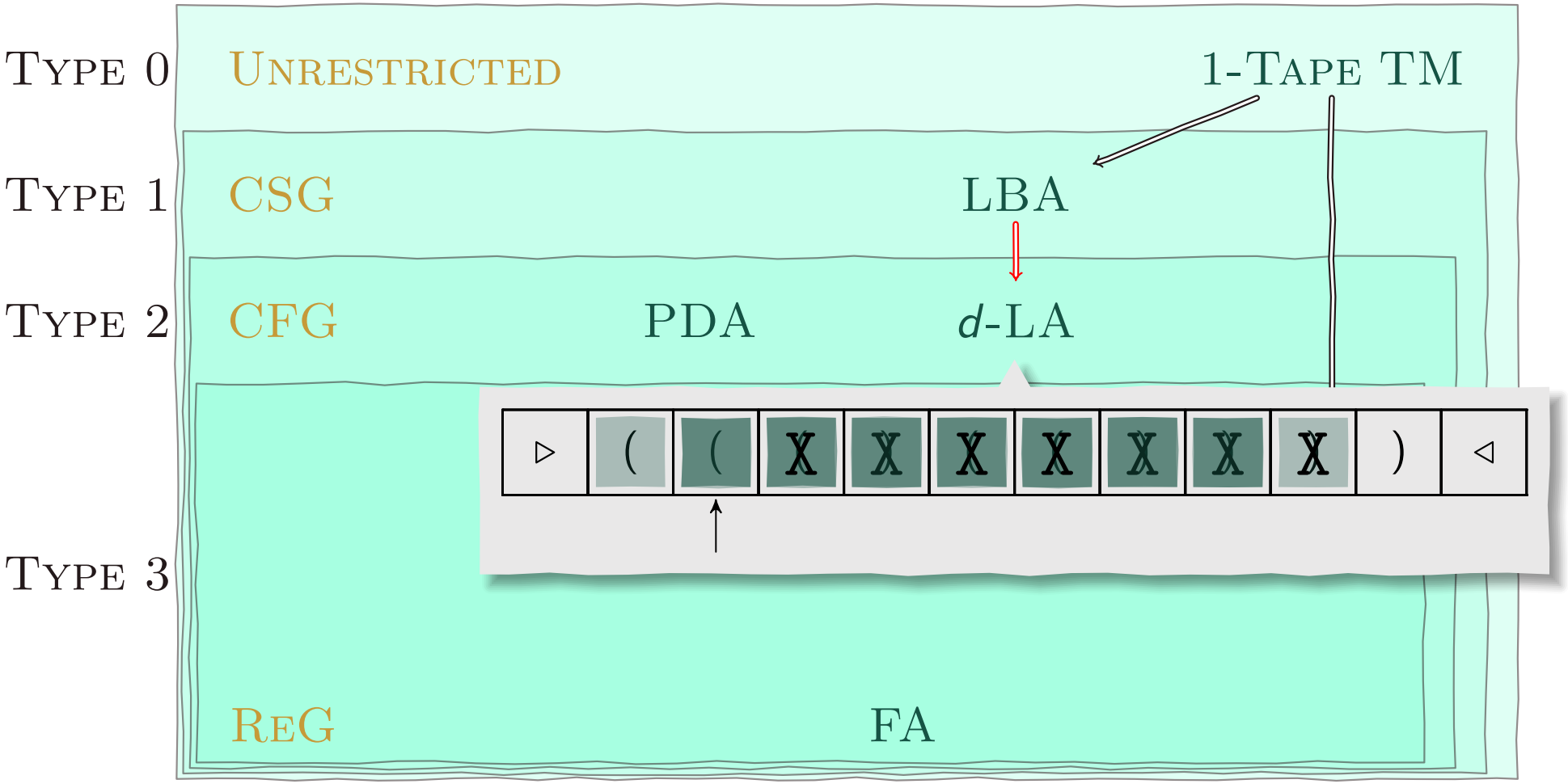
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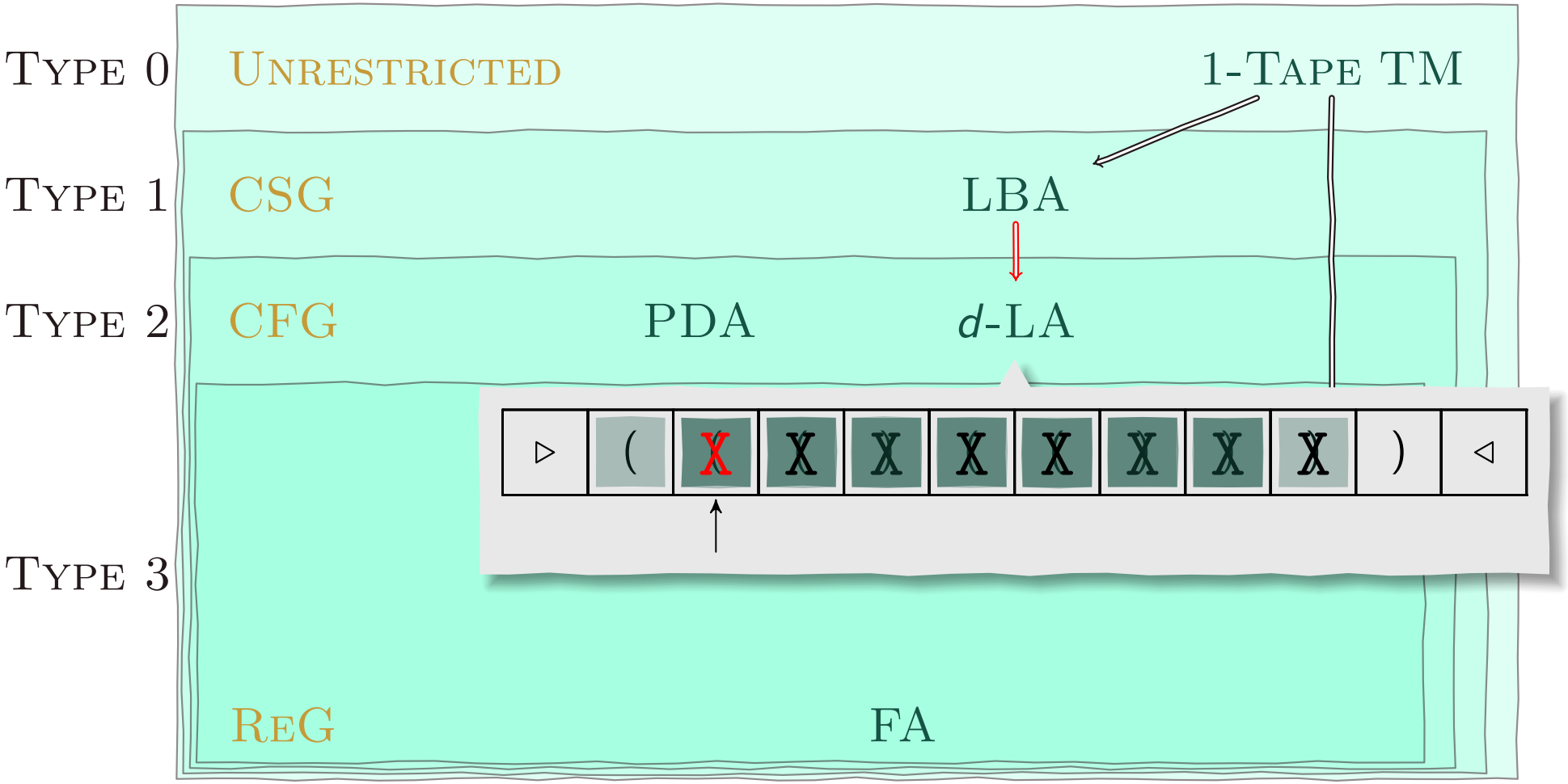
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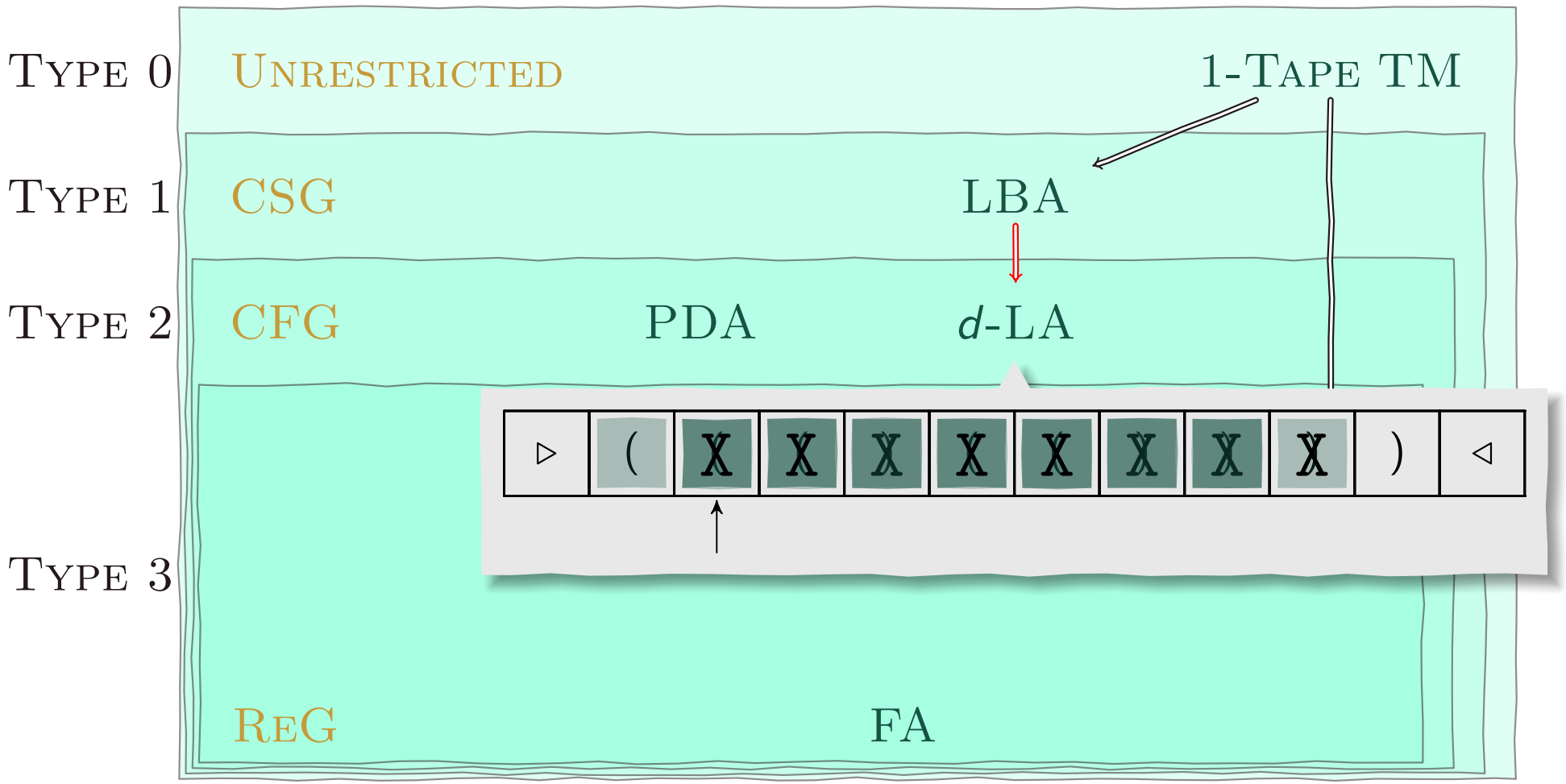
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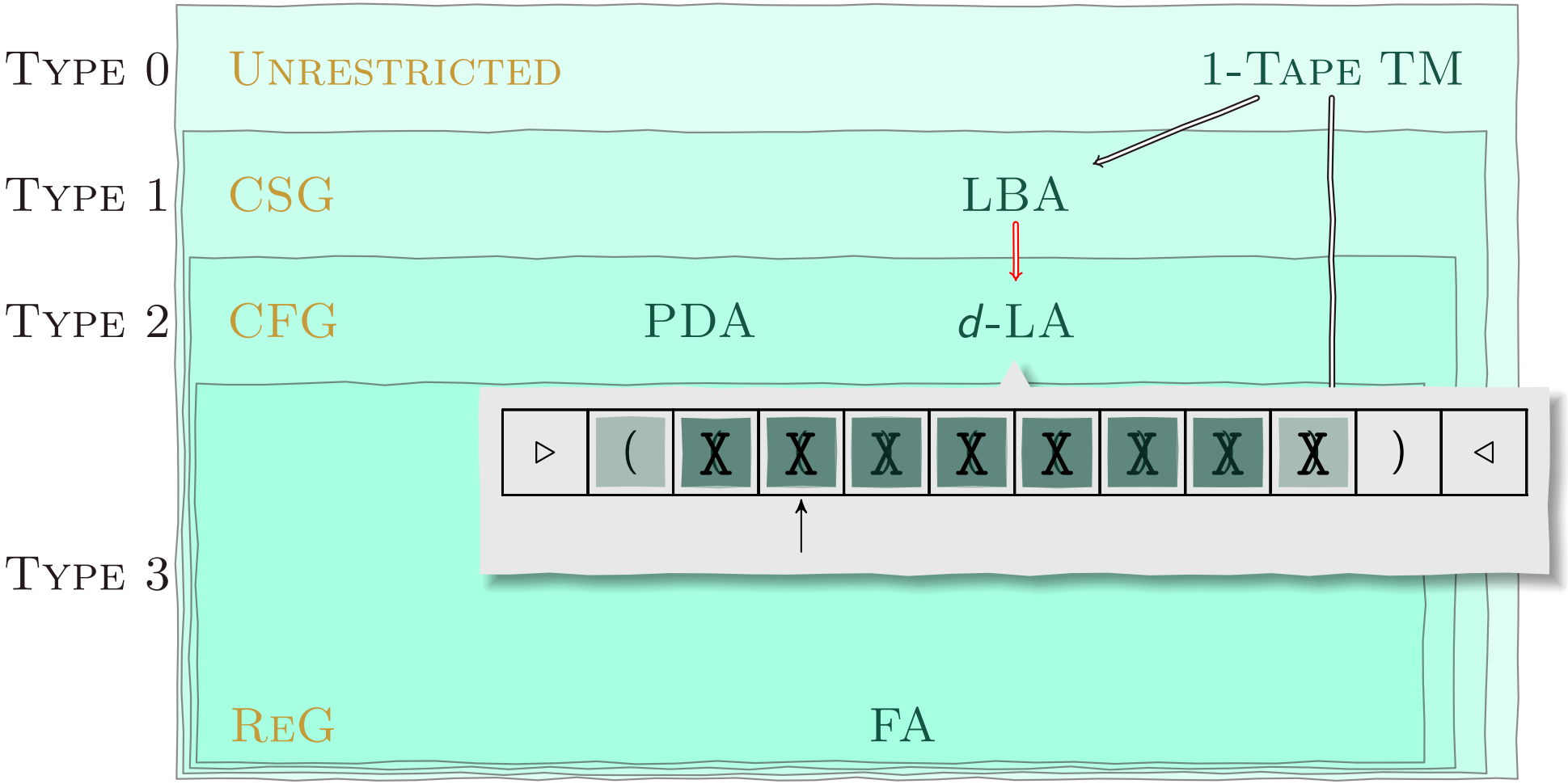
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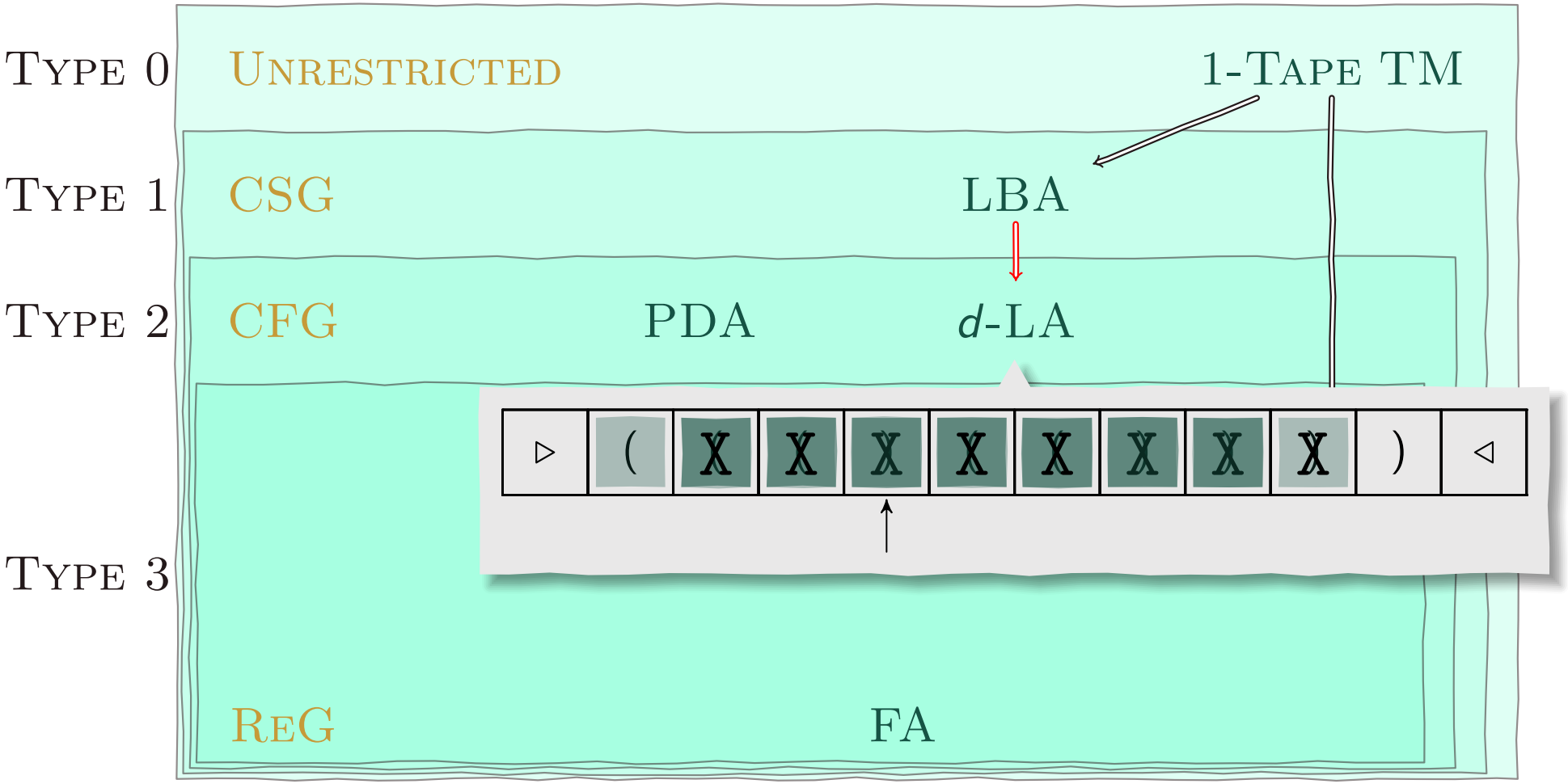
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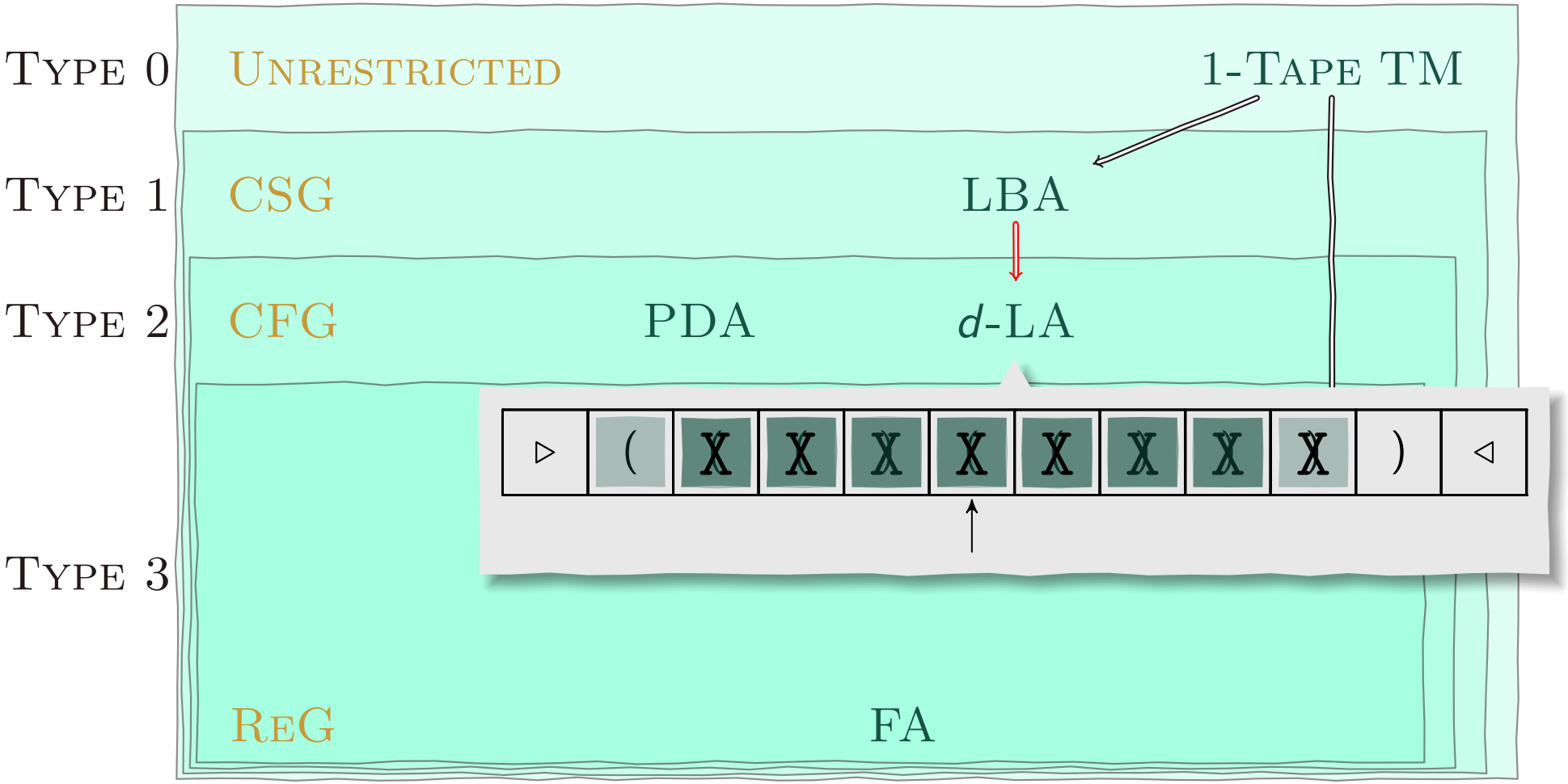
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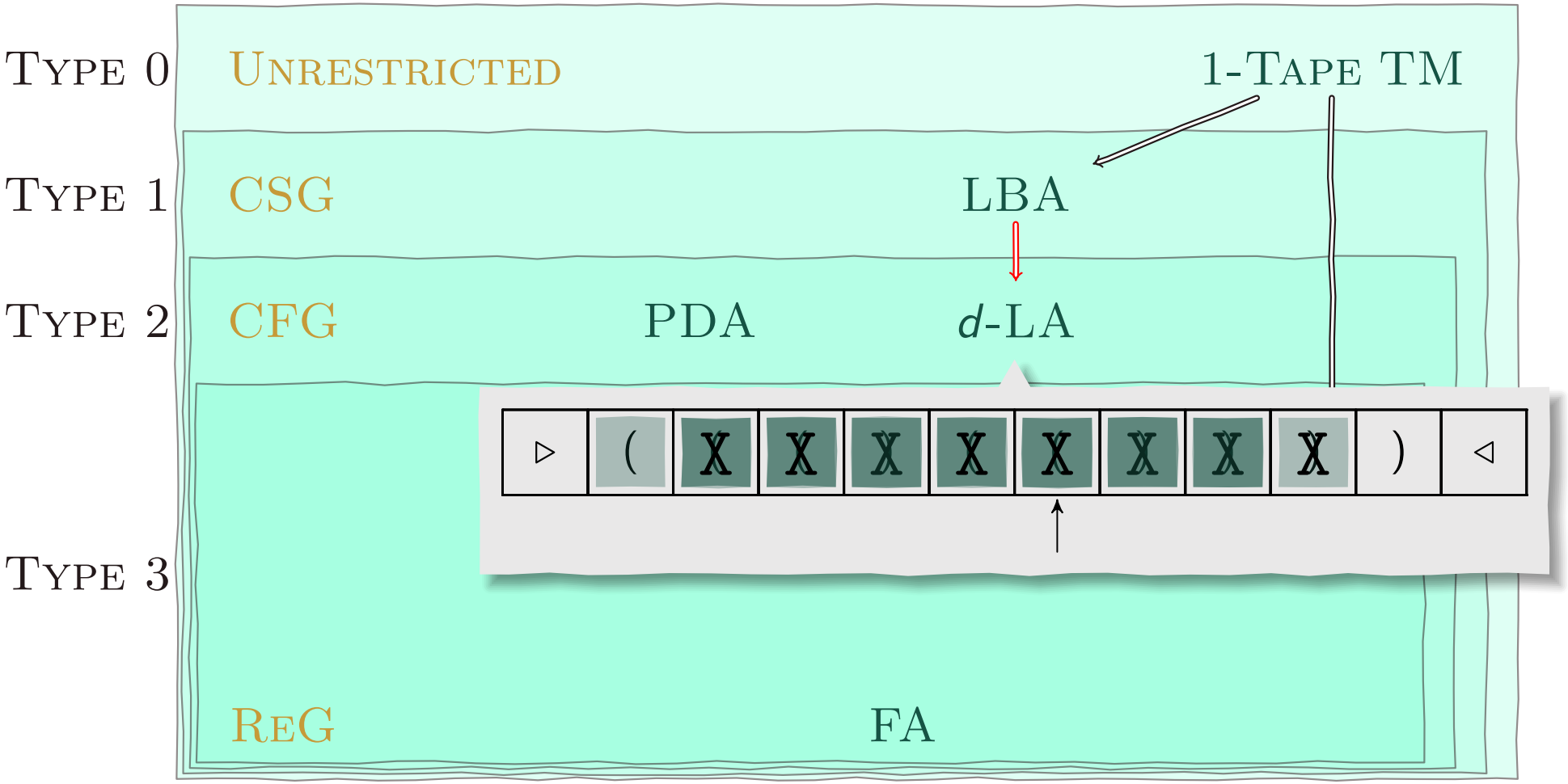
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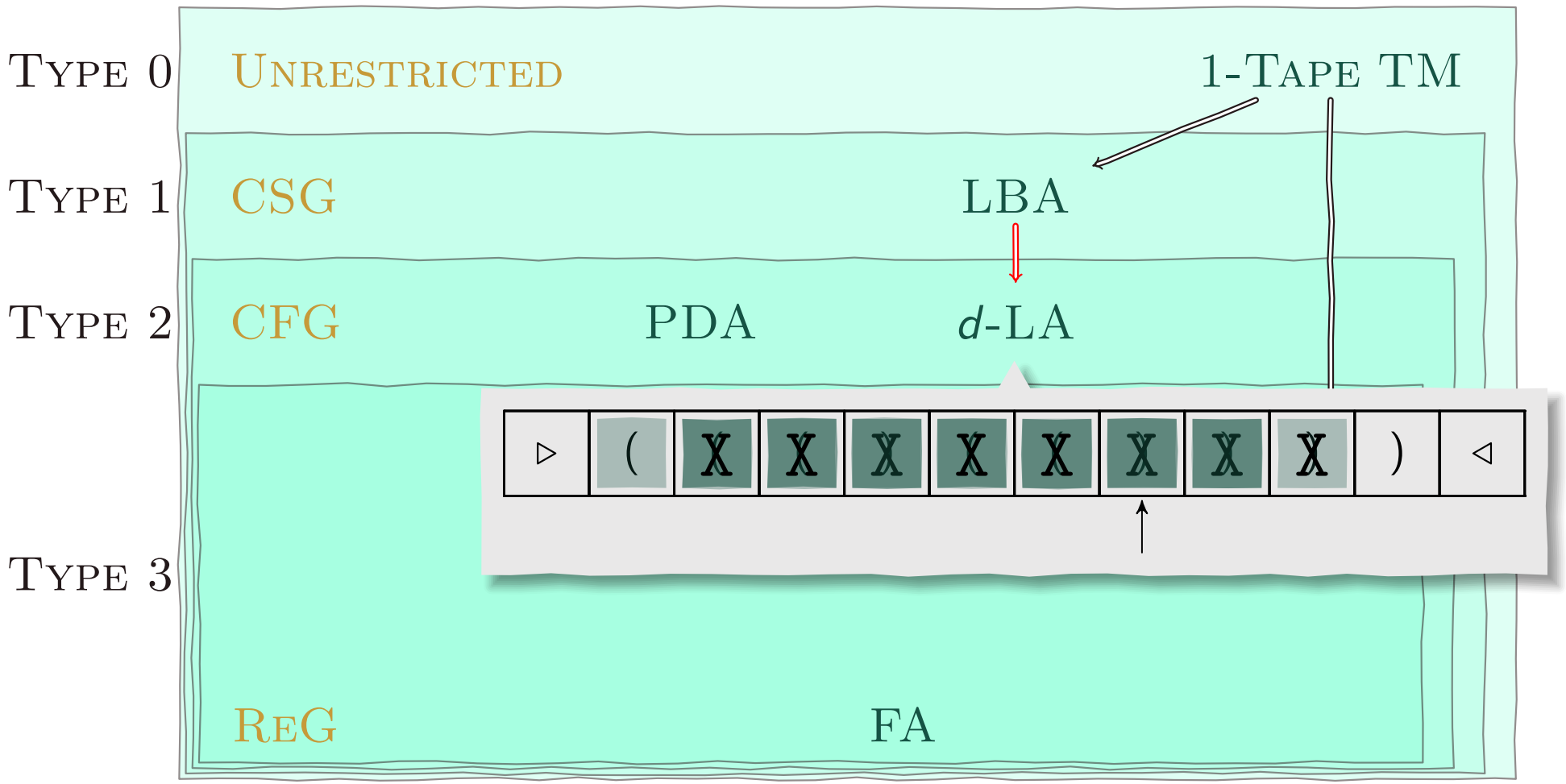
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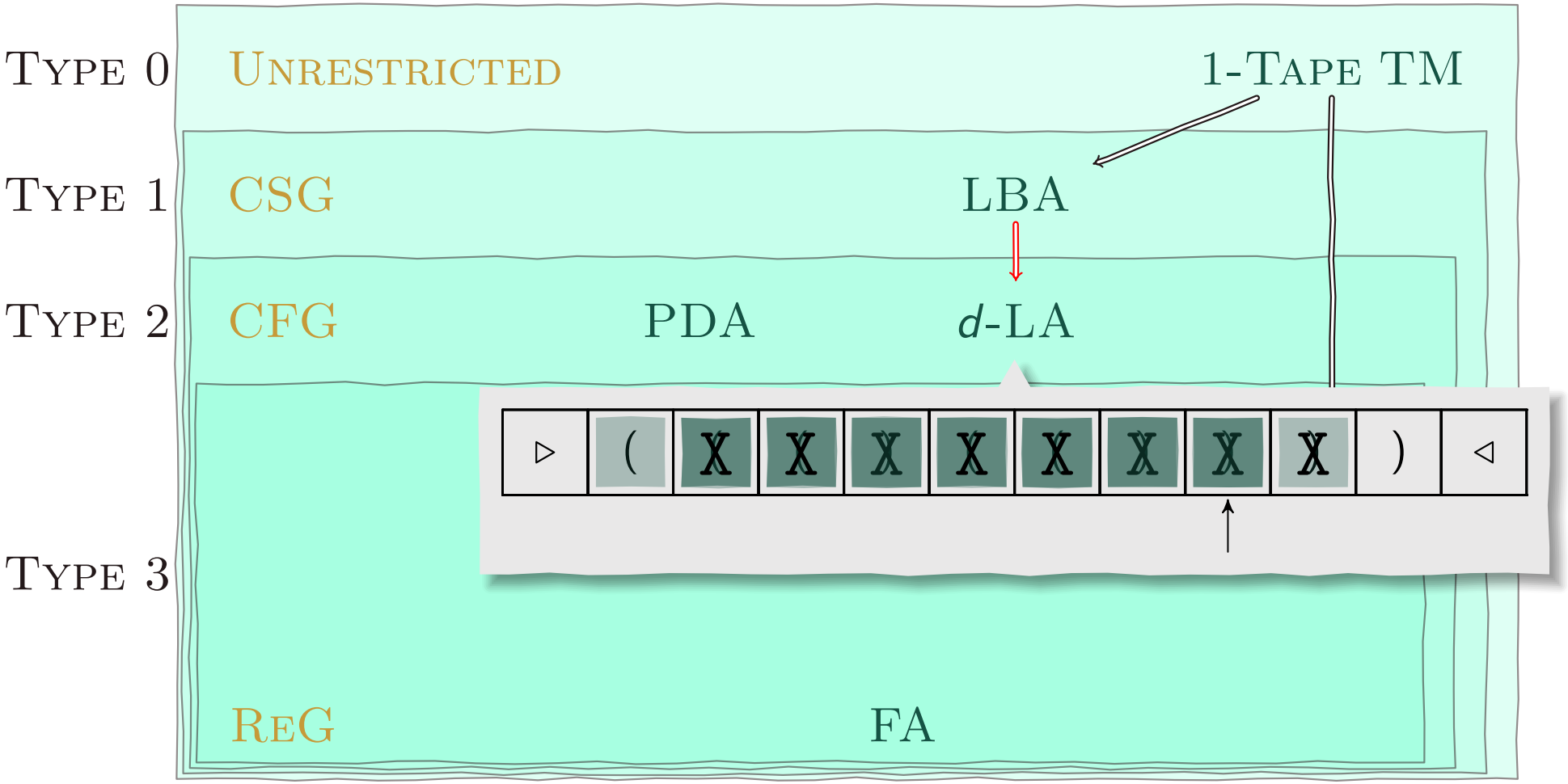
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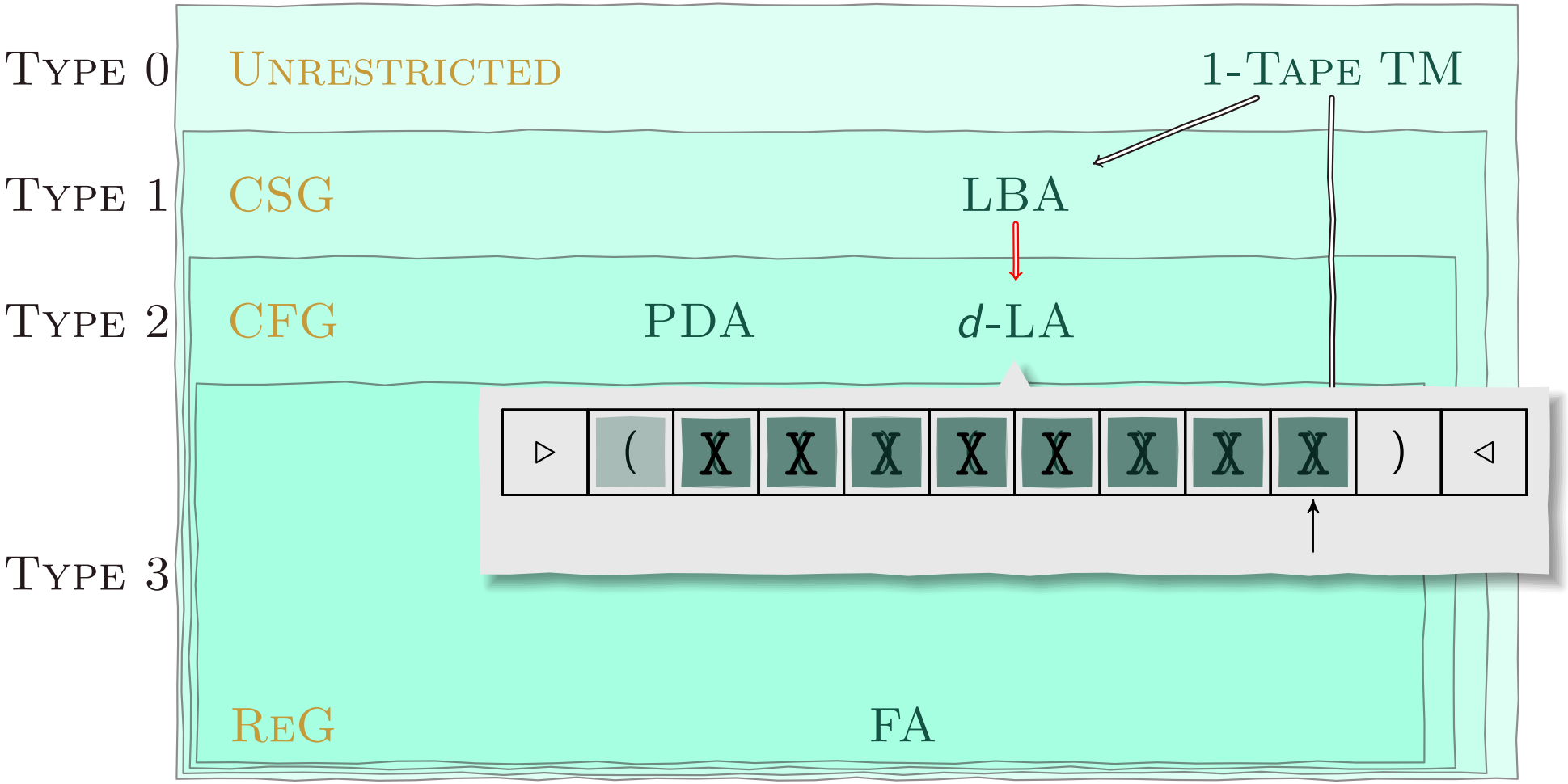
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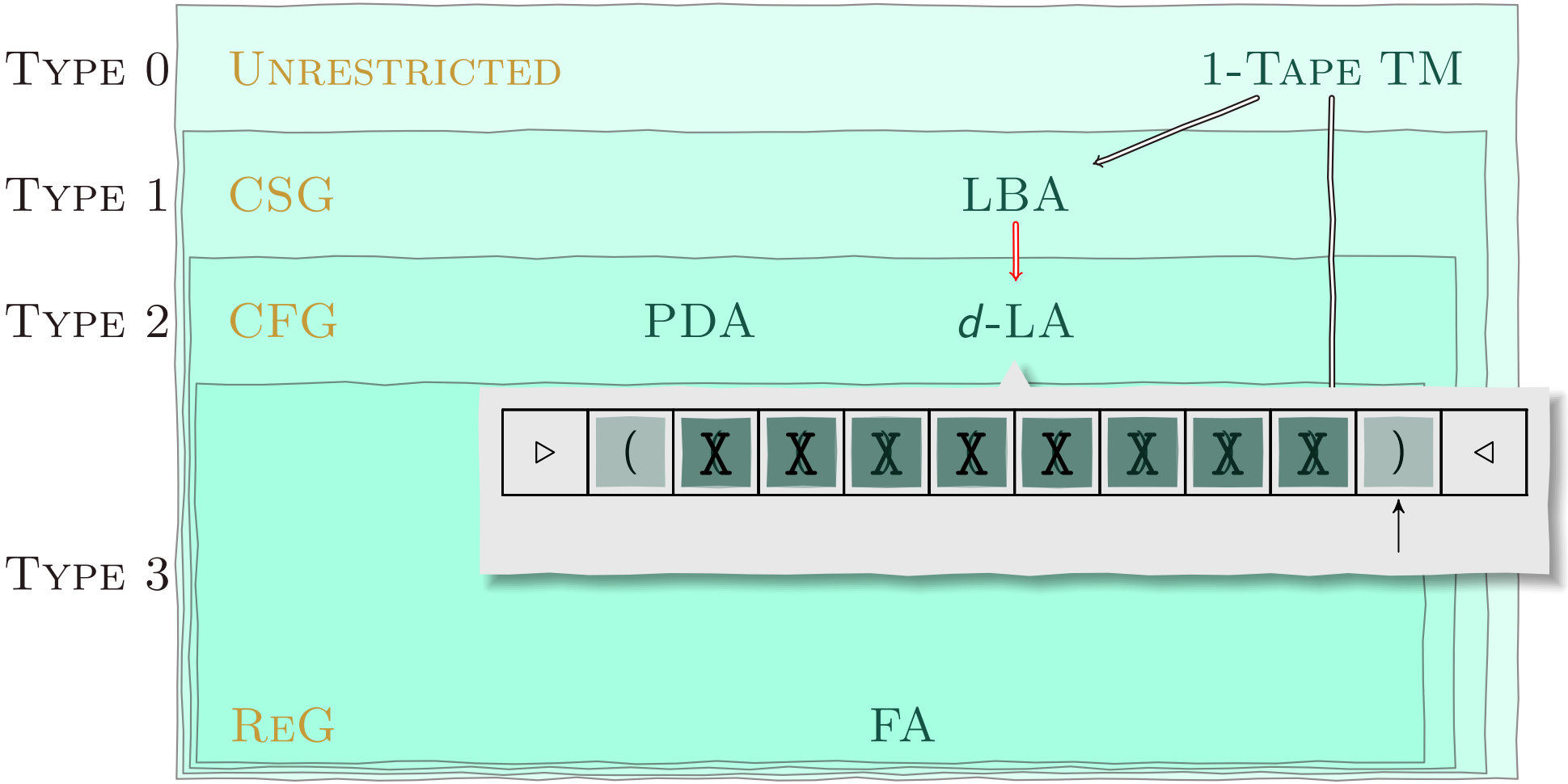
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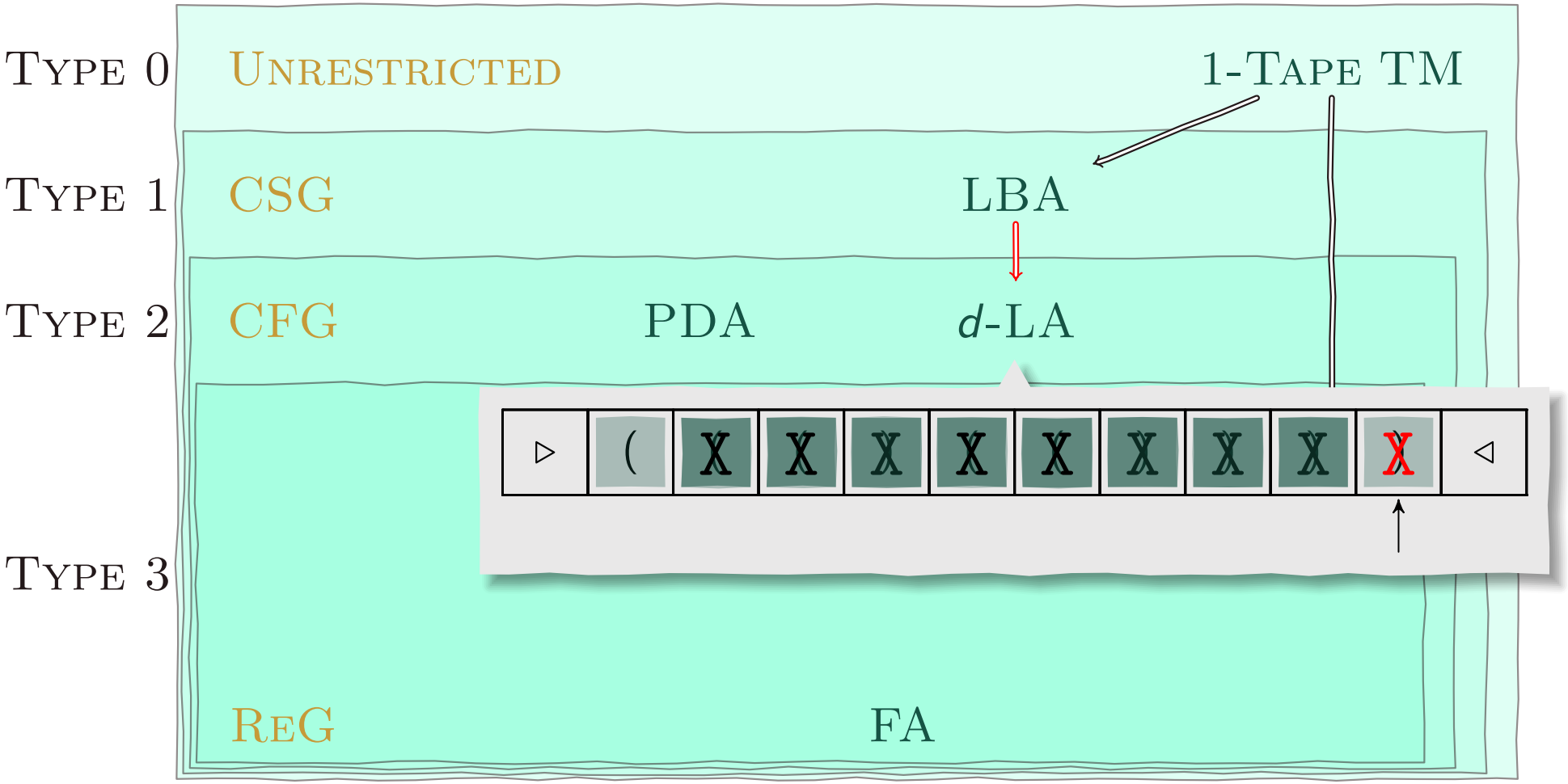
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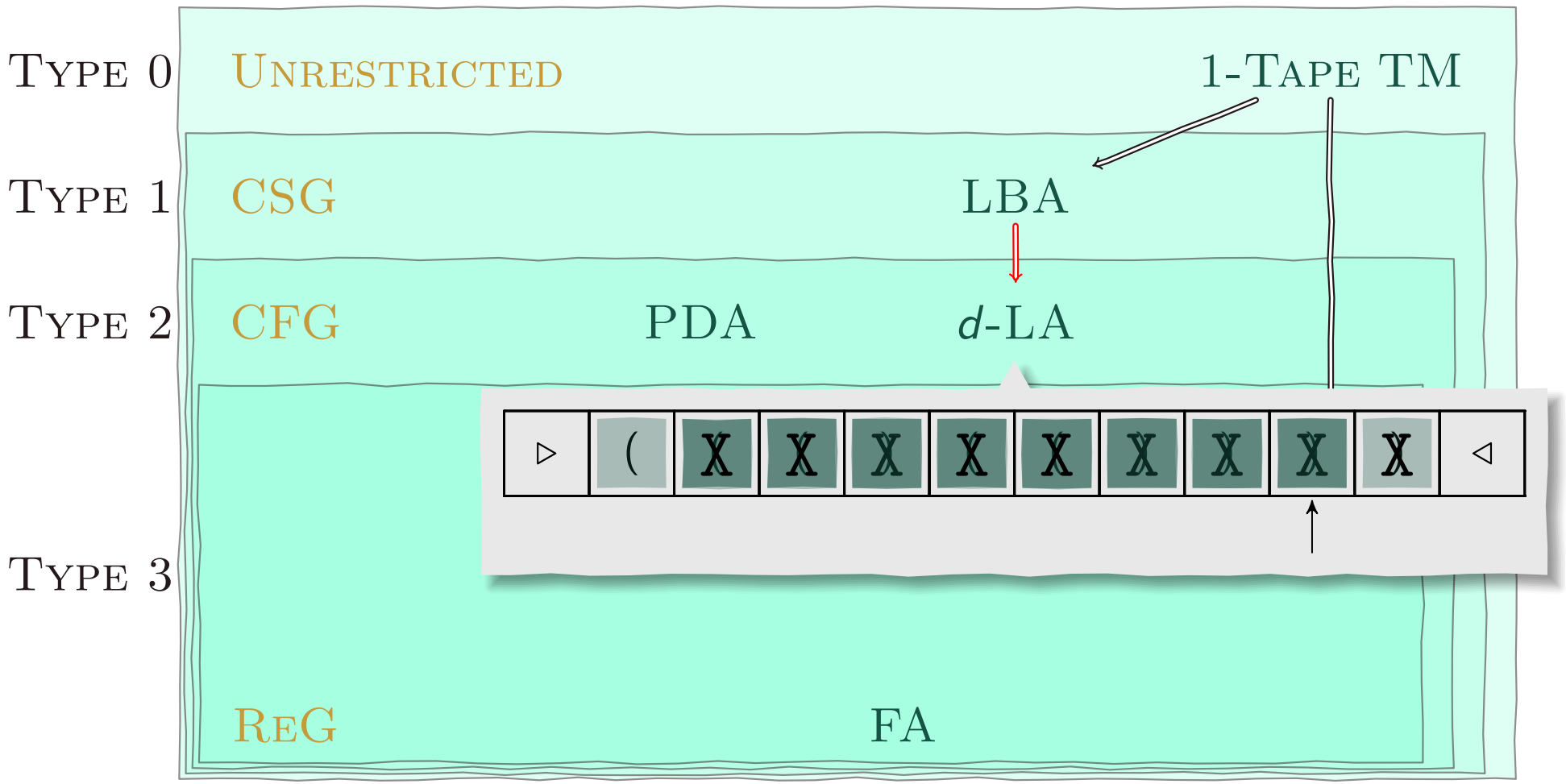
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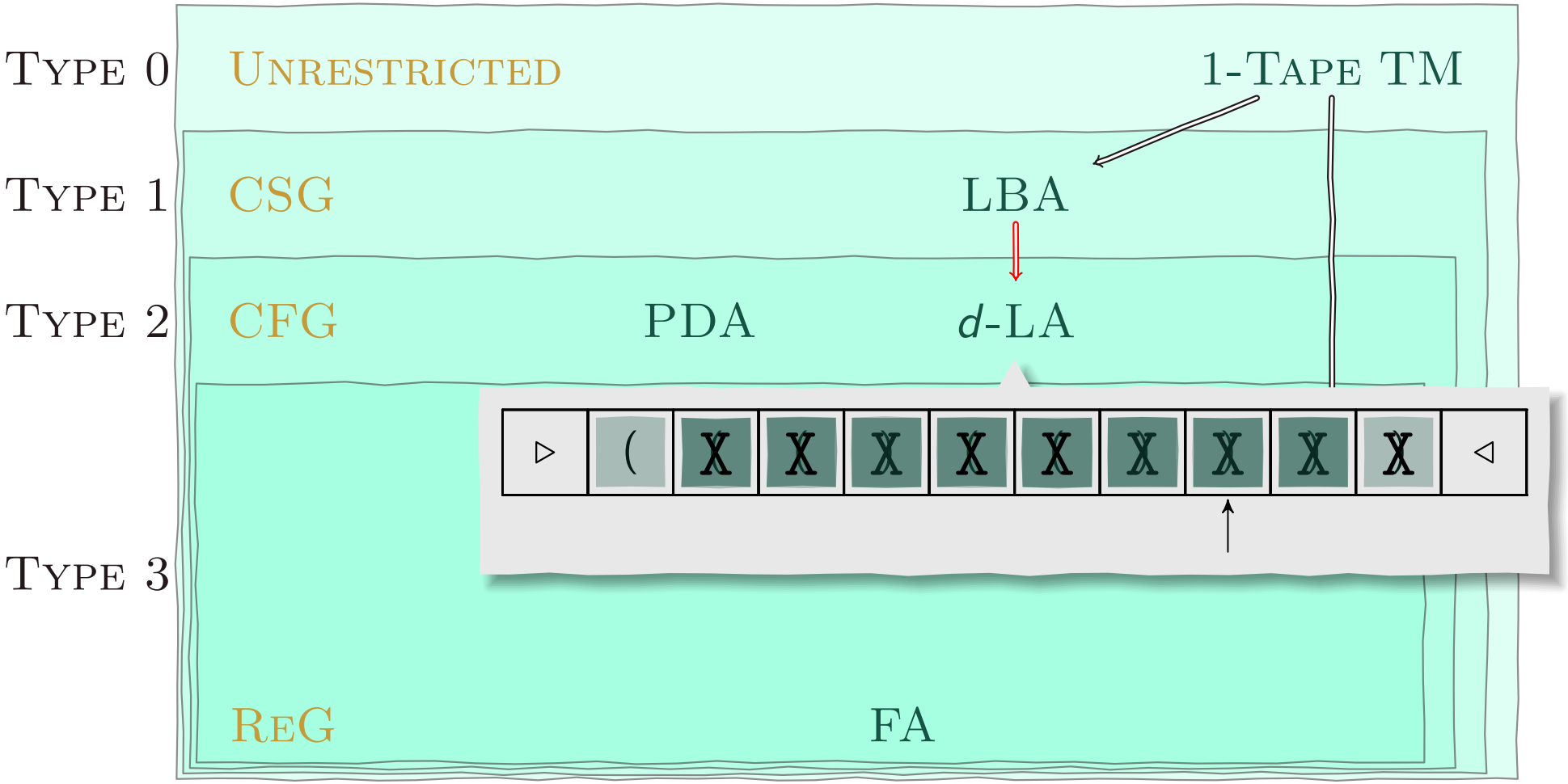
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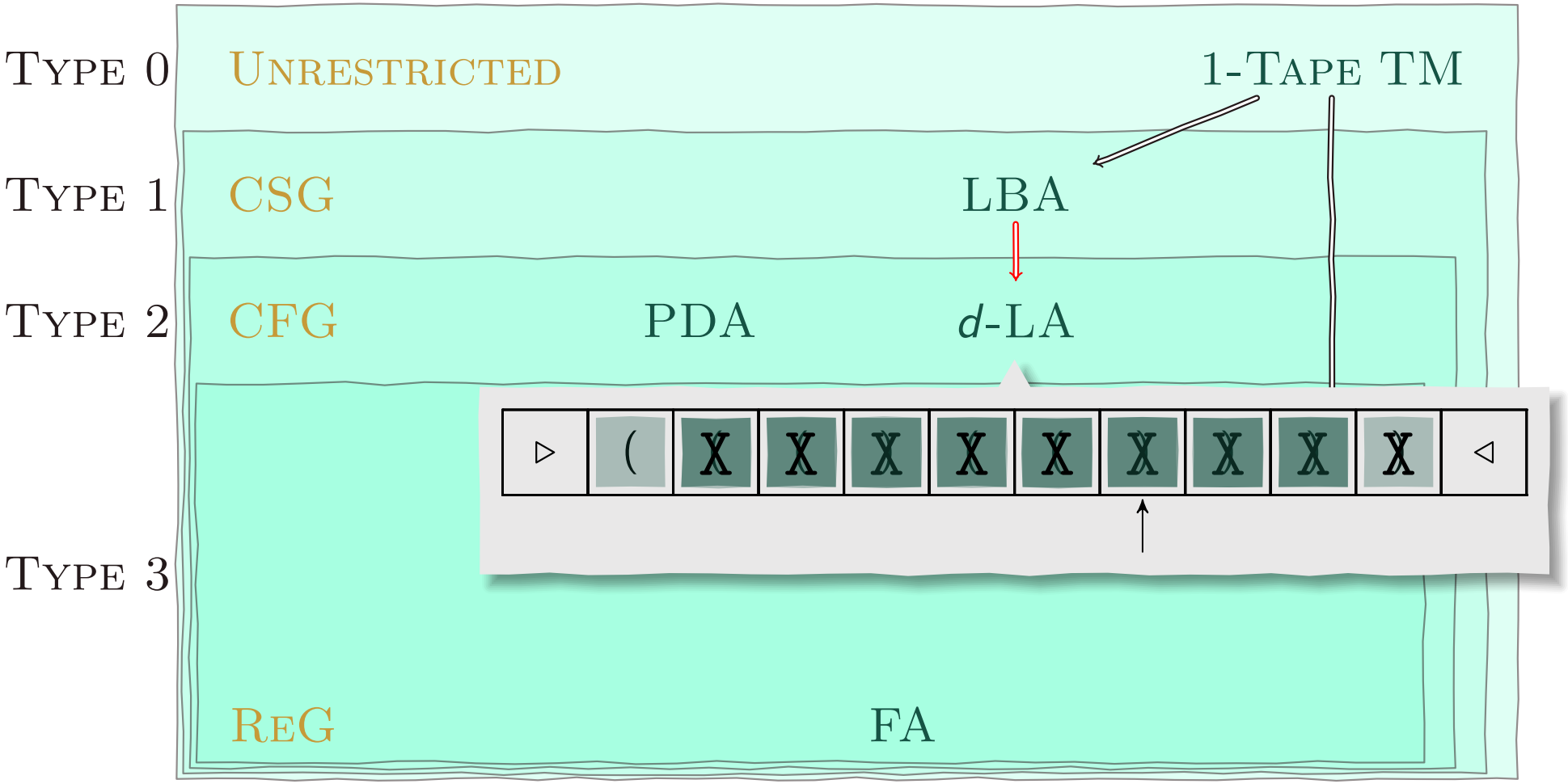
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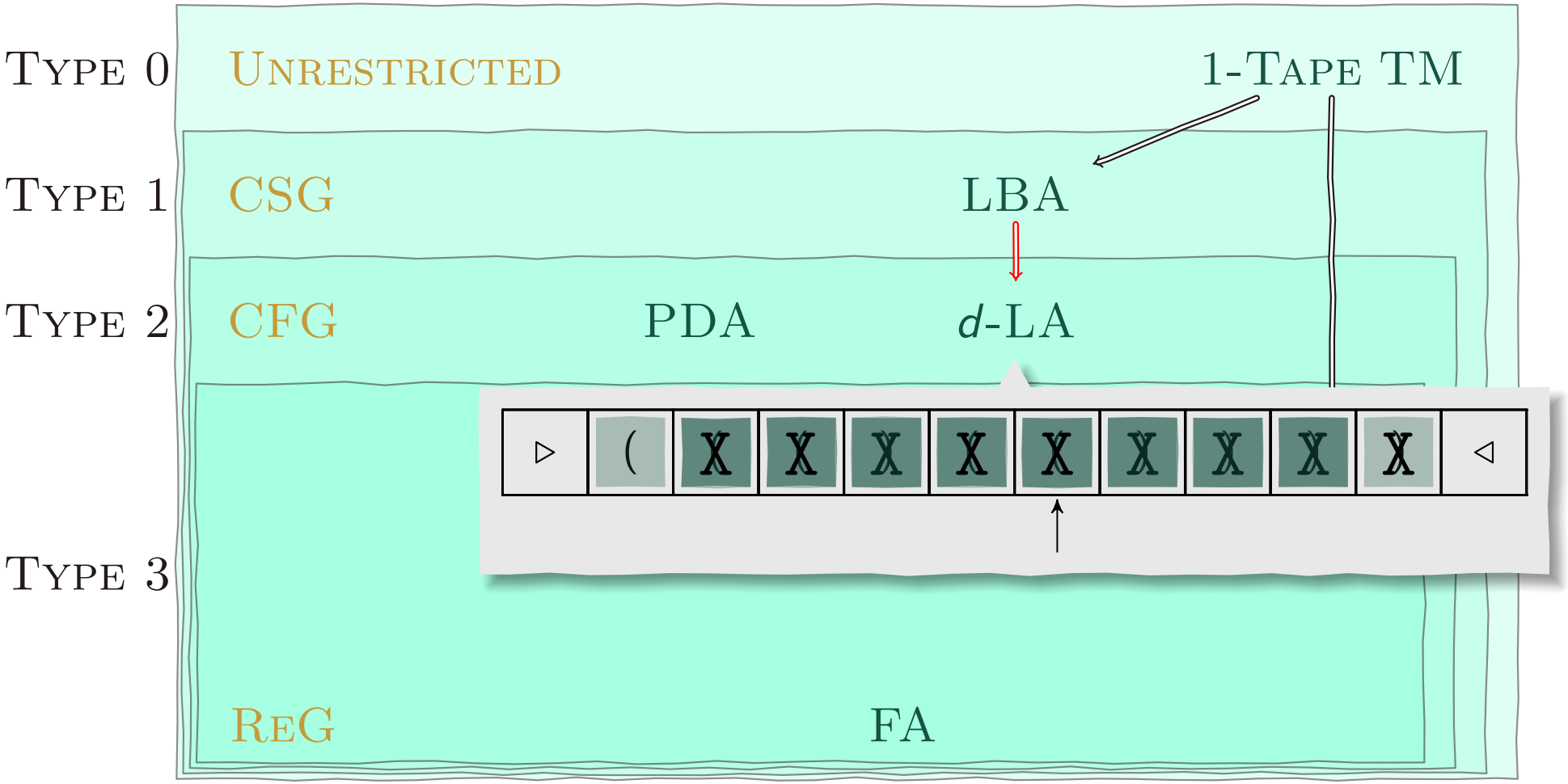
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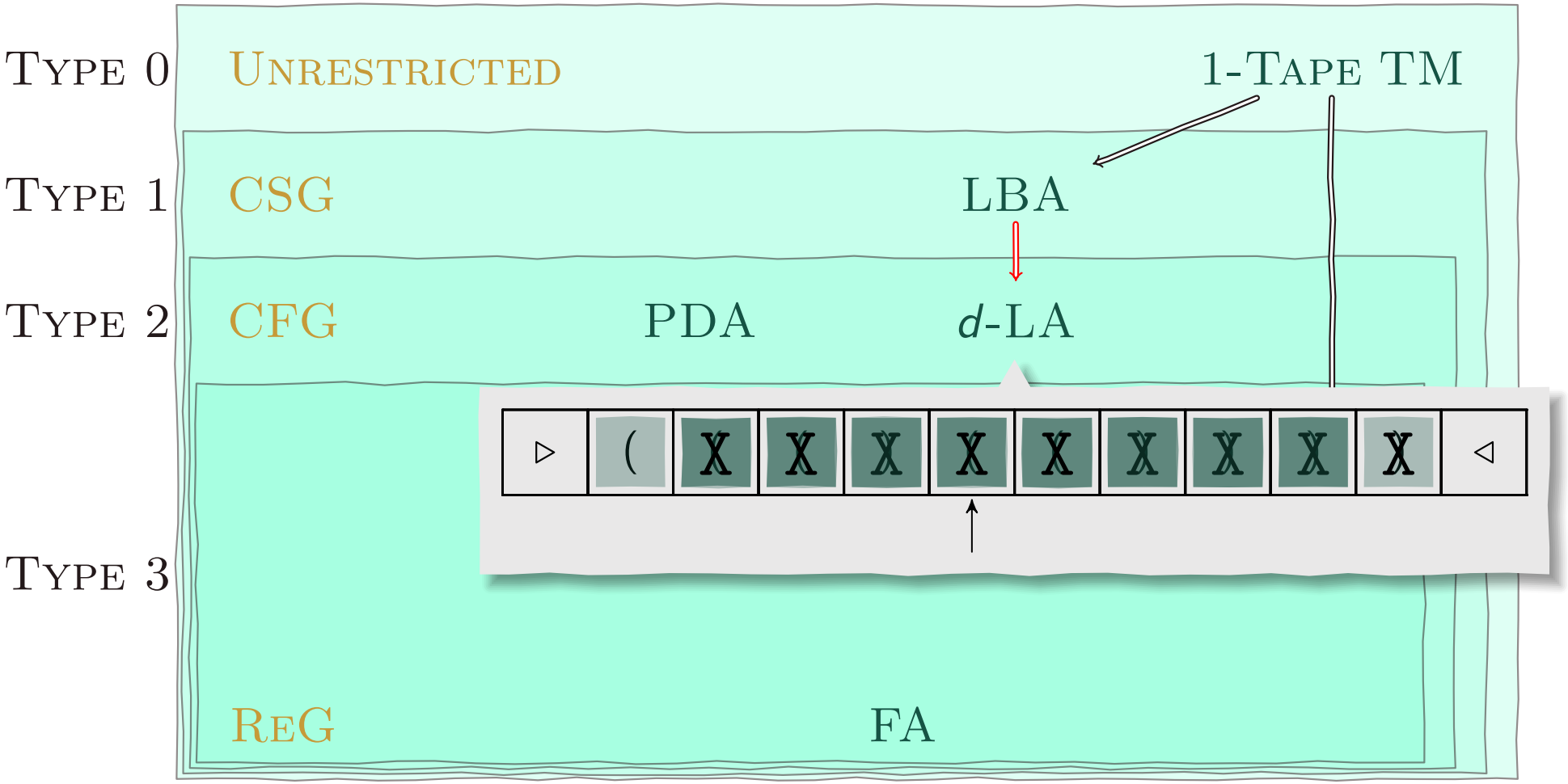
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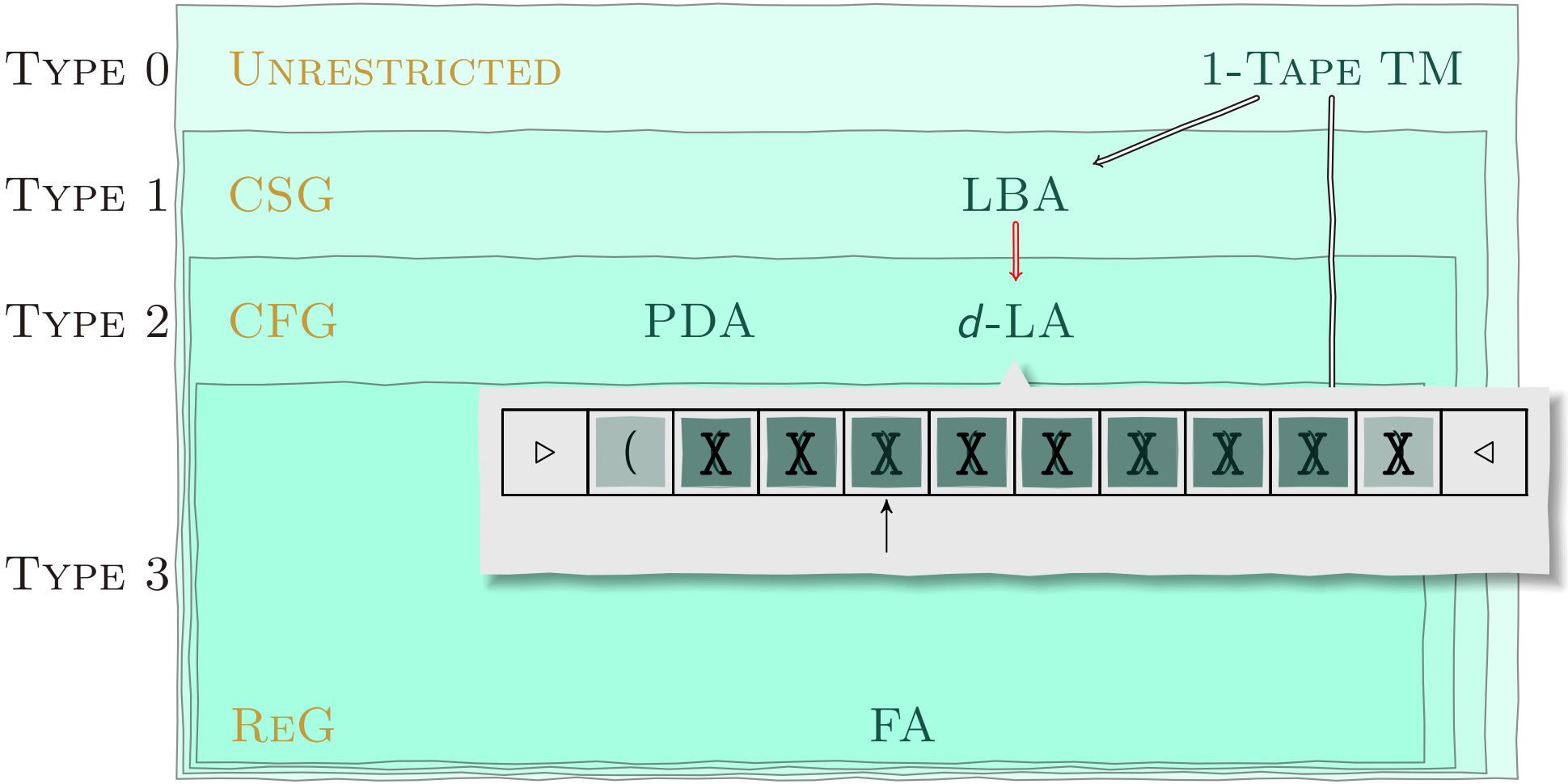
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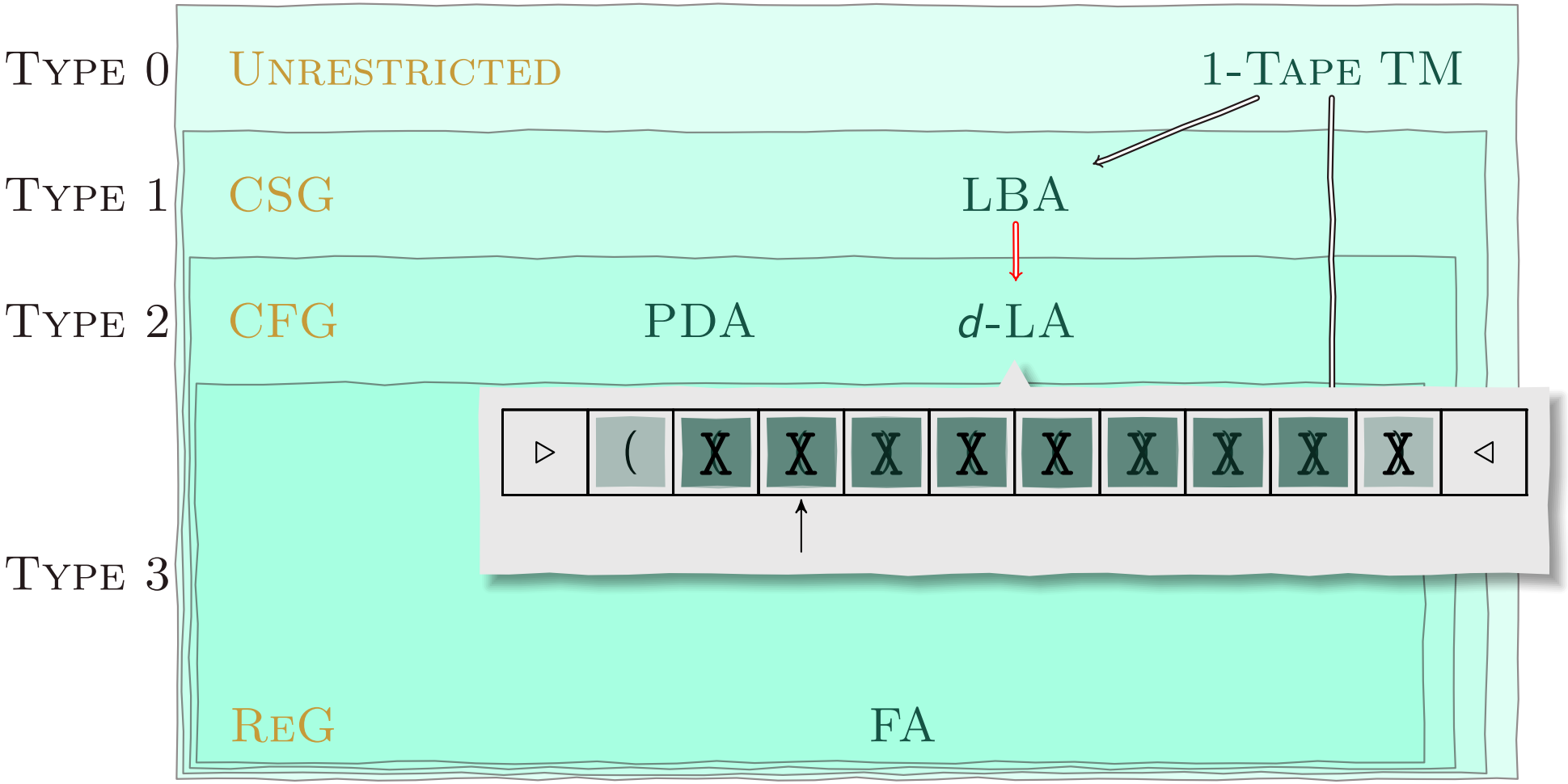
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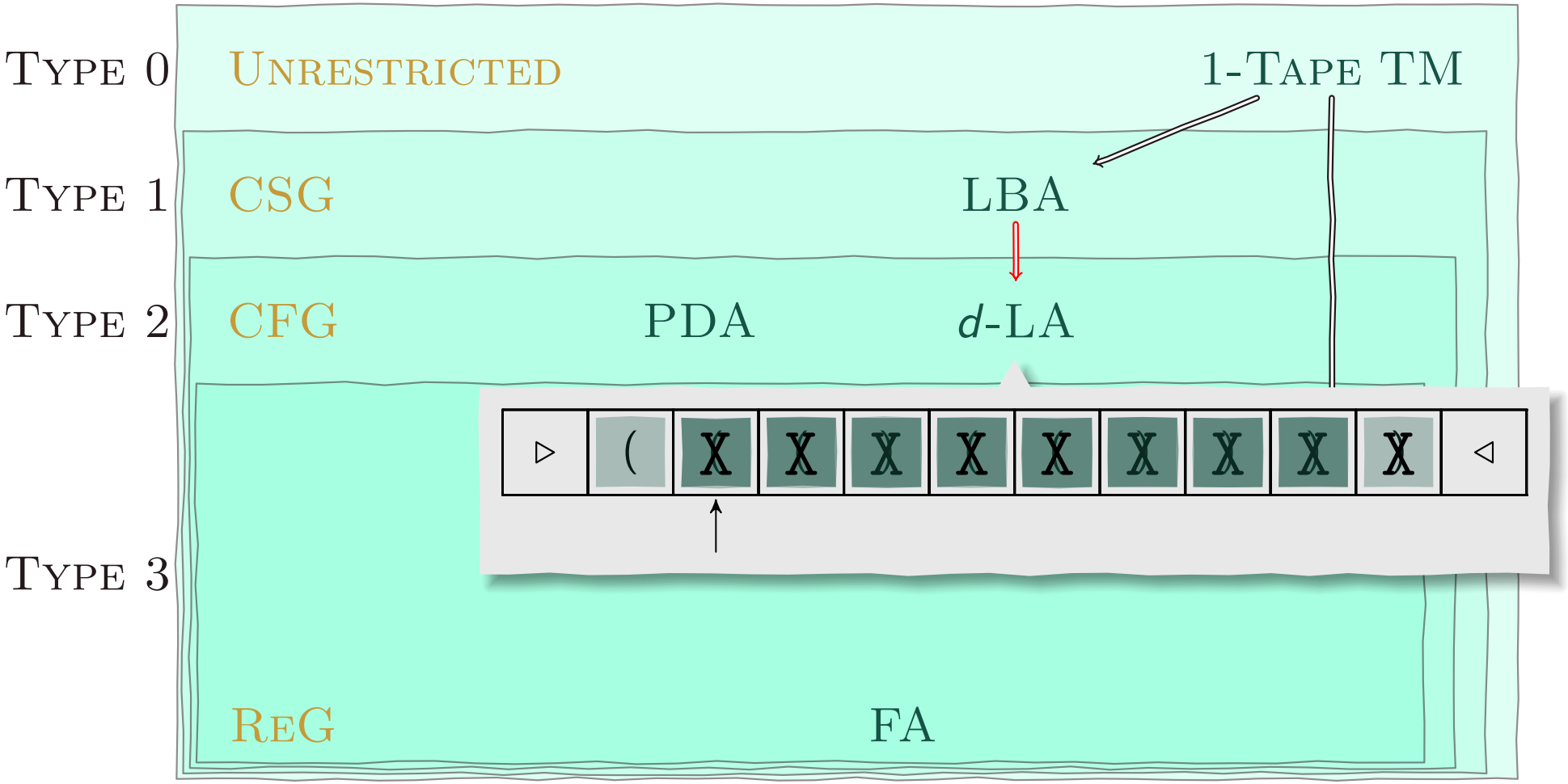
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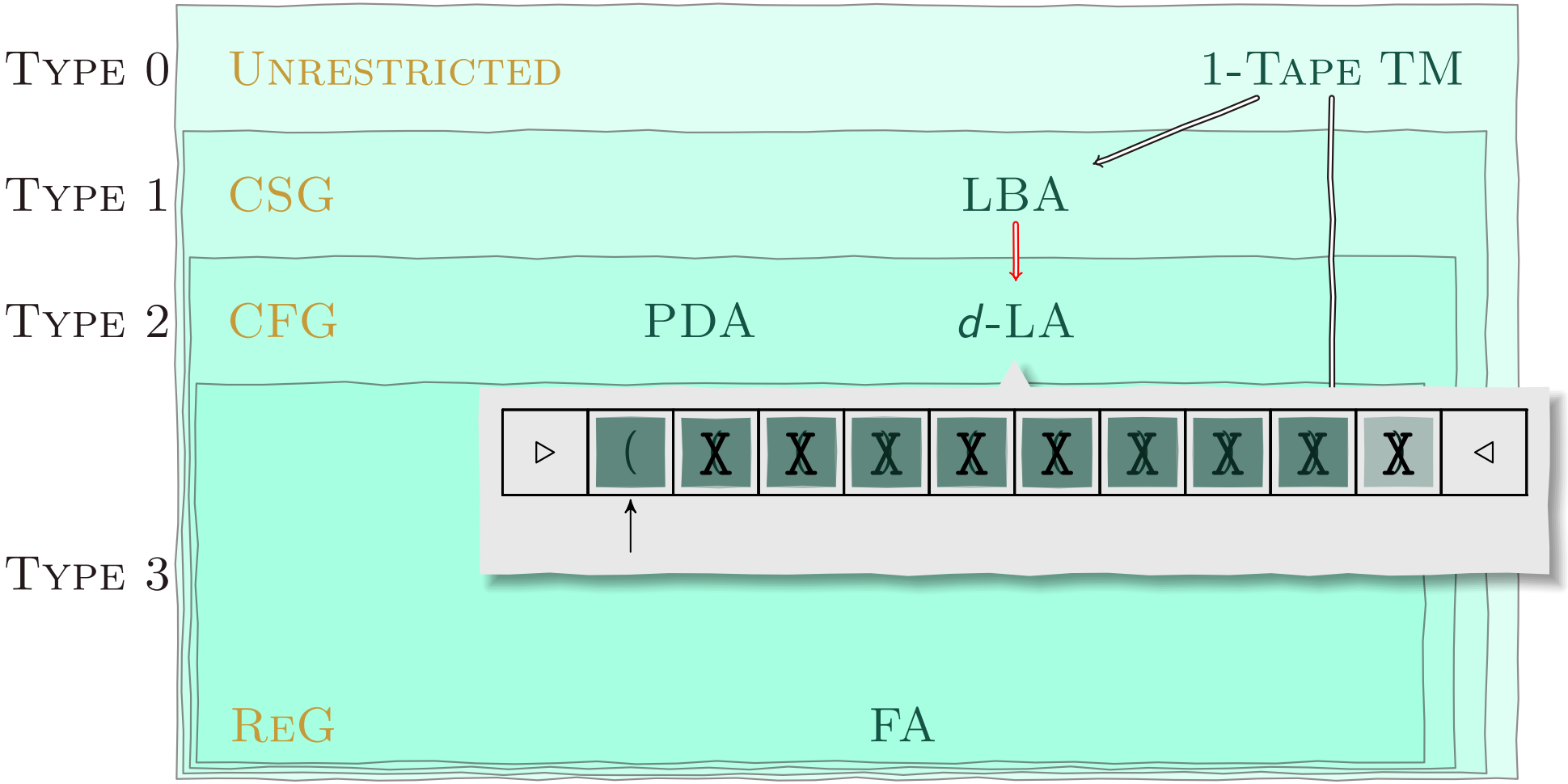
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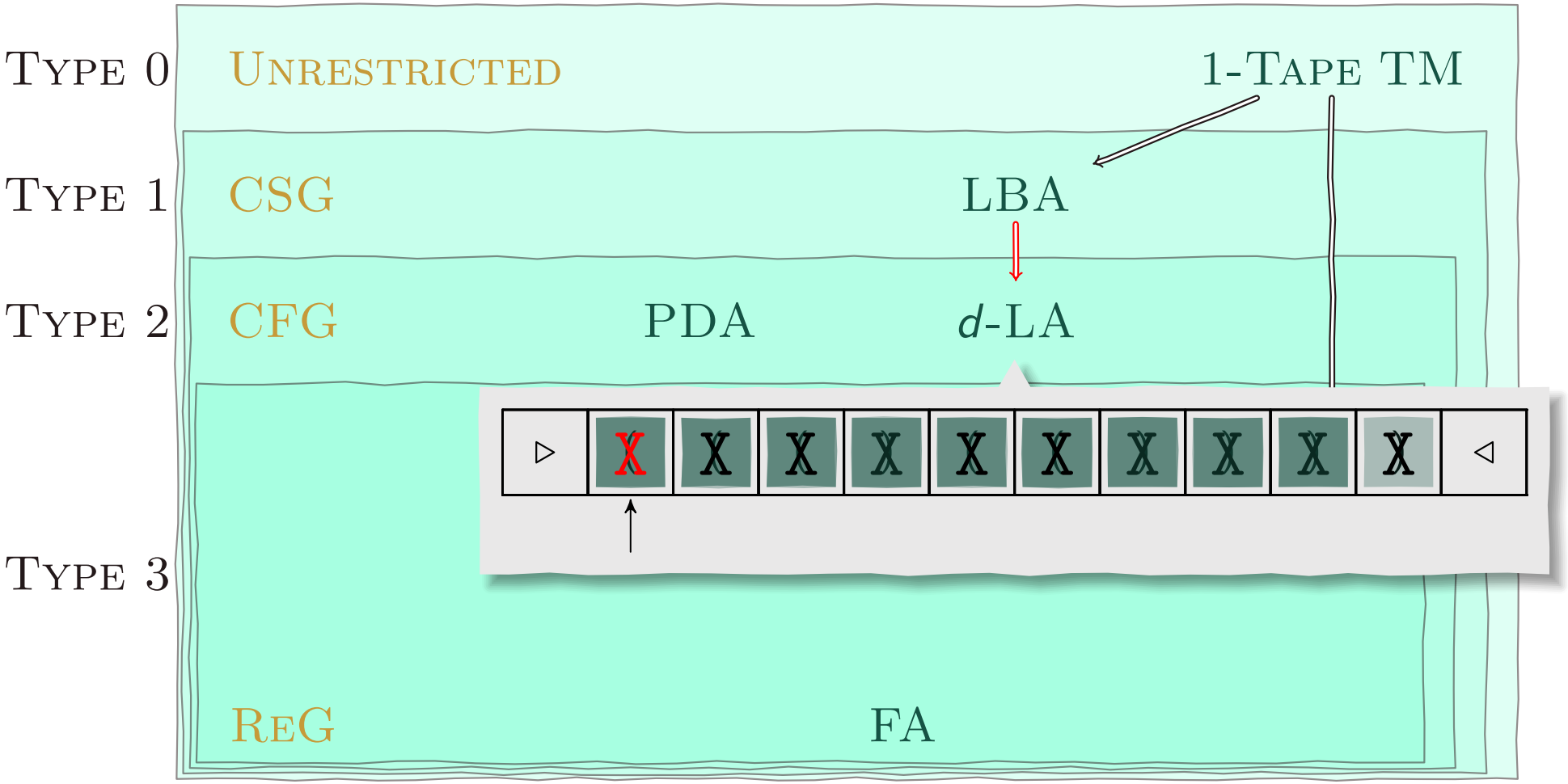
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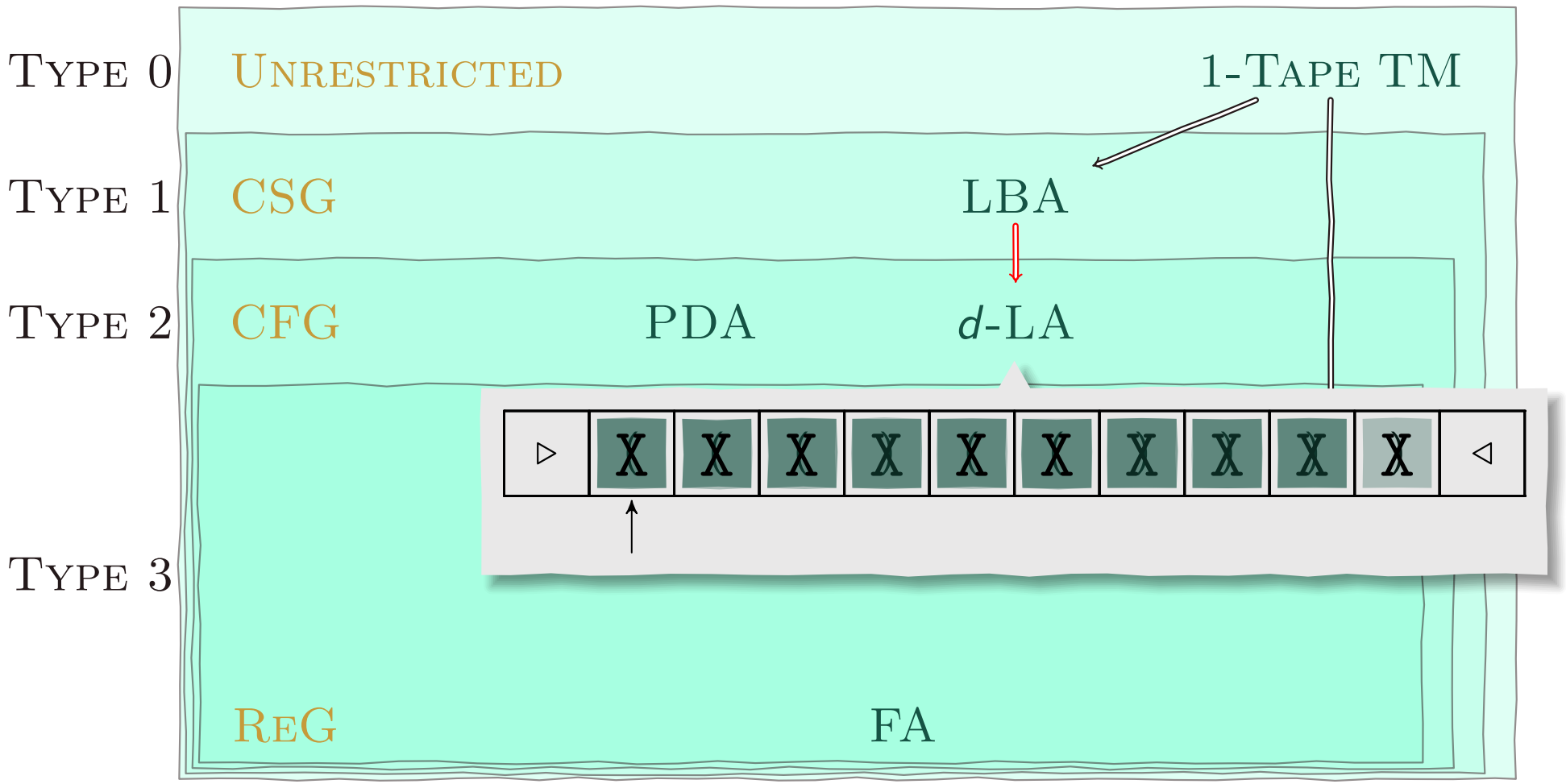
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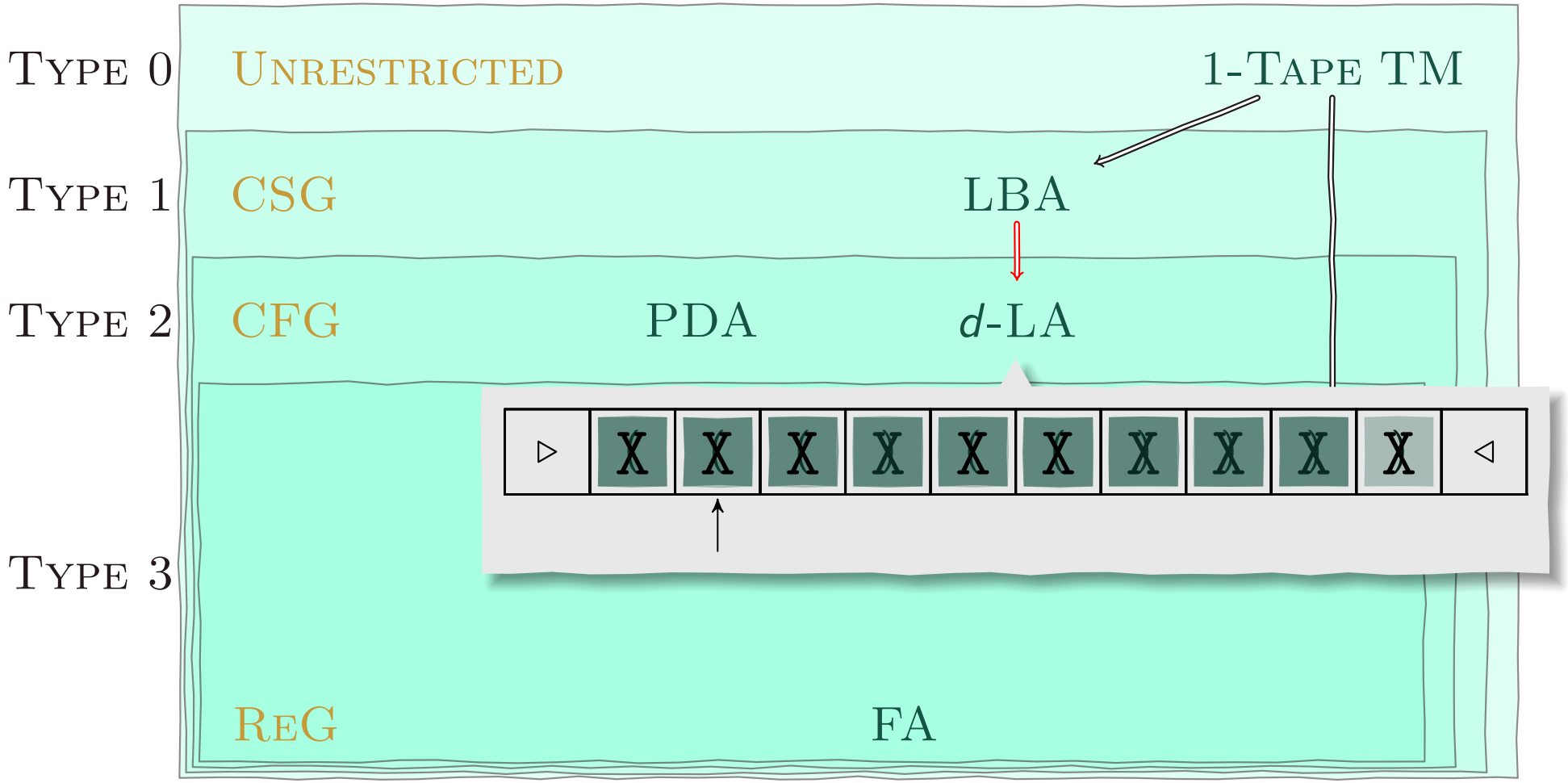
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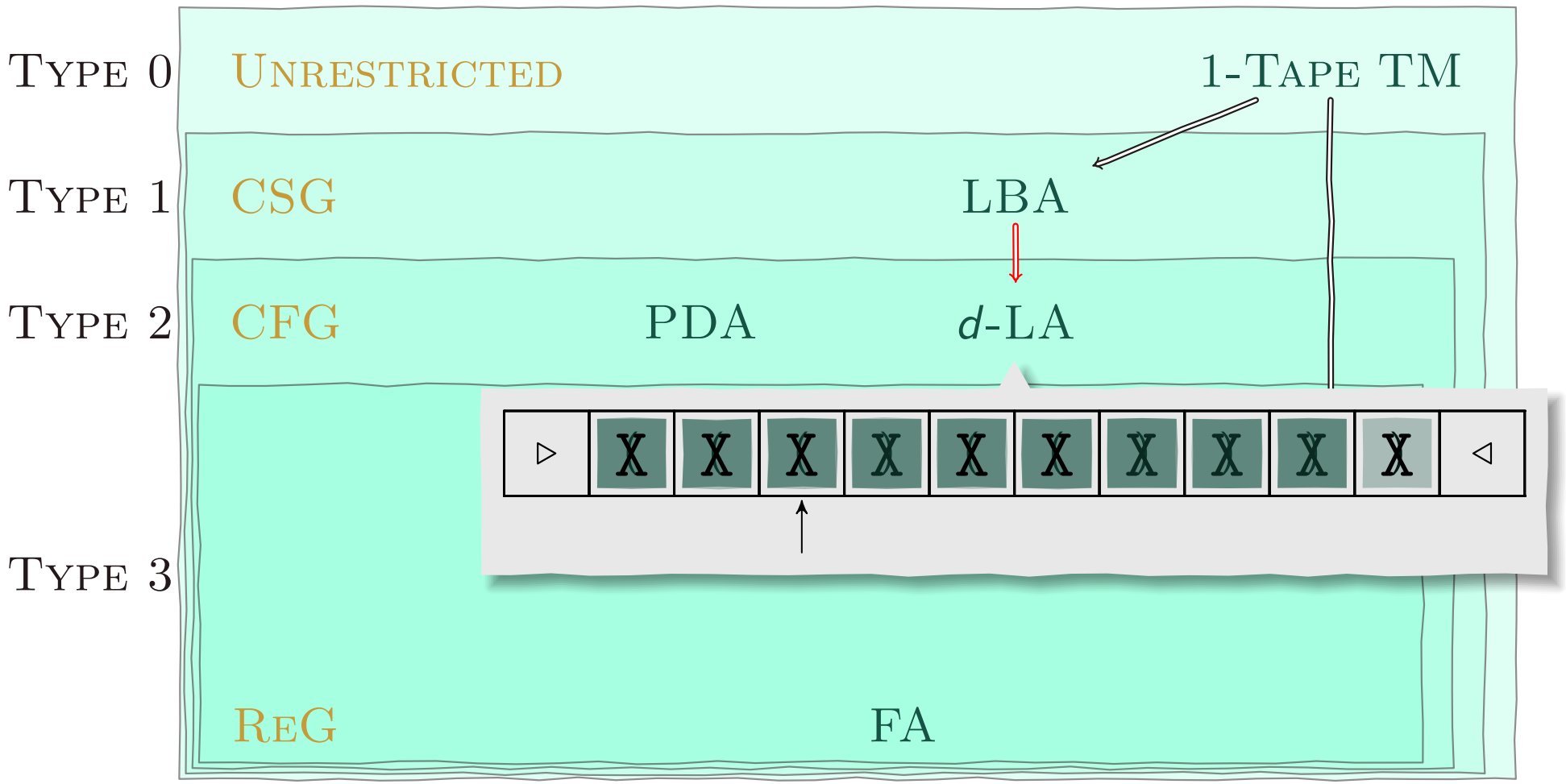
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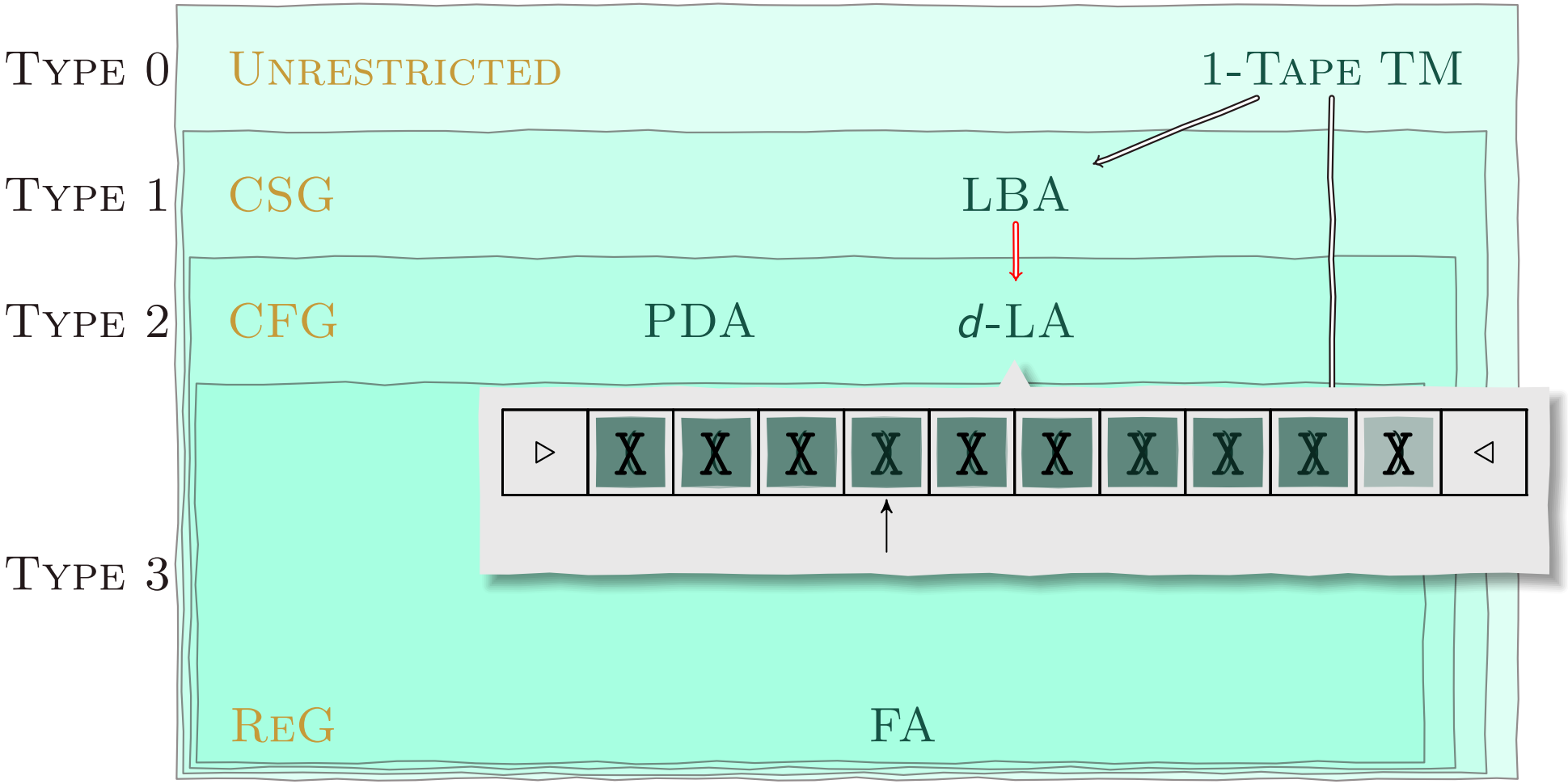
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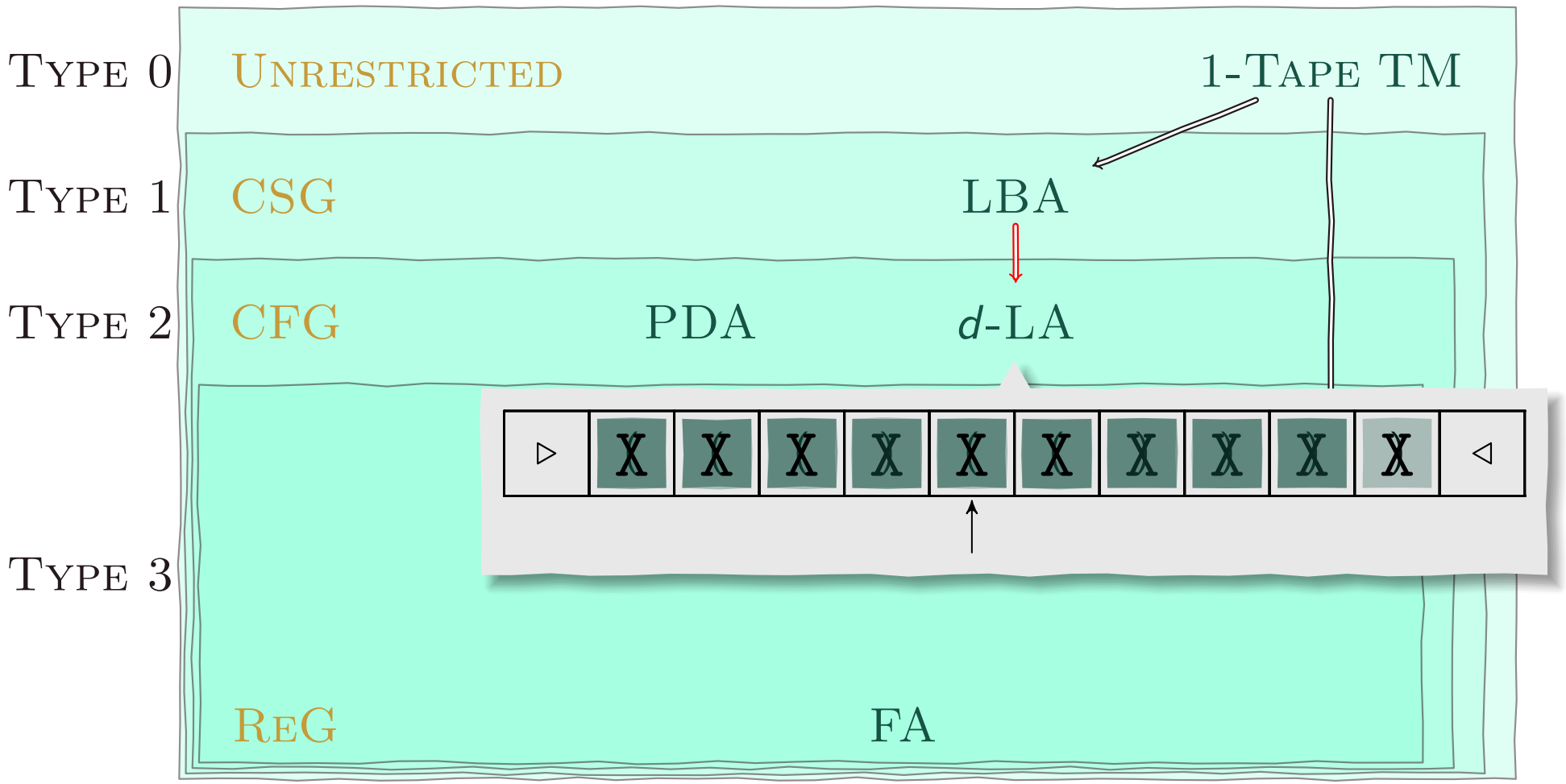
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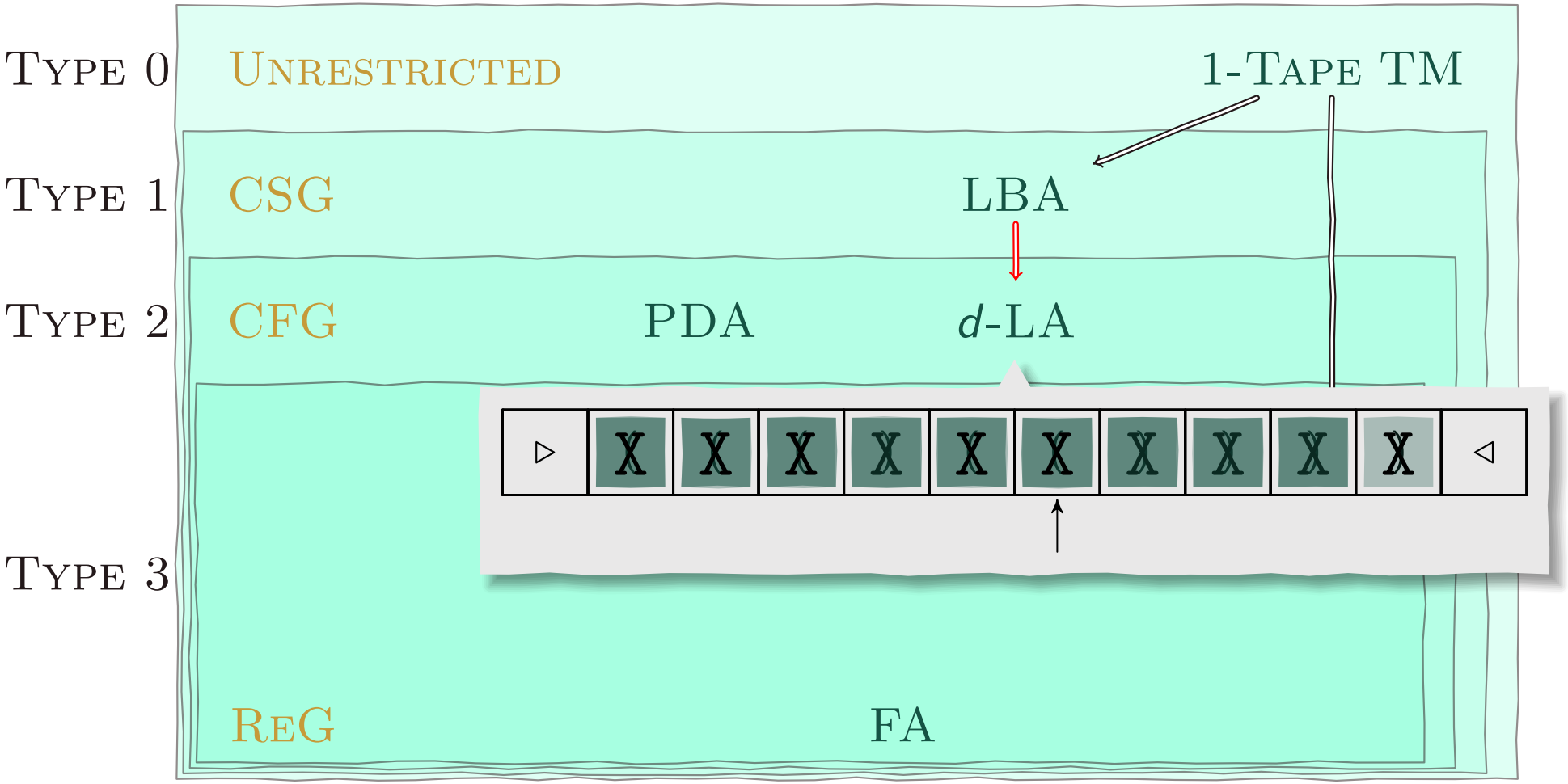
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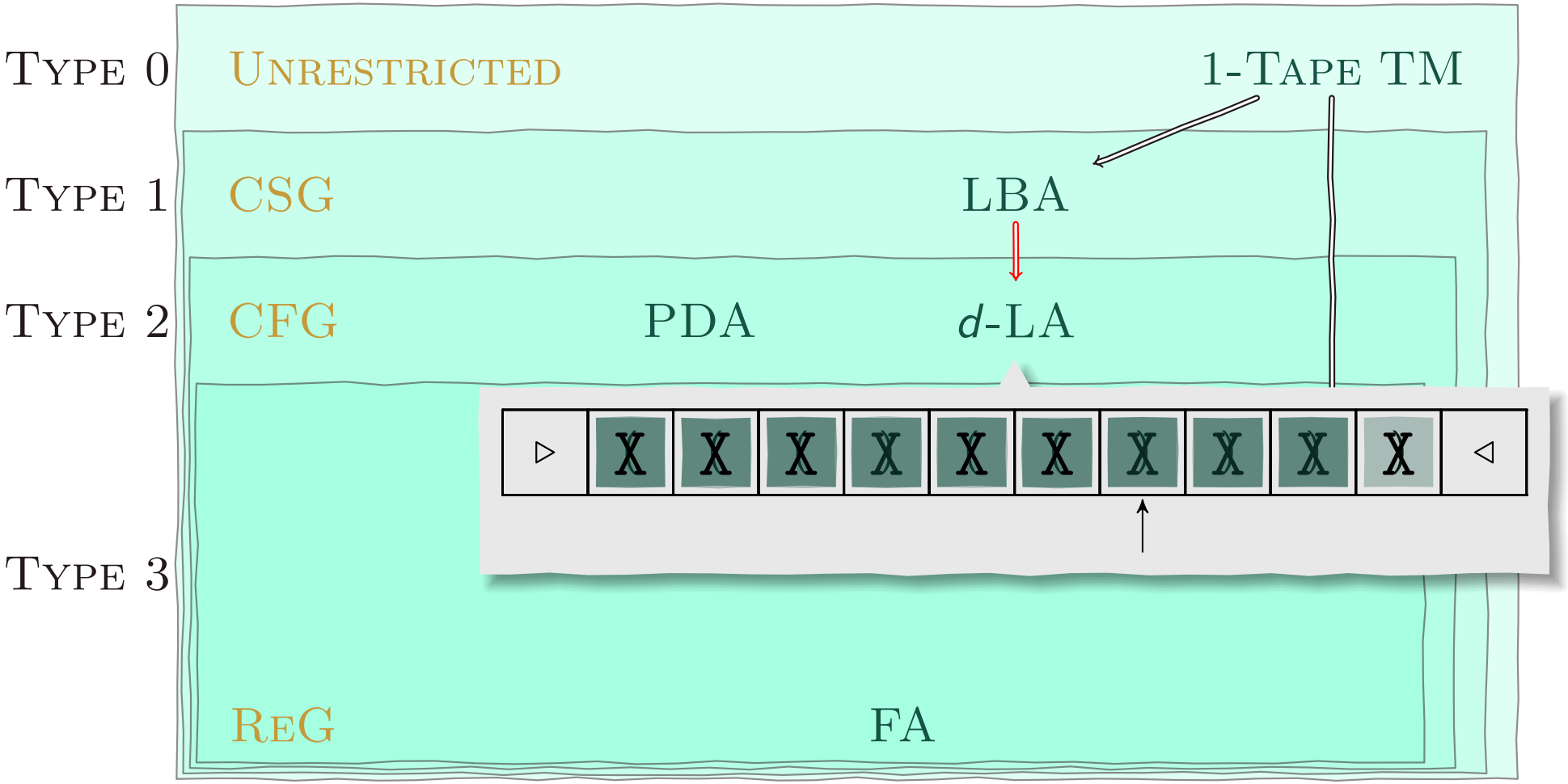
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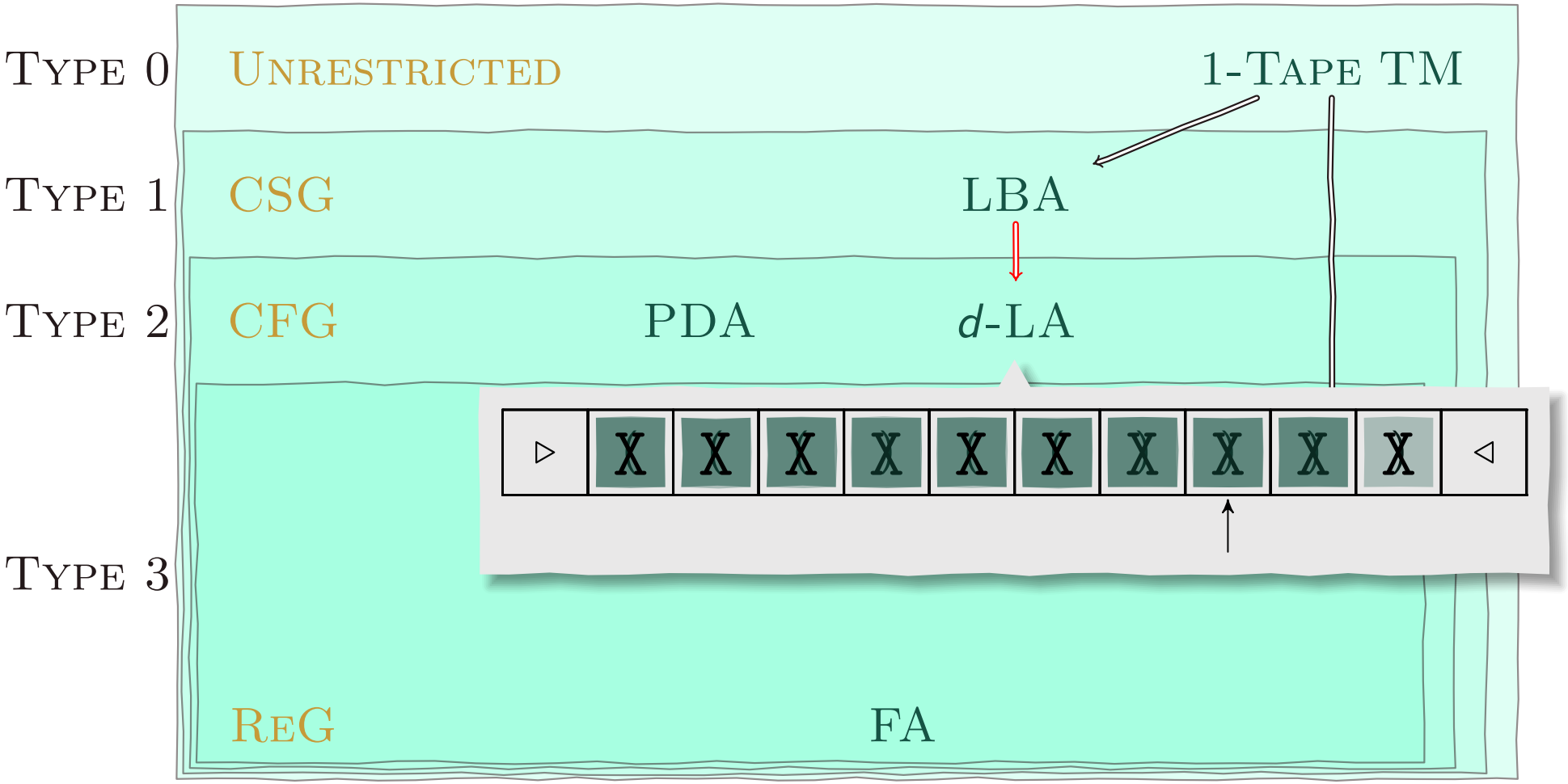
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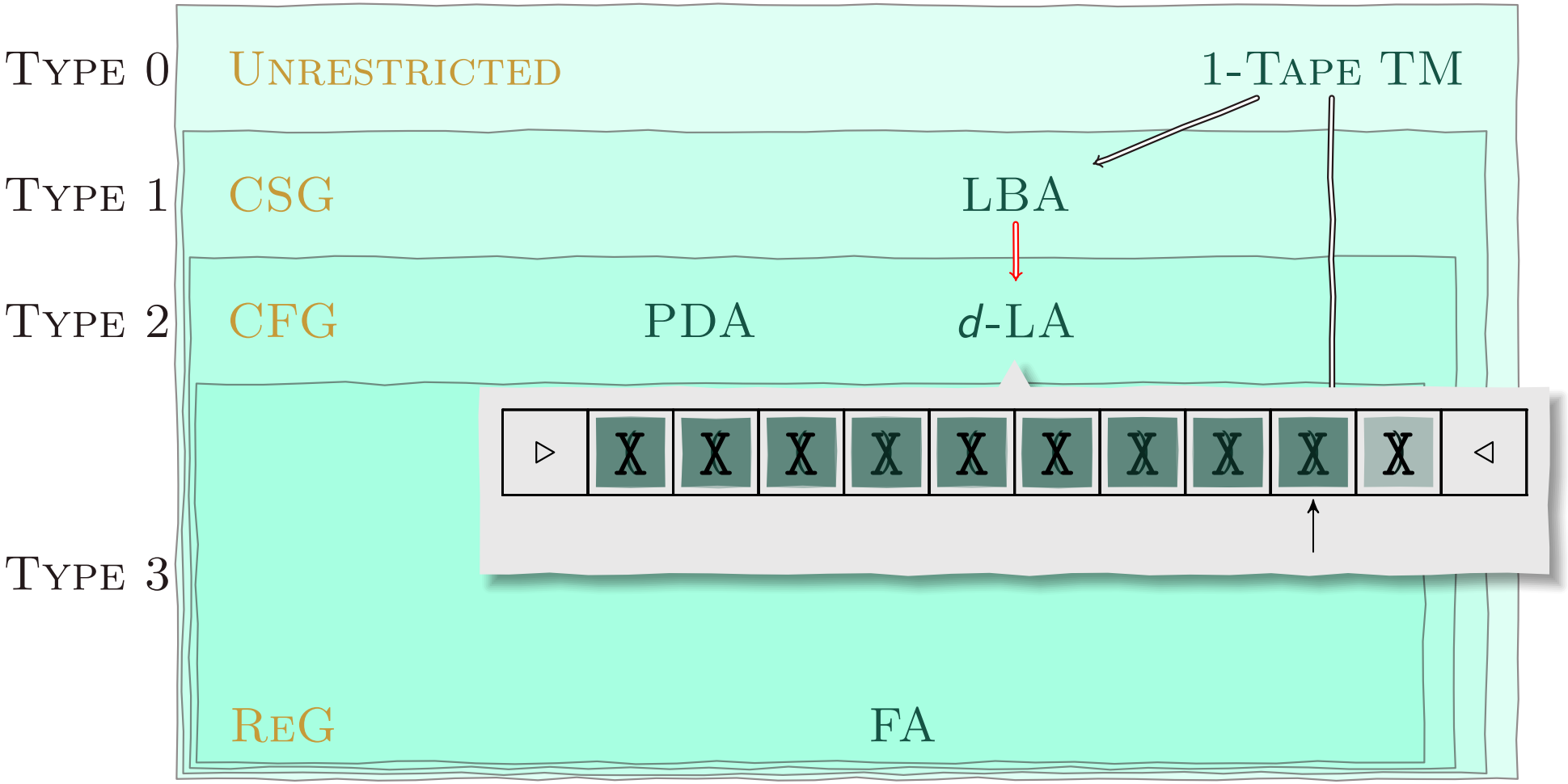
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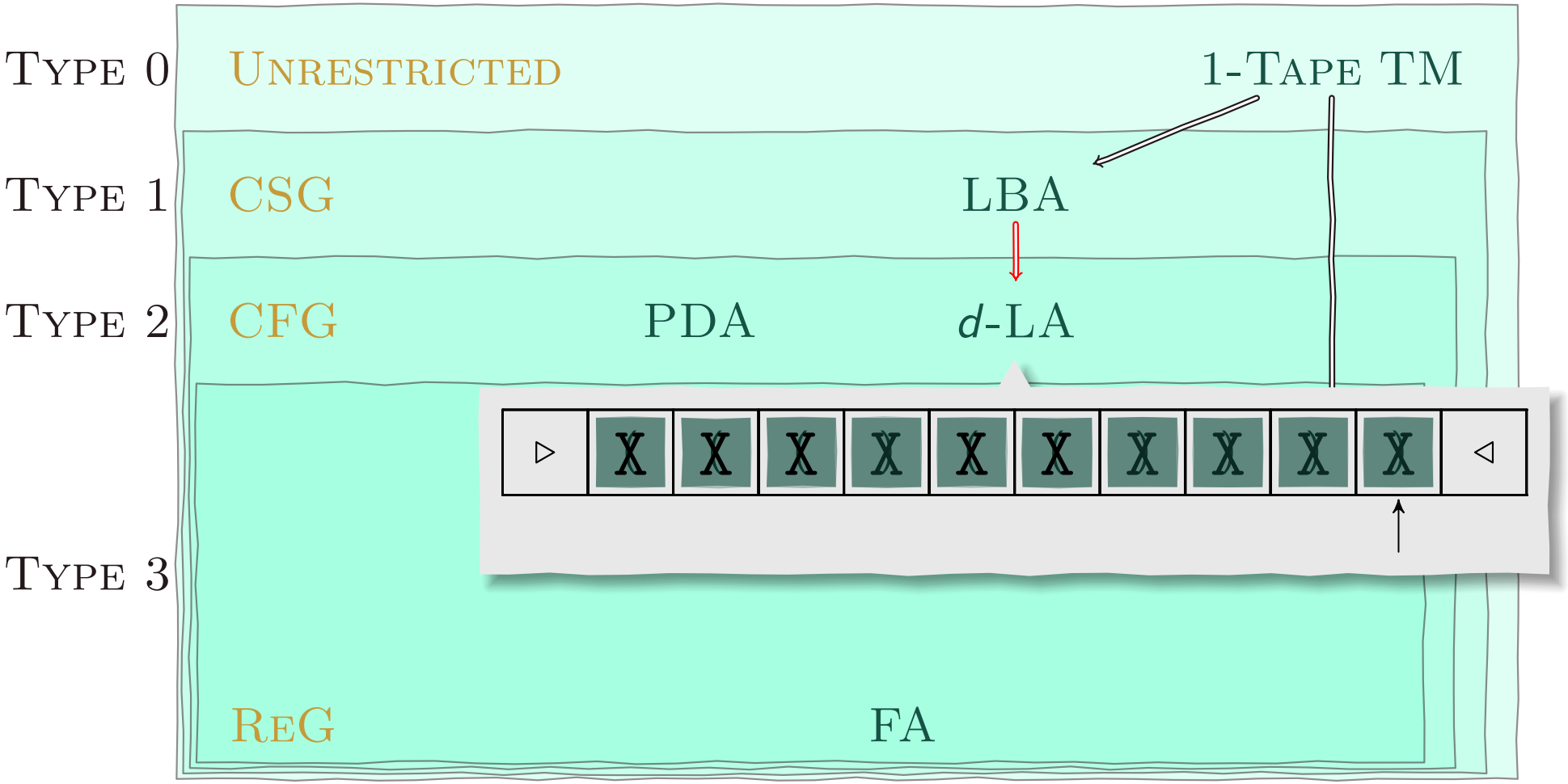
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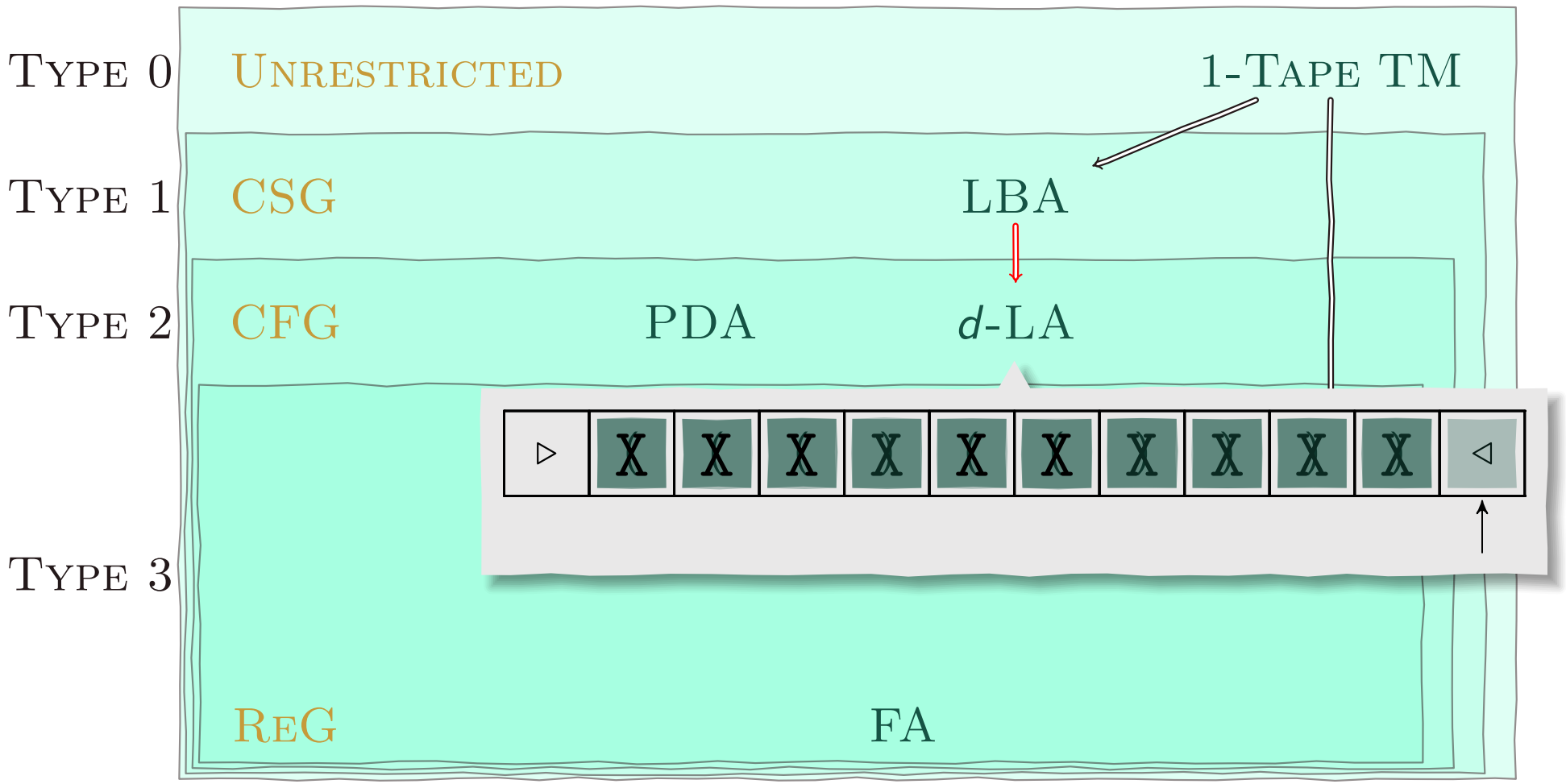
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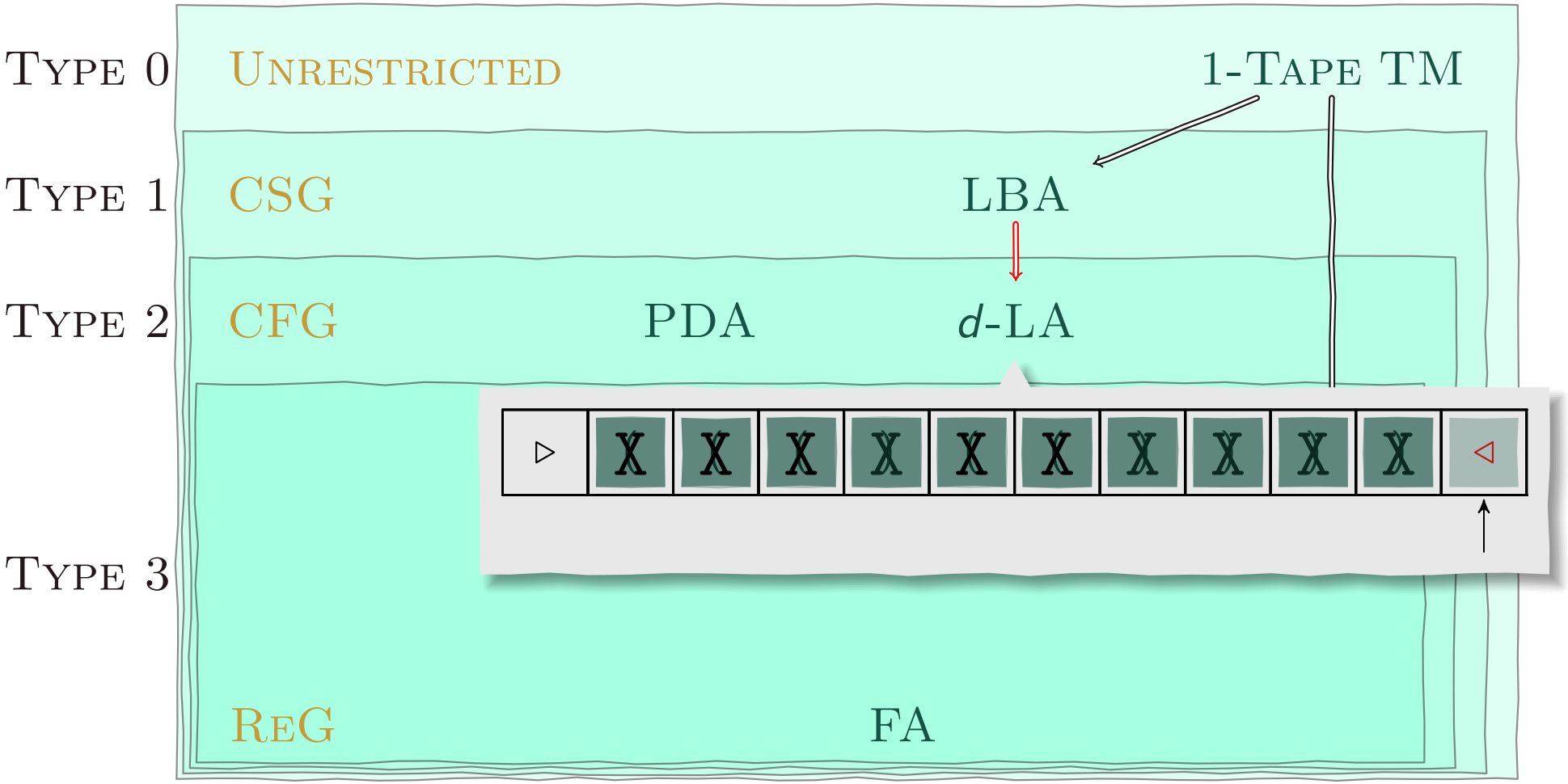
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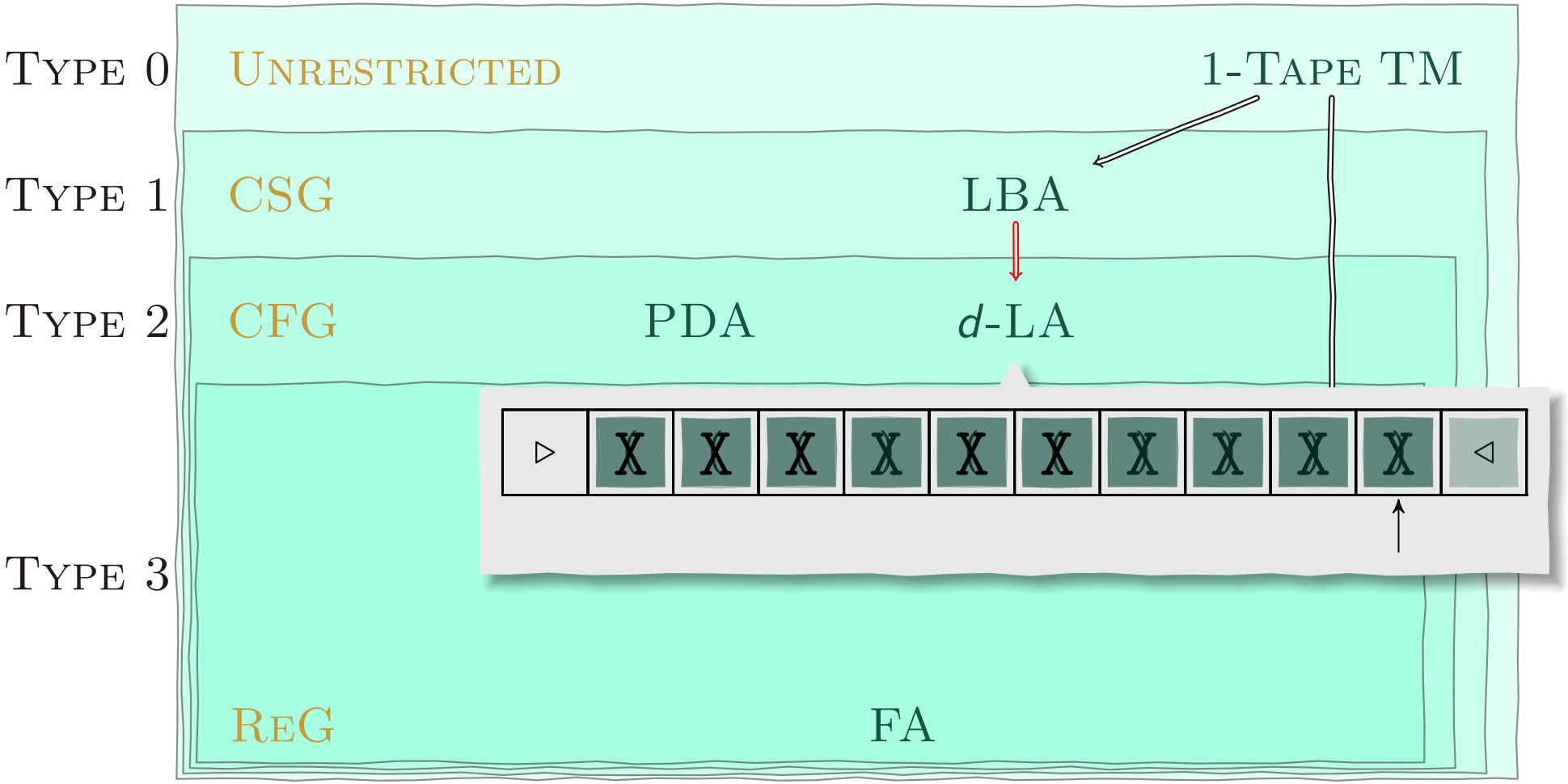
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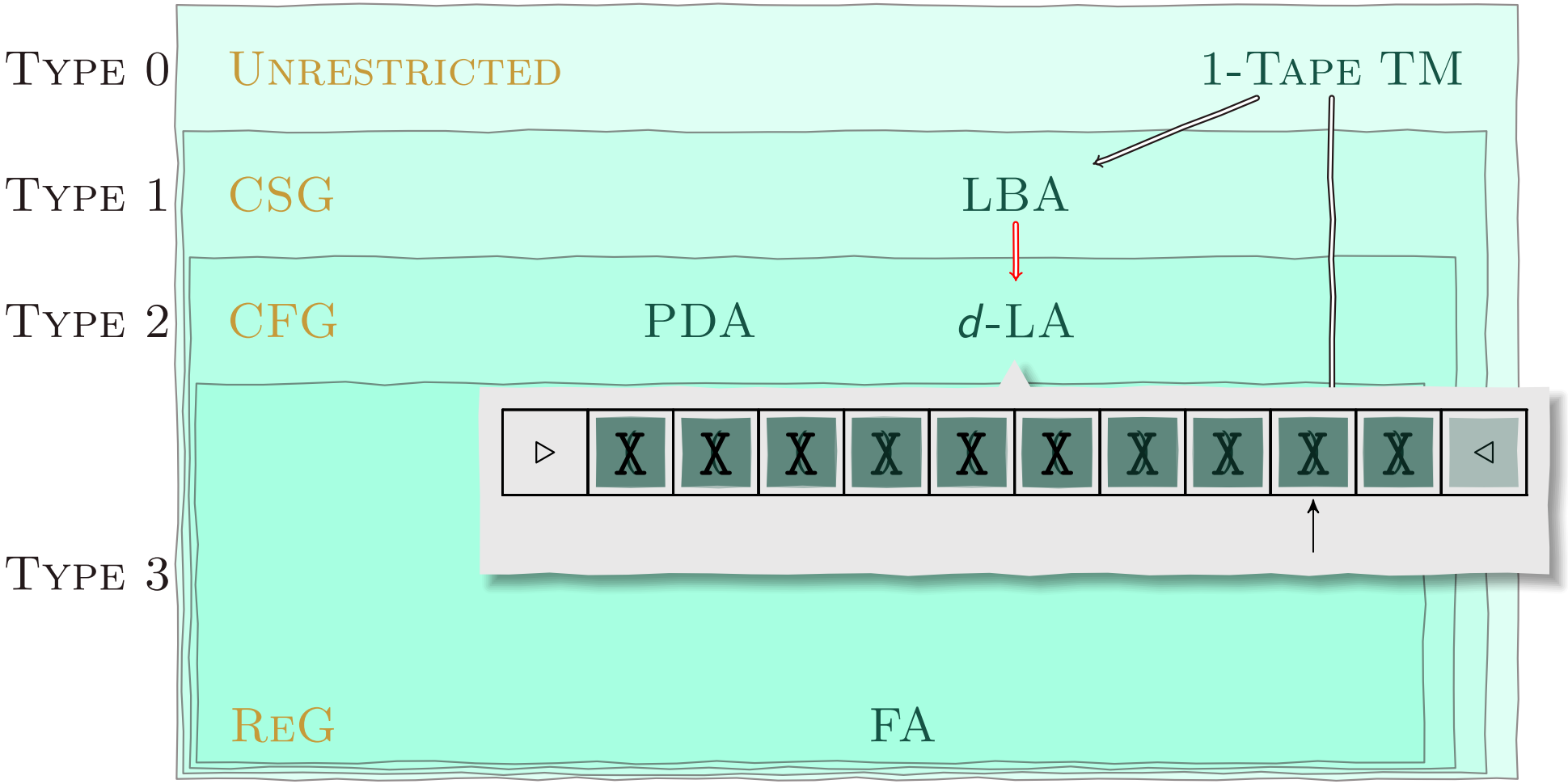
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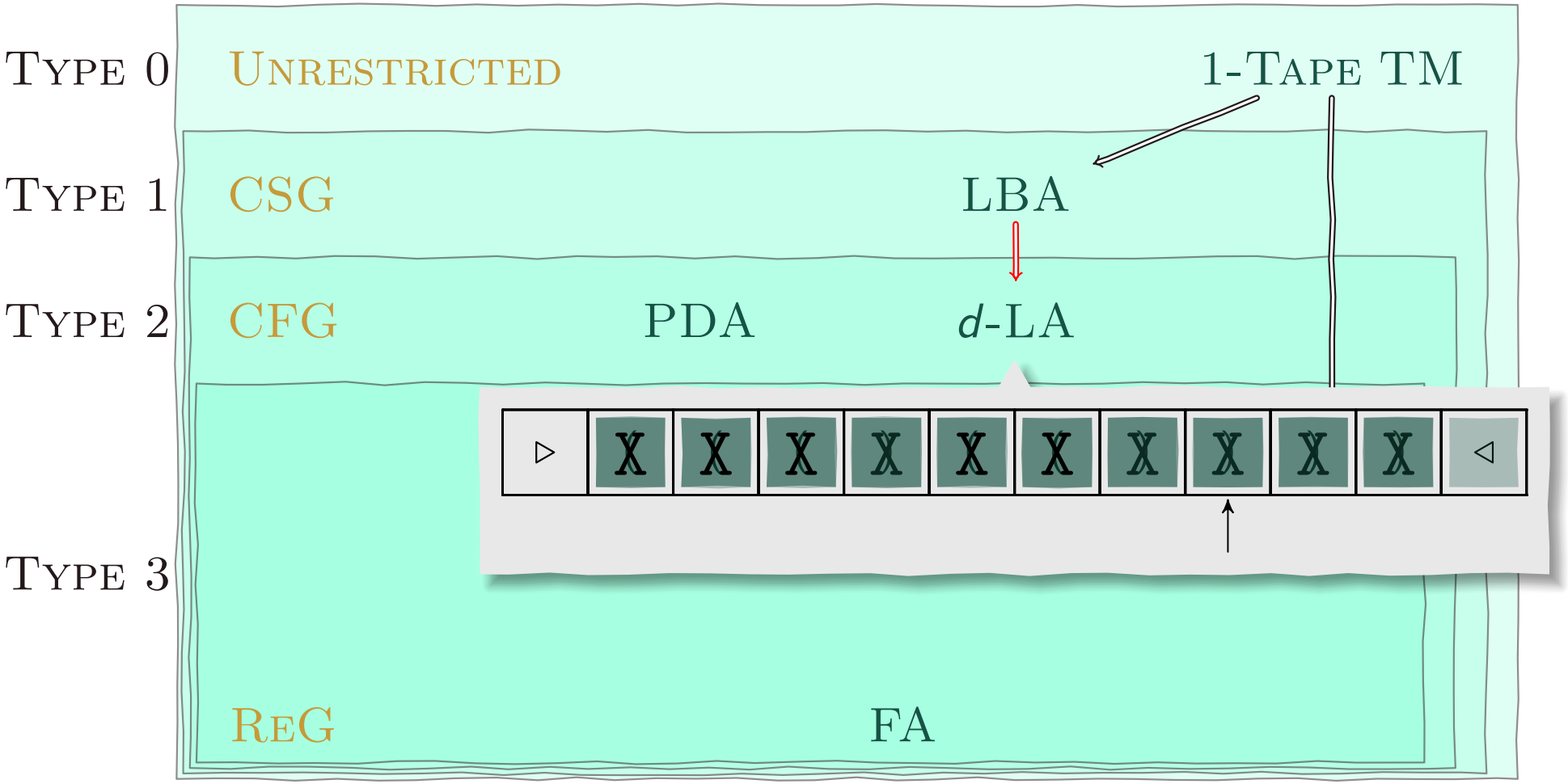
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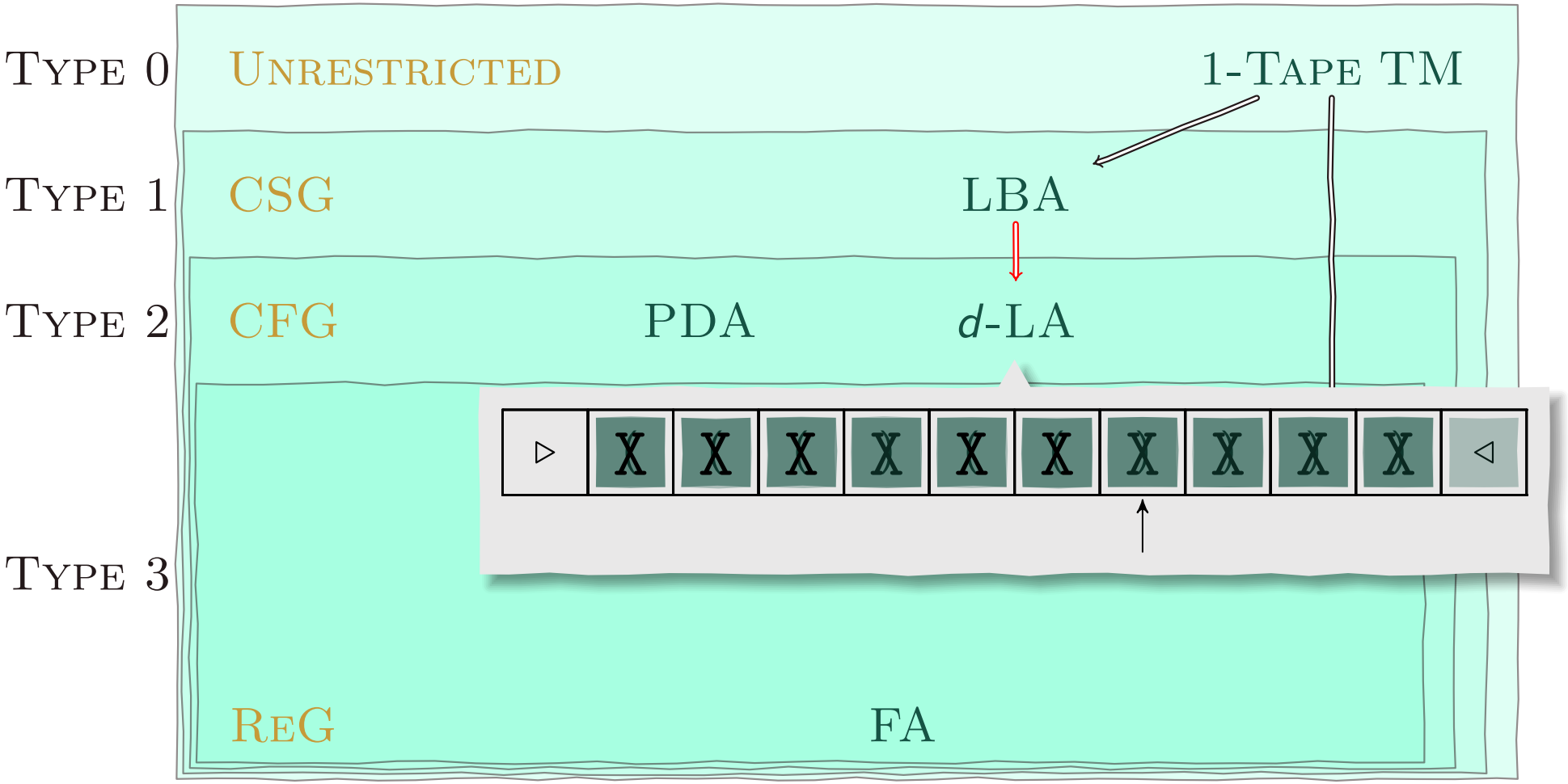
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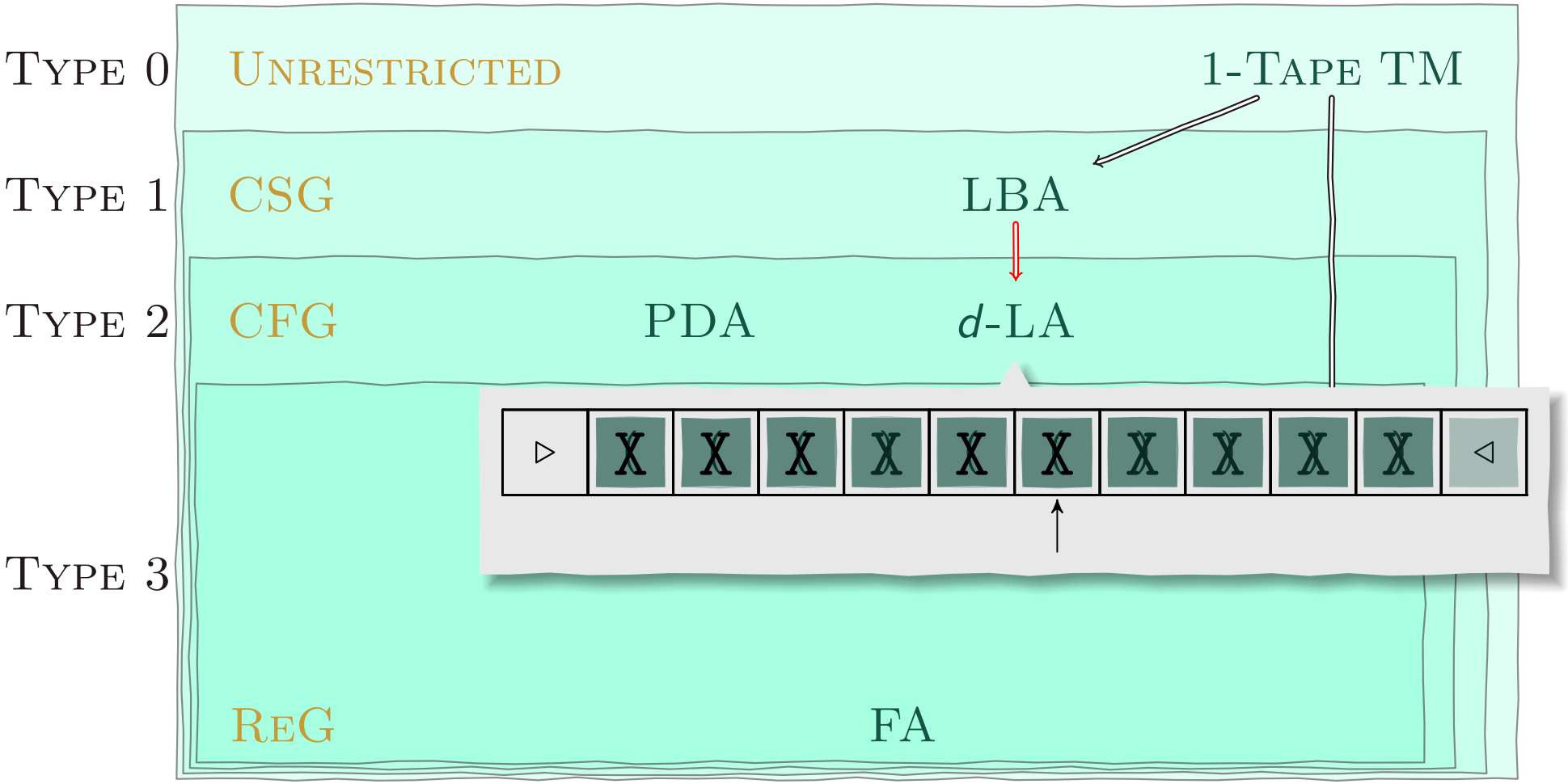
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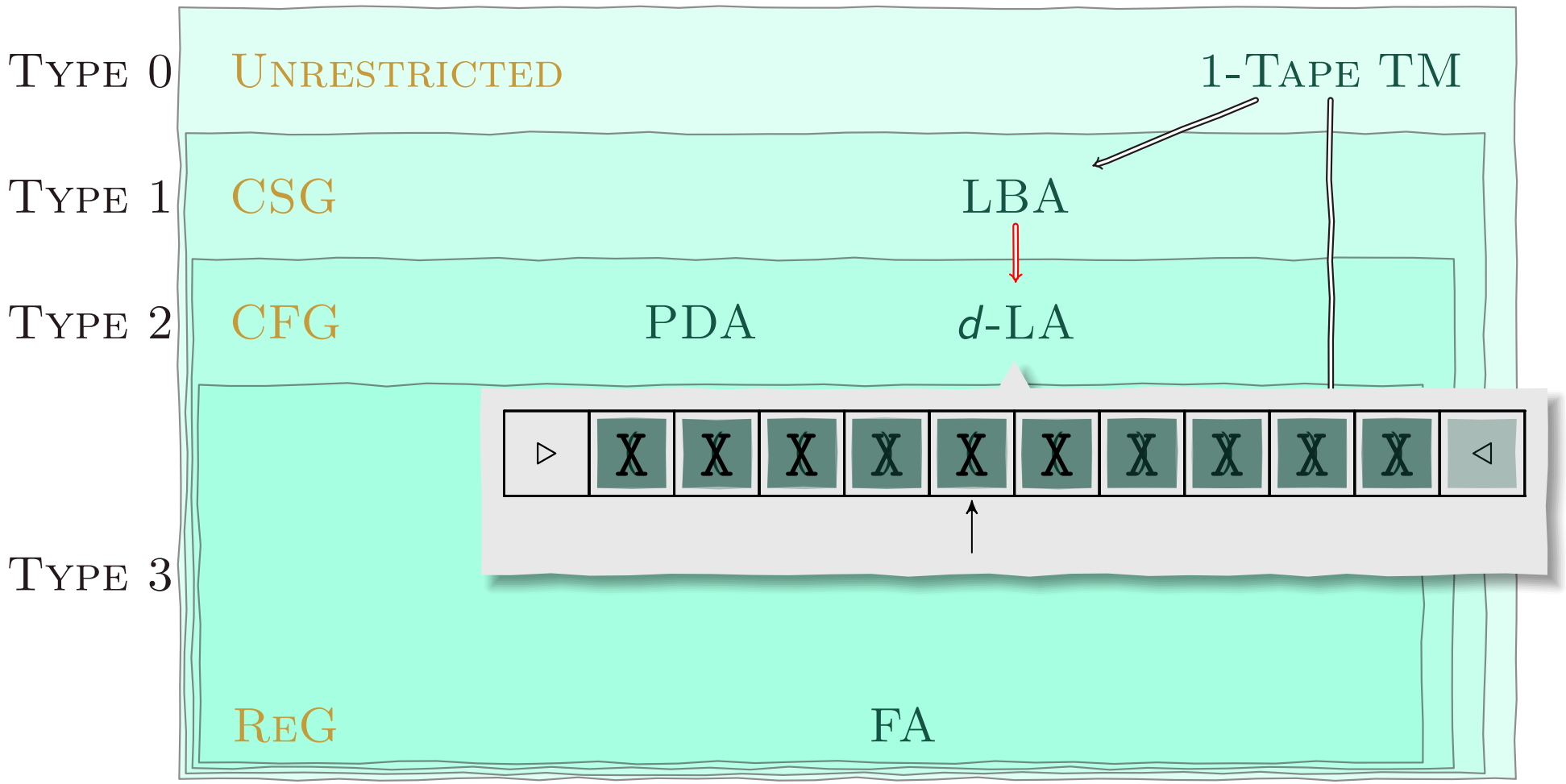
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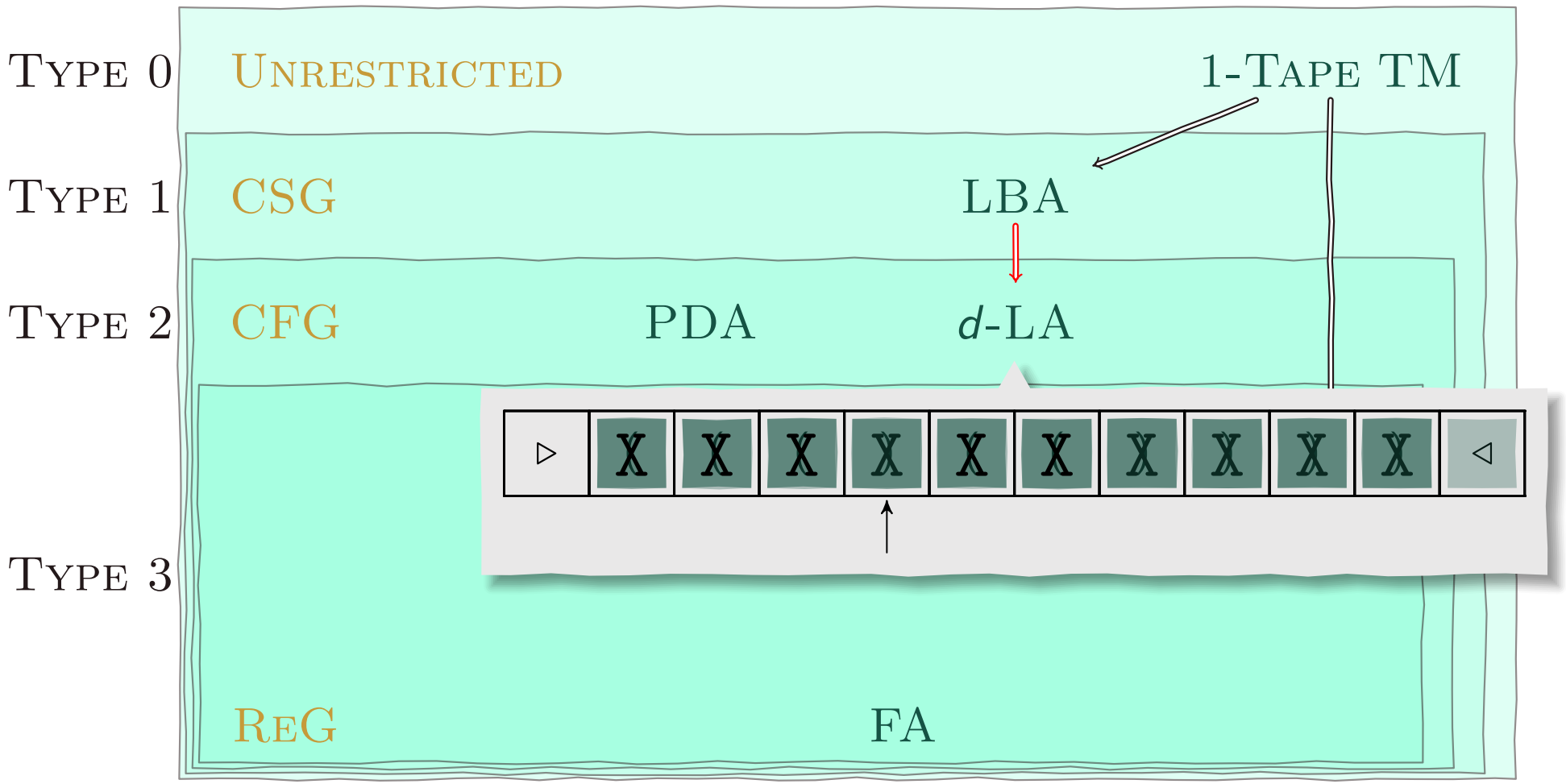
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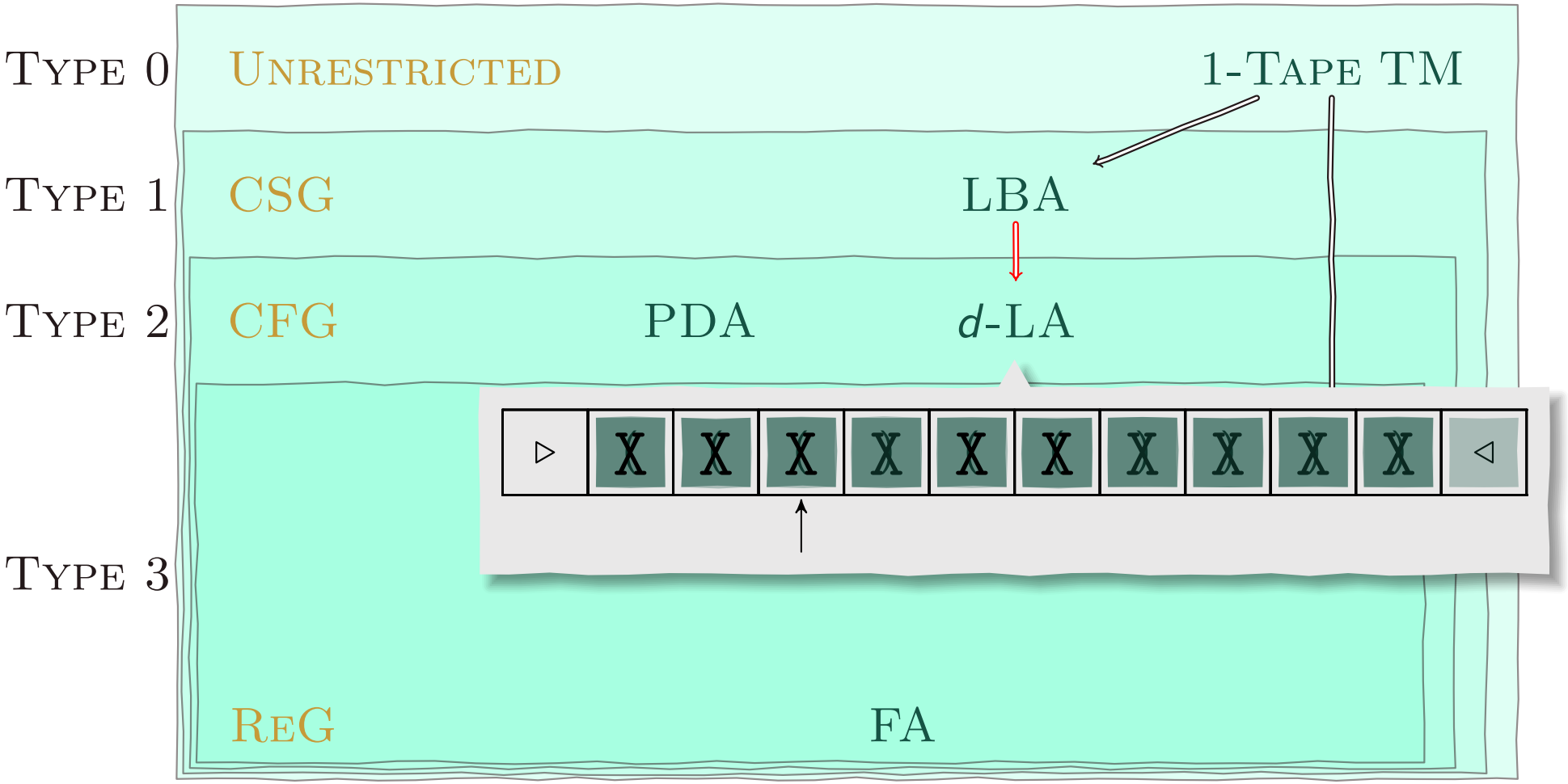
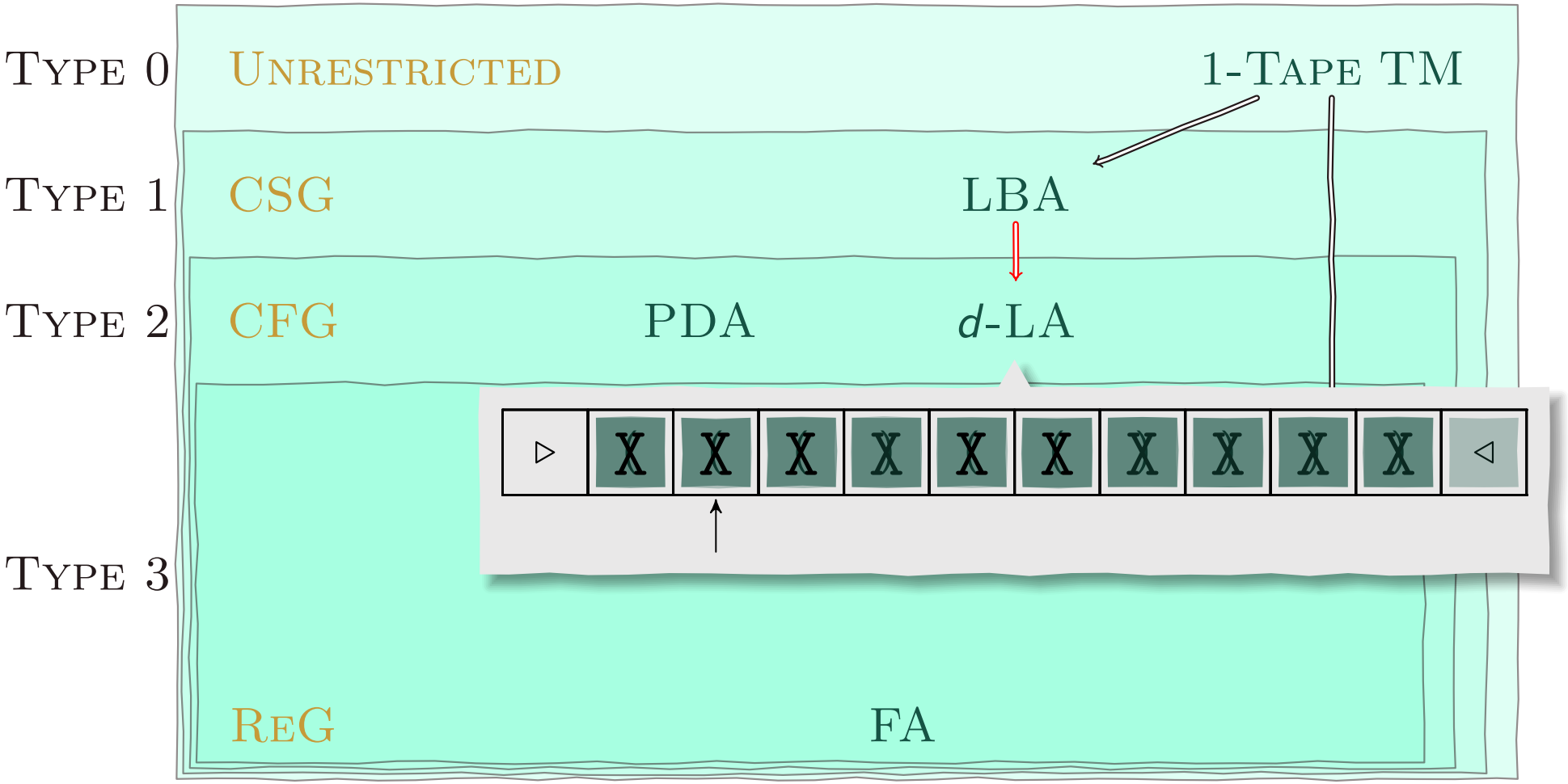

Type 0
Unrestricted
1-Tape TM
Type 1
CSG
LBA
Type 2
CFG
PDA
d-LA
Type 3
ReG
FA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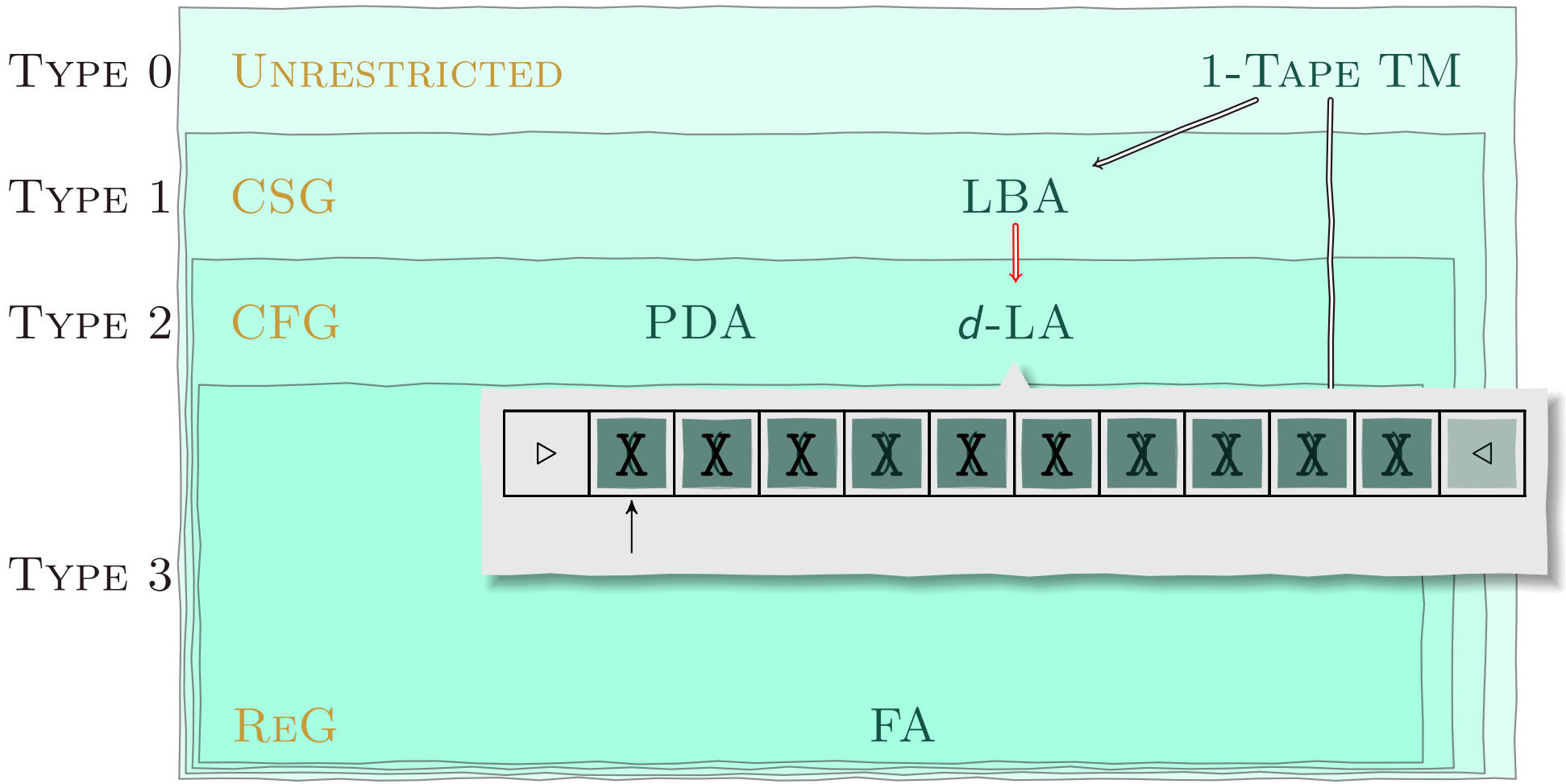
Unrestricted
1-Tape TM
Type 1
CSG
LBA
Type 2
CFG
PDA
d-LA
Type 3
ReG
FA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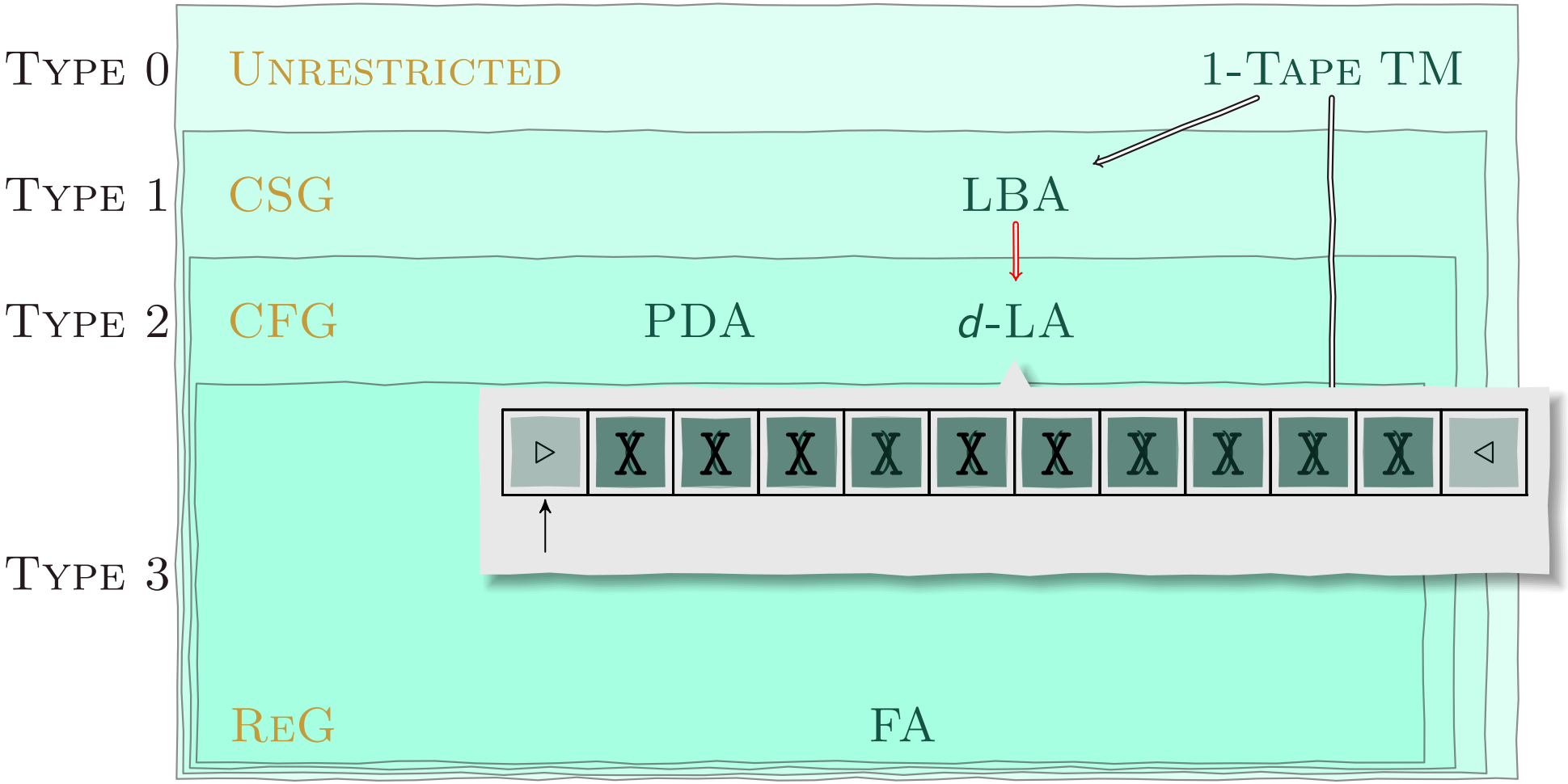
Unrestricted
1-Tape TM
Type 1
CSG
LBA
Type 2
CFG
PDA
d-LA
Type 3
ReG
FA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Unrestricted
1-Tape TM
Type 1
CSG
LBA
Type 2
CFG
PDA
d-LA
Type 3
ReG
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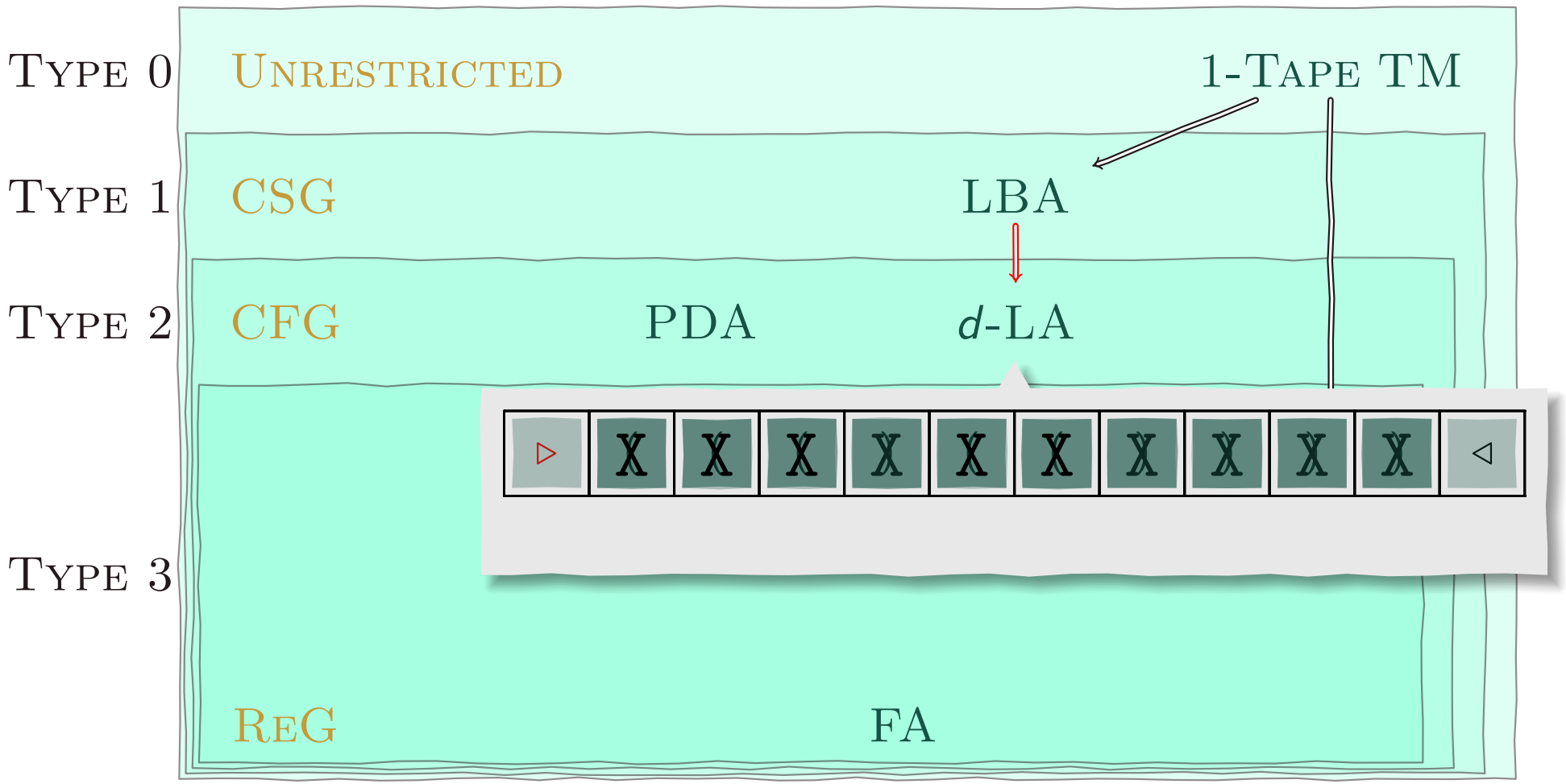
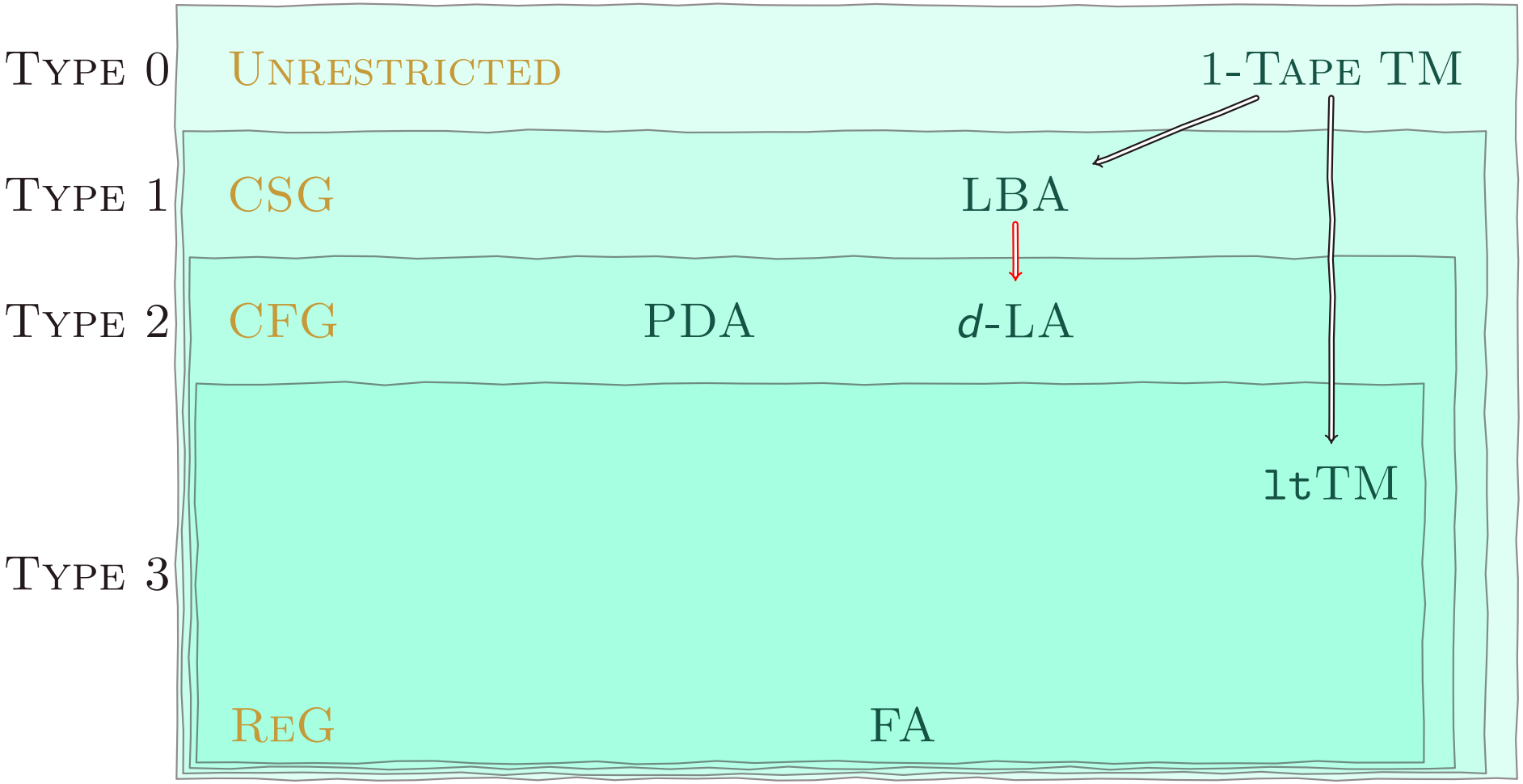

Type 0
Unrestricted
1-Tape TM
Type 1
CSG
LBA
Type 2
CFG
PDA
d-LA
Type 3
ReG
FA


Type 0
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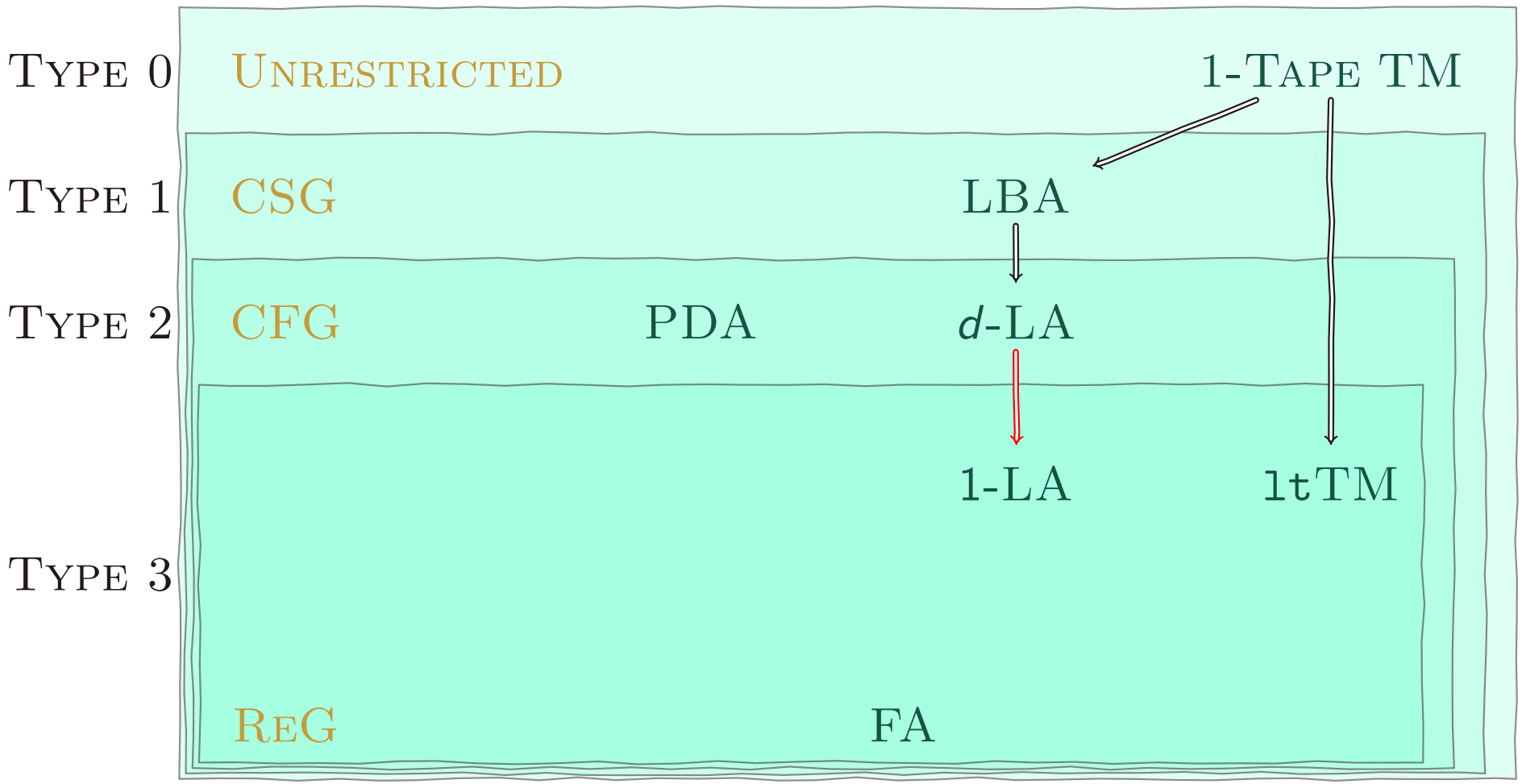
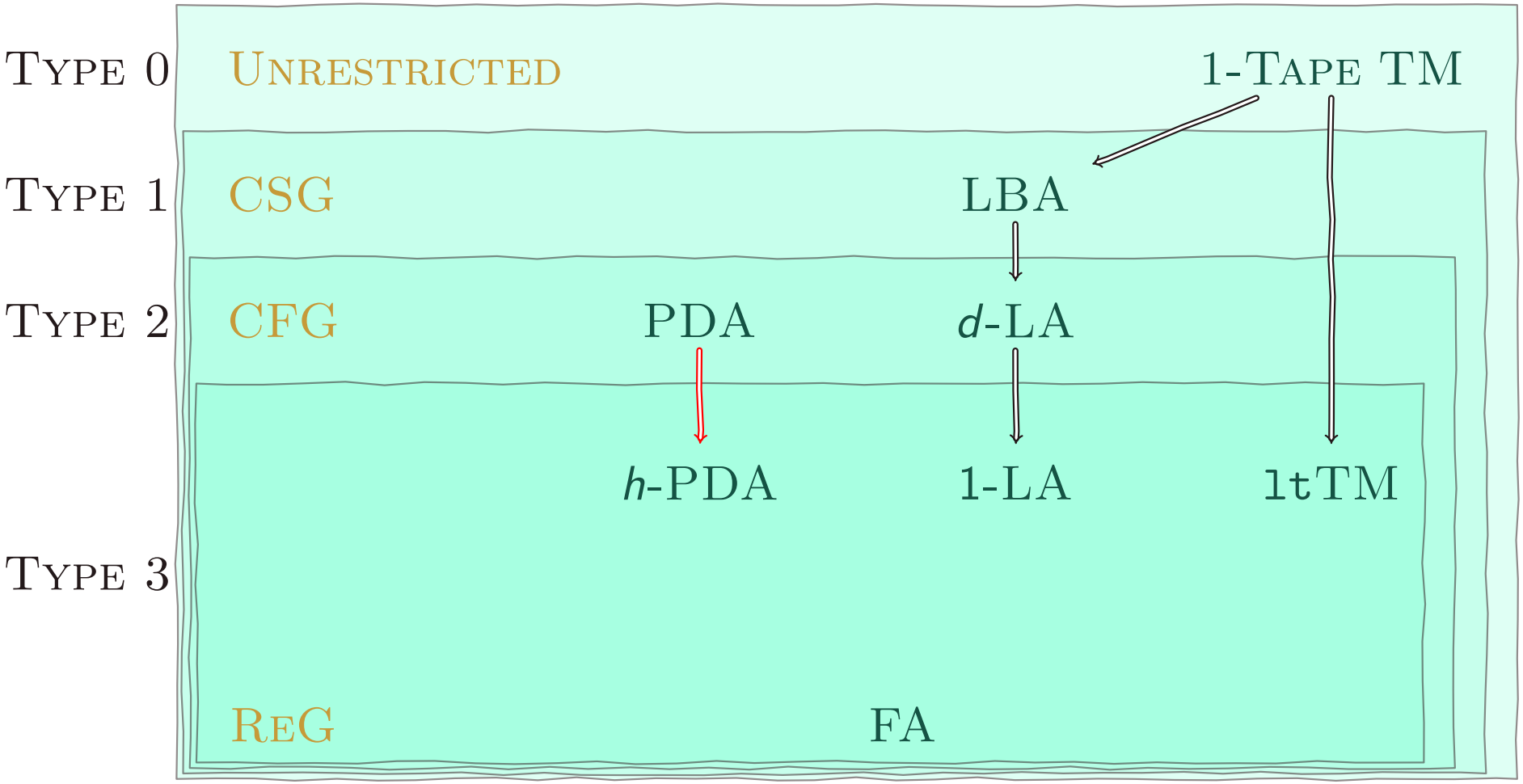
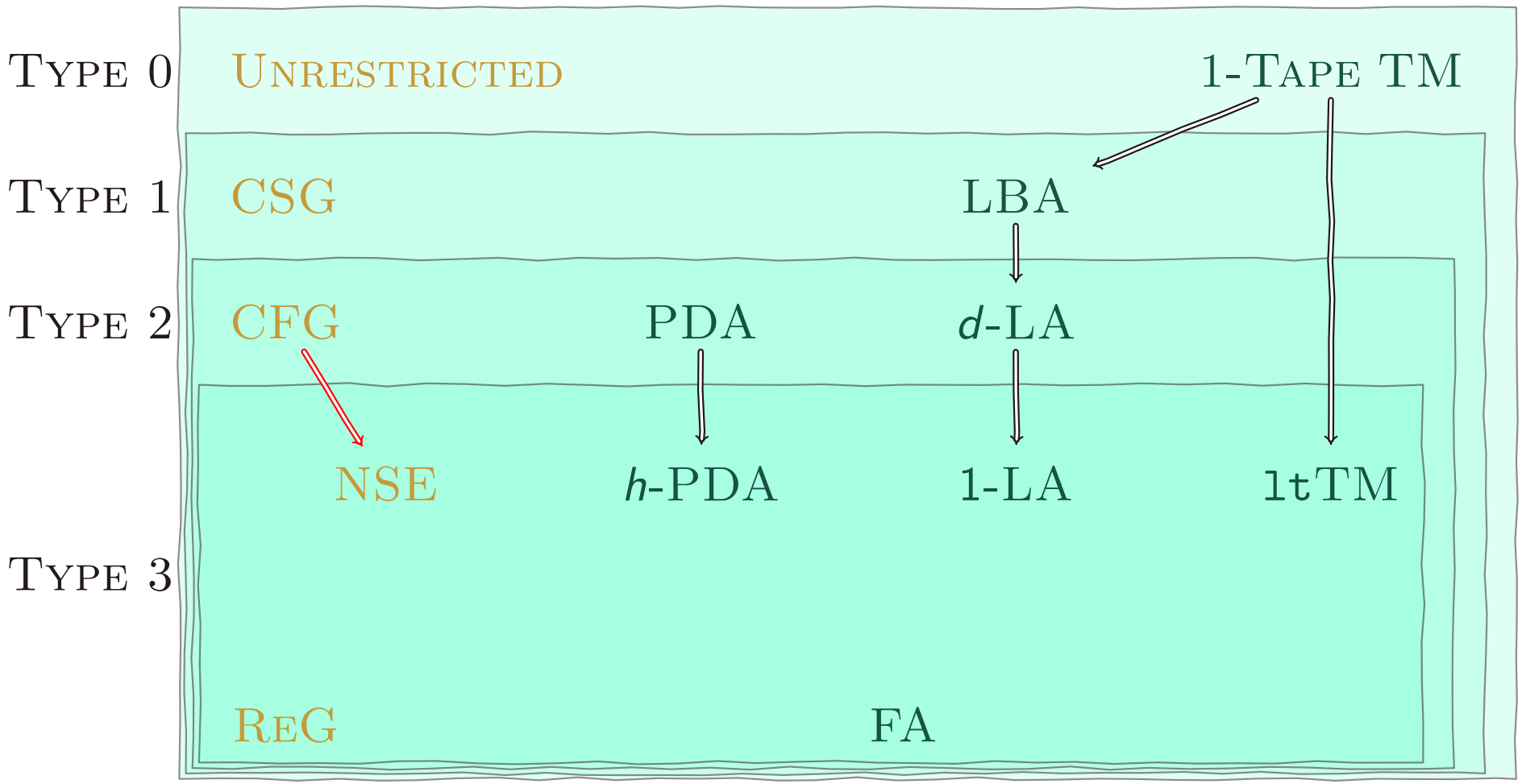
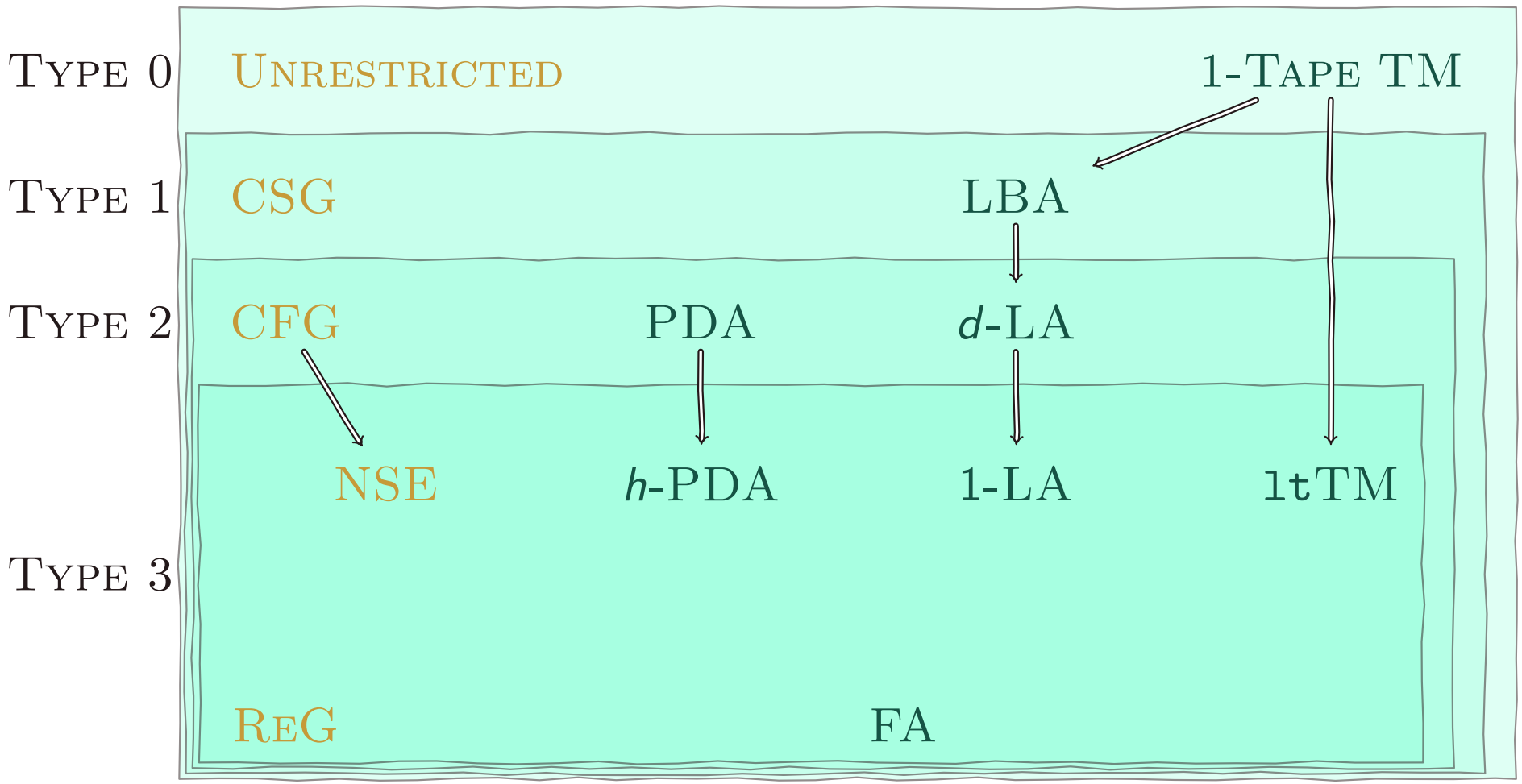
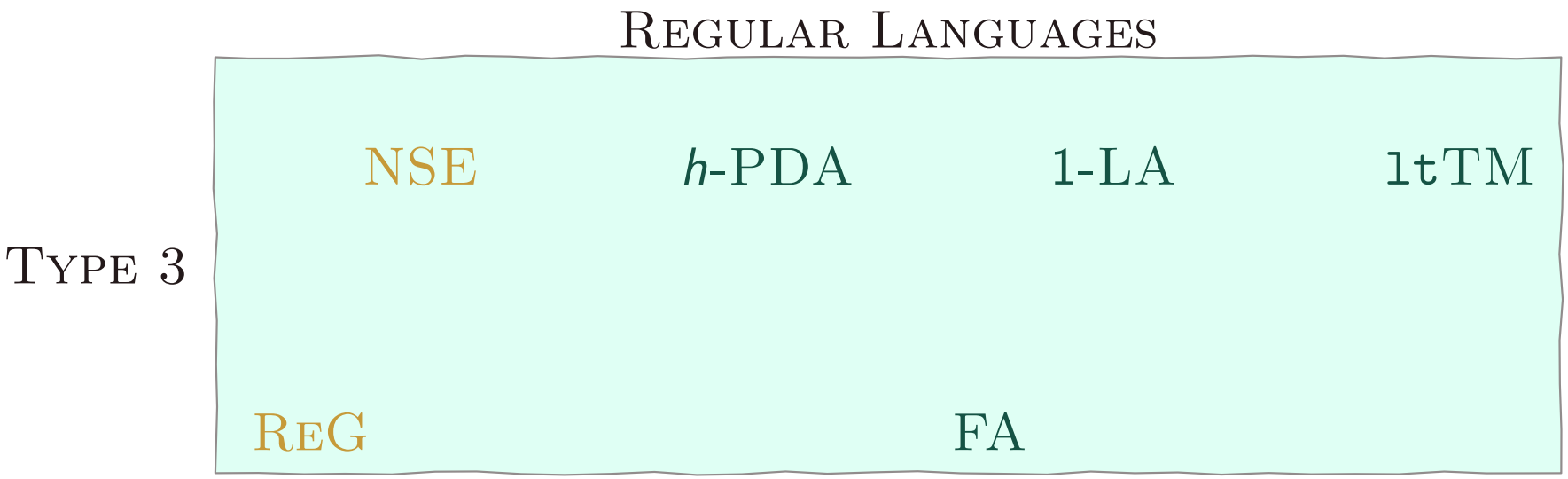
1-Tape TM
Type 1
CSG
LBA
Type 2
CFG
PDA
d-LA
ltTM
Type 3
ReG
FA


Type 0
Unrestricted
1-Tape TM
Type 1
CSG
LBA
Type 2
CFG
PDA
d-LA
1-LA
ltTM
Type 3
ReG
FA


Type 0
Unrestricted
1-Tape TM
Type 1
CSG
LBA
Type 2
CFG
PDA
d-LA
h-PDA
1-LA
1tTM
Type 3
ReG
FA


Type 0
Unrestricted
1-Tape TM
Type 1
CSG
LBA
Type 2
CFG
PDA
d-LA
NSE
h-PDA
1-LA
1tTM
Type 3
ReG
FA

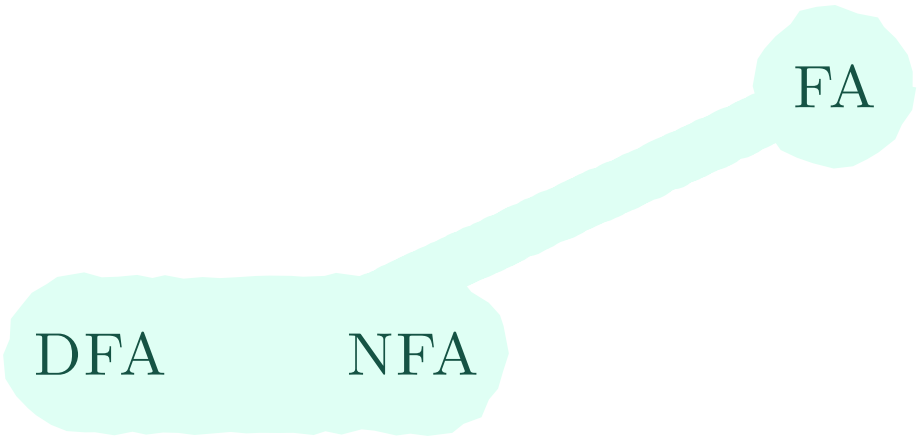

Type 0
Unrestricted
1-Tape TM
Type 1
CSG
LBA
Type 2
CFG
PDA
d-LA
NSE
h-PDA
1-LA
1tTM
Type 3
ReG
FA


Regular Languages
Type 3
NSE
h-PDA
1-LA
1tTM
ReG
FA

NSE h-PDA 1-LA `1tTM`

FA

NSE h-PDA 1-LA `1tTM`



DFA NFA

$$\left\{ w \mid \text{the } 3^{rd}\text{-last symbol of } w \text{ is } 1 \right\}$$

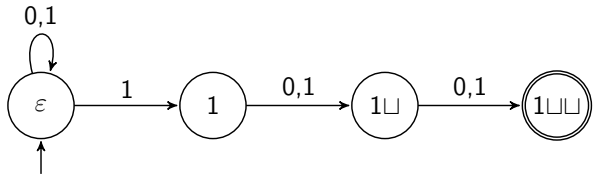
DFA NFA

$$\{\, w \mid \text{the } 3^{rd}\text{-last symbol of } w \text{ is } 1 \,\}$$

0	0	0	1	1	1	0	1	0	0

DFA NFA

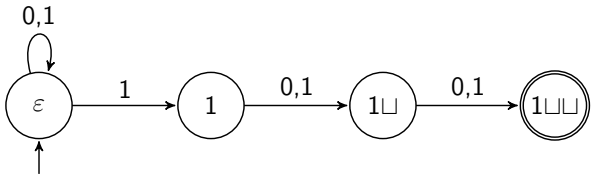
$$\left\{ w \mid \text{the } 3^{rd}\text{-last symbol of } w \text{ is } 1 \right\}$$

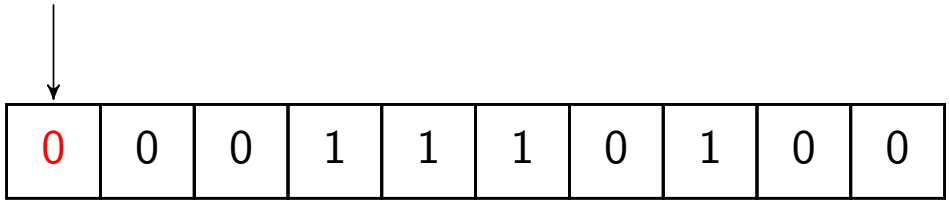


0	0	0	1	1	1	0	1	0	0

DFA NFA

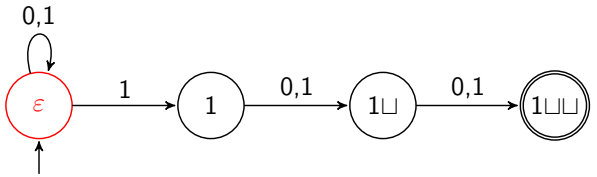
$$\{\, w \mid \text{the } 3^{rd}\text{-last symbol of } w \text{ is } 1 \,\}$$

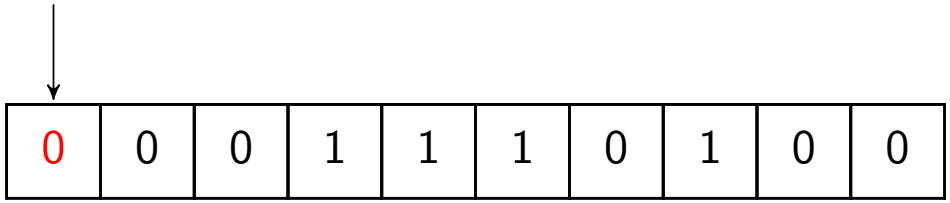




DFA NFA

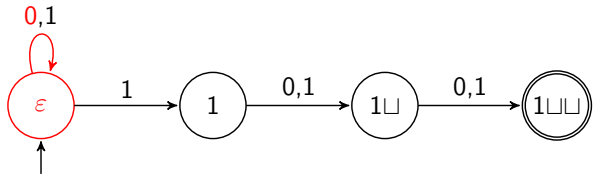
$$\{\, w \mid \text{the } 3^{rd}\text{-last symbol of } w \text{ is } 1 \,\}$$

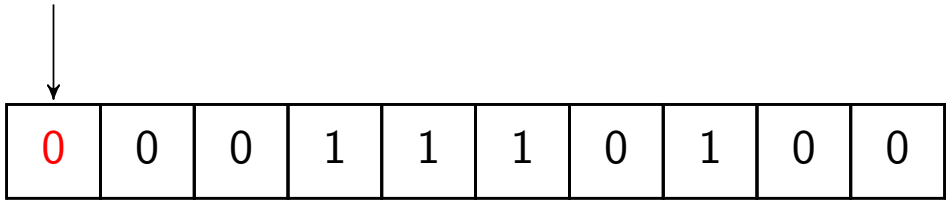




DFA NFA

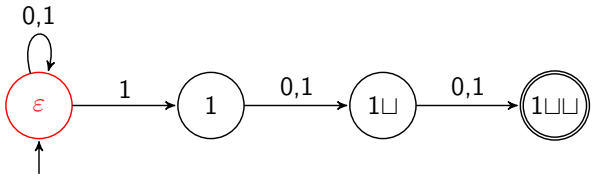
$$\{\, w \mid \text{the } 3^{rd}\text{-last symbol of } w \text{ is } 1 \,\}$$

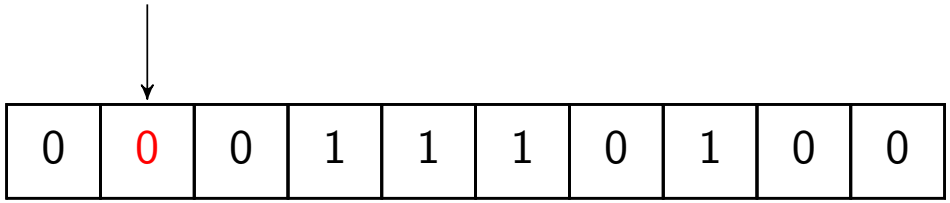




DFA NFA

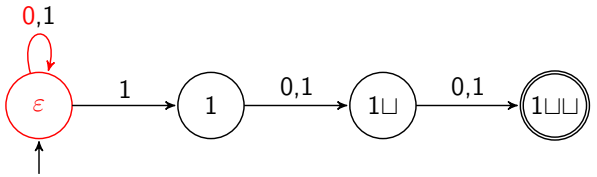
$$\{\, w \mid \text{the } 3^{rd}\text{-last symbol of } w \text{ is } 1 \,\}$$

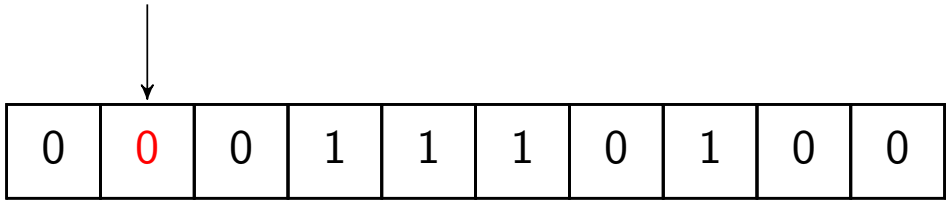




DFA NFA

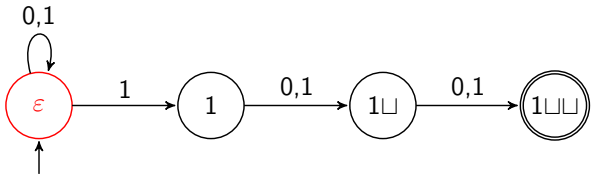
$$\{\, w \mid \text{the } 3^{rd}\text{-last symbol of } w \text{ is } 1 \,\}$$

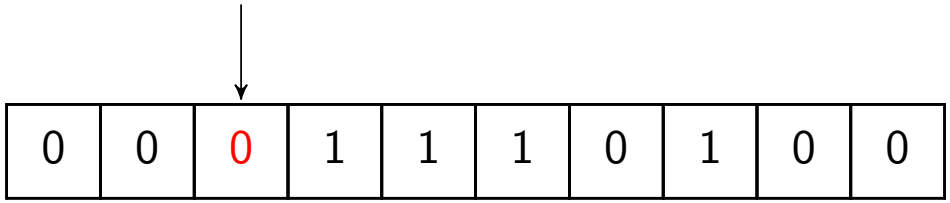




DFA NFA

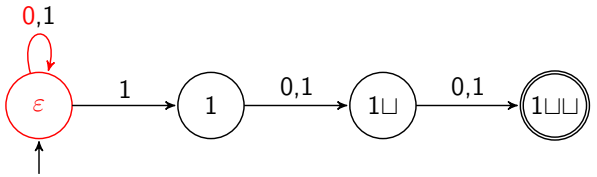
$$\{\, w \mid \text{the } 3^{rd}\text{-last symbol of } w \text{ is } 1 \,\}$$

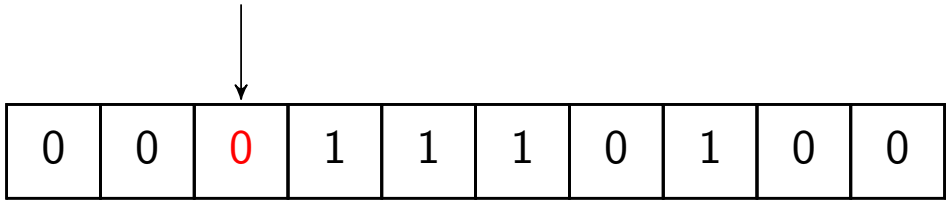




DFA NFA

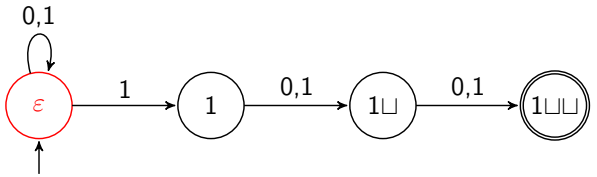
$$\{\, w \mid \text{the } 3^{rd}\text{-last symbol of } w \text{ is } 1 \,\}$$

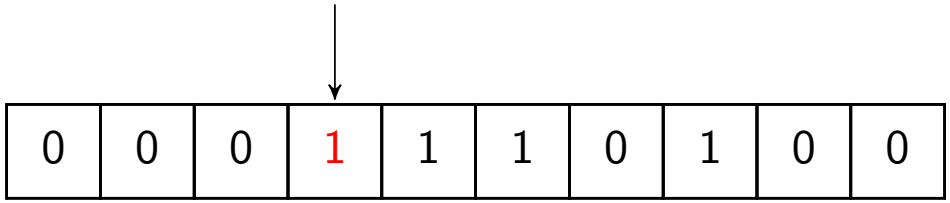




DFA NFA

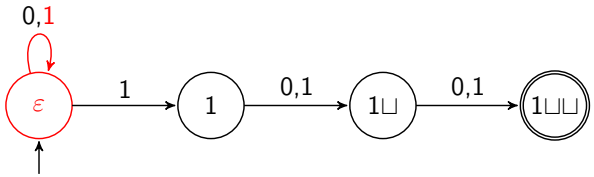
$$\{\, w \mid \text{the } 3^{rd}\text{-last symbol of } w \text{ is } 1 \,\}$$

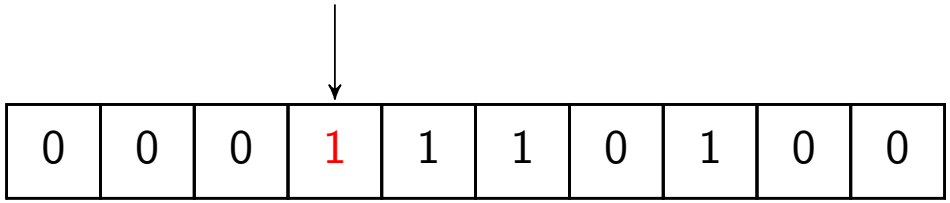




DFA NFA

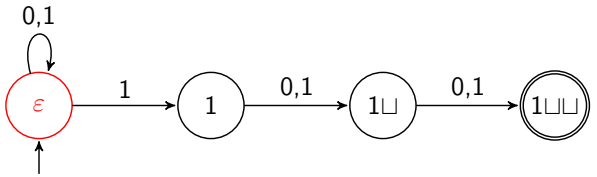
$$\{\, w \mid \text{the } 3^{rd}\text{-last symbol of } w \text{ is } 1 \,\}$$

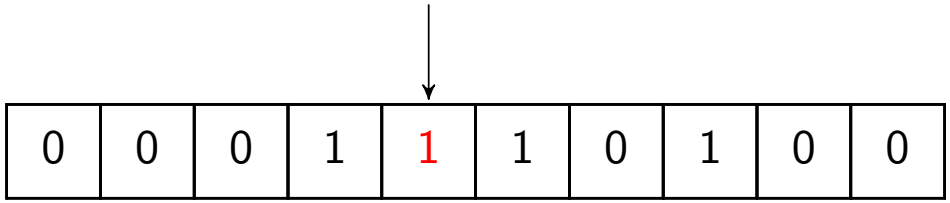




DFA NFA

$$\{ w \mid \text{the } 3^{rd}\text{-last symbol of } w \text{ is } 1 \}$$





DFA NFA

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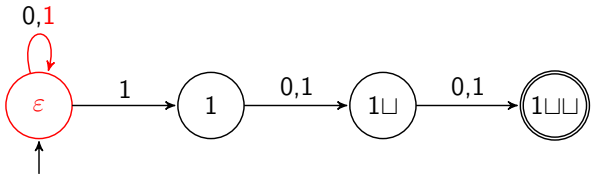
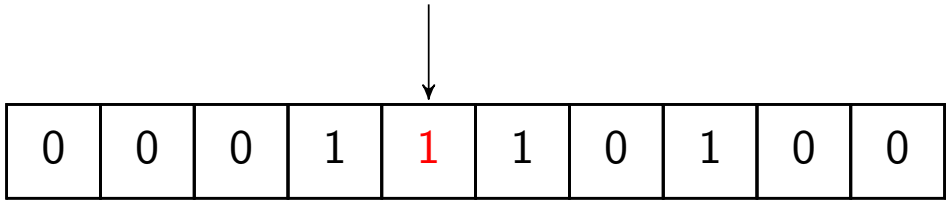
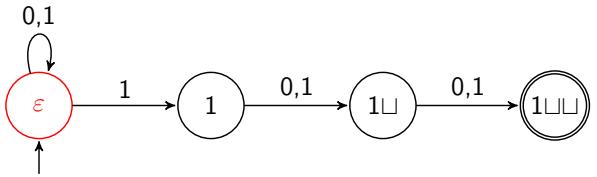
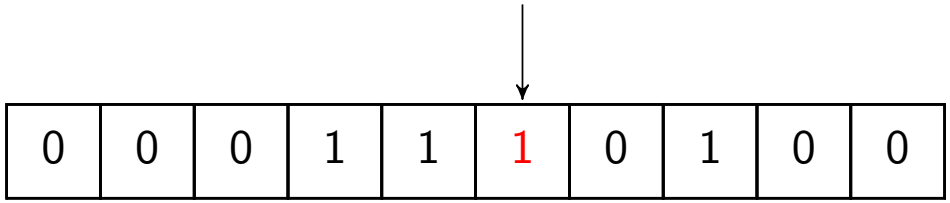




DFA NFA

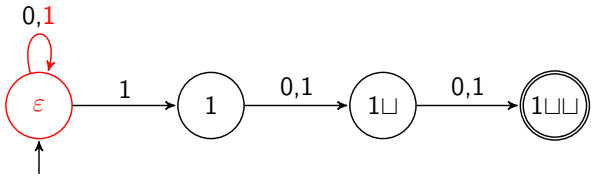
$$\{ w \mid \text{the } 3^{rd}\text{-last symbol of } w \text{ is } 1 \}$$

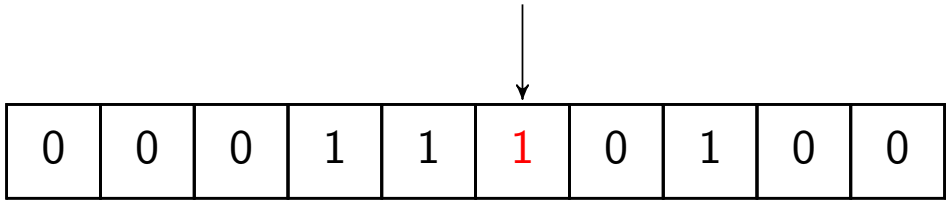




DFA NFA

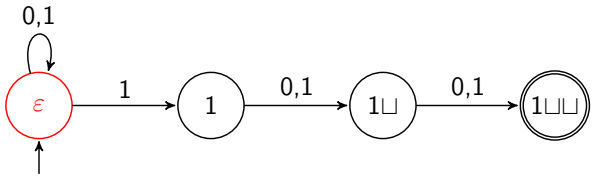
$$\{\, w \mid \text{the } 3^{rd}\text{-last symbol of } w \text{ is } 1 \,\}$$

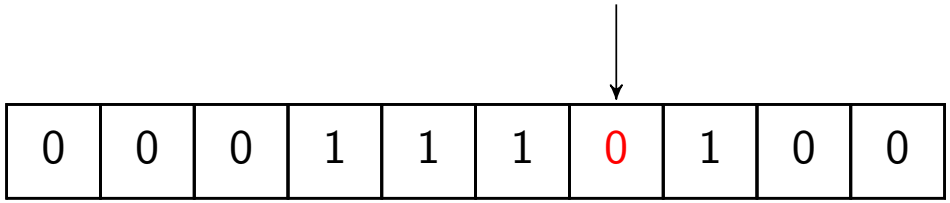




DFA NFA

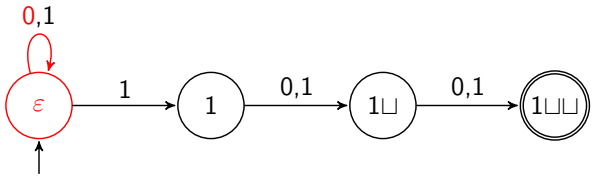
$$\{\, w \mid \text{the } 3^{rd}\text{-last symbol of } w \text{ is } 1 \,\}$$

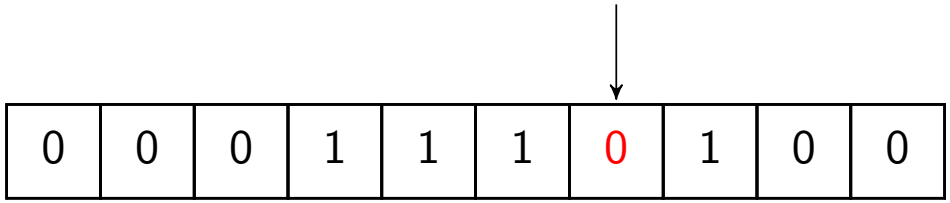




DFA NFA

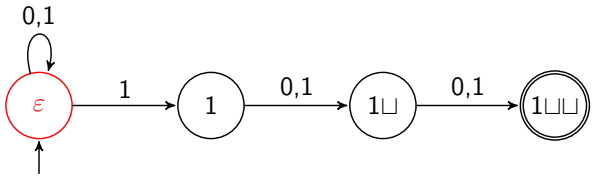
$$\{\, w \mid \text{the } 3^{rd}\text{-last symbol of } w \text{ is } 1 \,\}$$

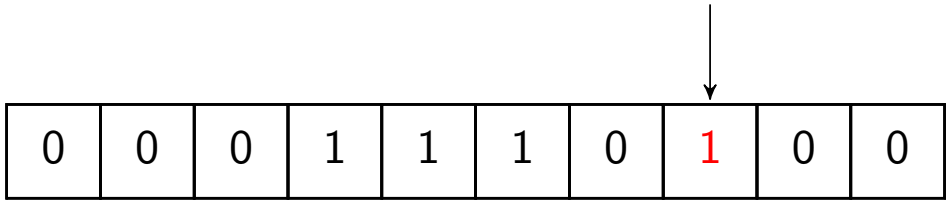




DFA NFA

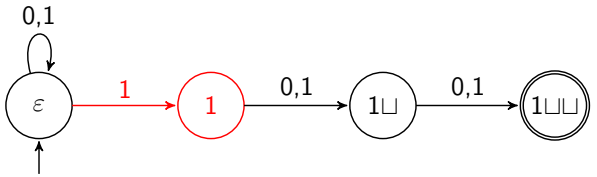
$$\{\, w \mid \text{the } 3^{rd}\text{-last symbol of } w \text{ is } 1 \,\}$$

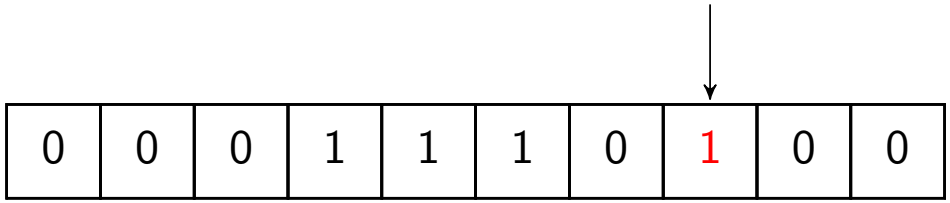




DFA NFA

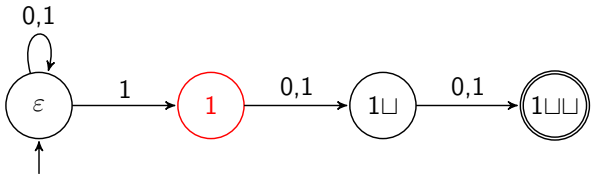
$$\{\, w \mid \text{the } 3^{rd}\text{-last symbol of } w \text{ is } 1 \,\}$$

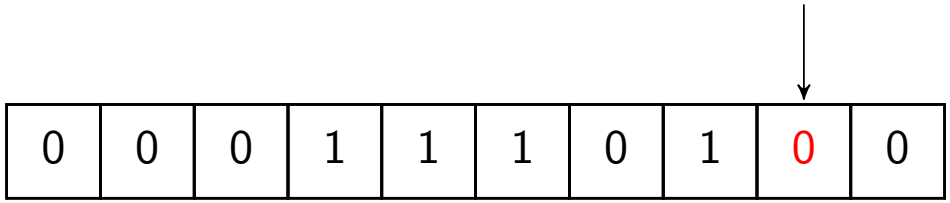




DFA NFA

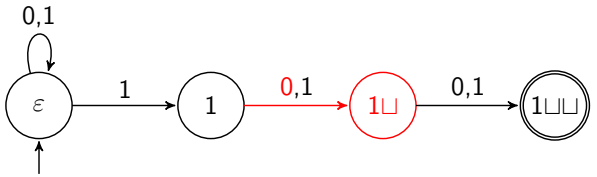
$$\{\, w \mid \text{the } 3^{rd}\text{-last symbol of } w \text{ is } 1 \,\}$$

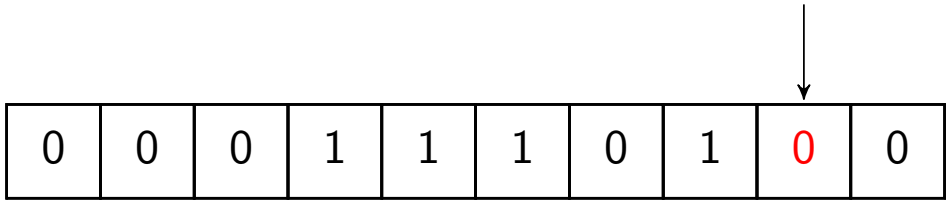




DFA NFA

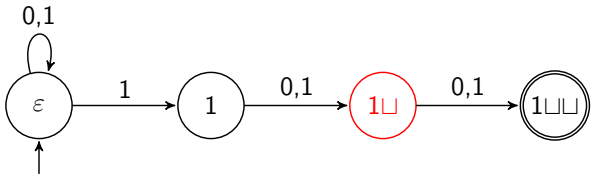
$$\{\, w \mid \text{the } 3^{rd}\text{-last symbol of } w \text{ is } 1 \,\}$$

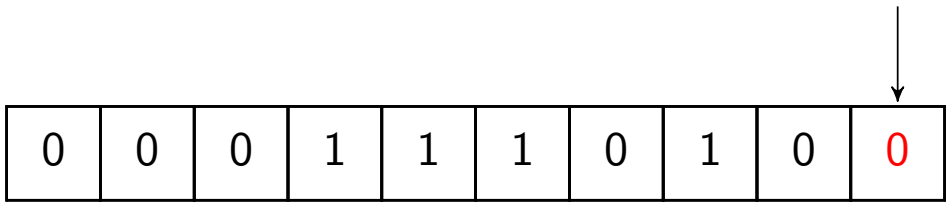




DFA NFA

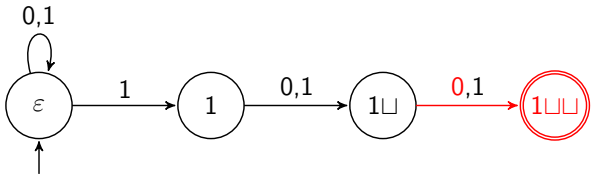
$$\{\, w \mid \text{the } 3^{rd}\text{-last symbol of } w \text{ is } 1 \,\}$$

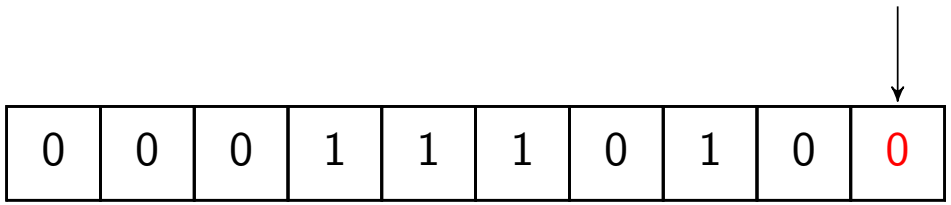




DFA NFA

$$\{\, w \mid \text{the } 3^{rd}\text{-last symbol of } w \text{ is } 1 \,\}$$





DFA NFA

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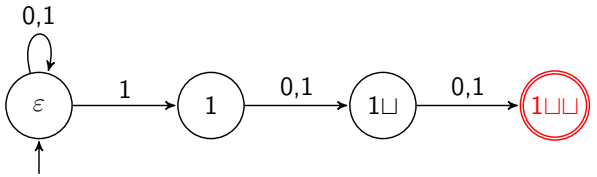
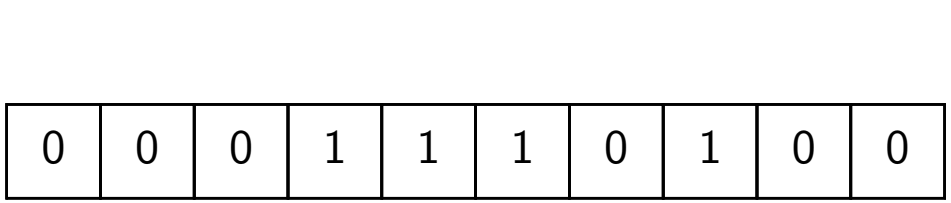
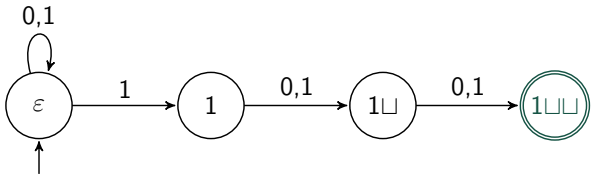
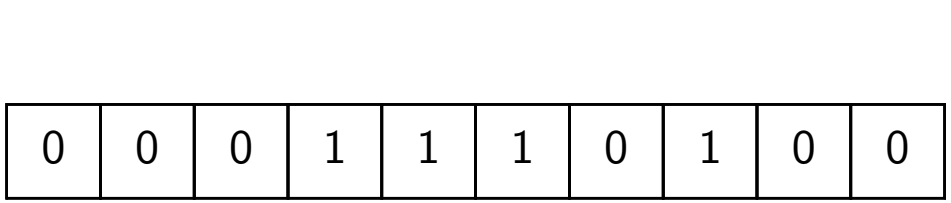




DFA NFA

$$\{\, w \mid \text{the } 3^{rd}\text{-last symbol of } w \text{ is } 1 \,\}$$





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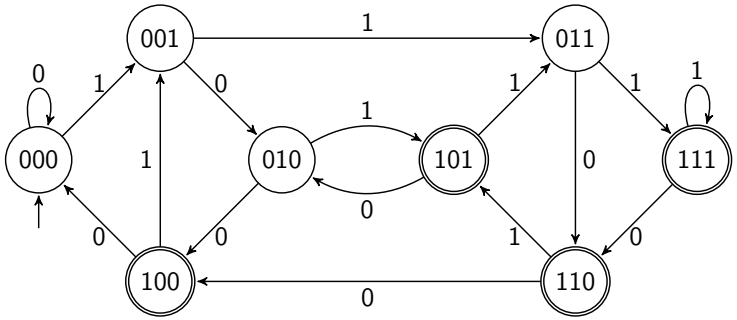

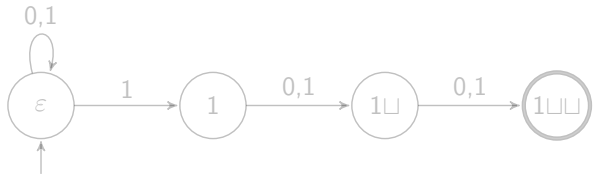

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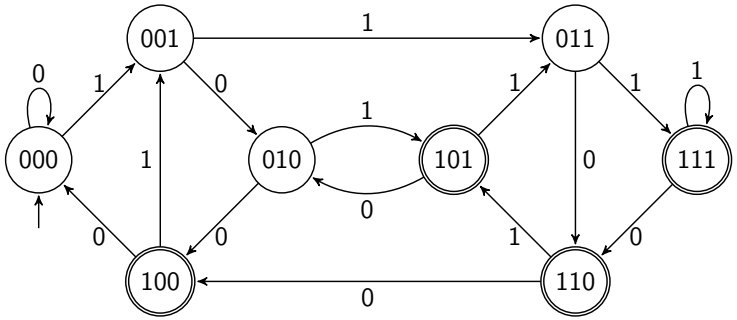
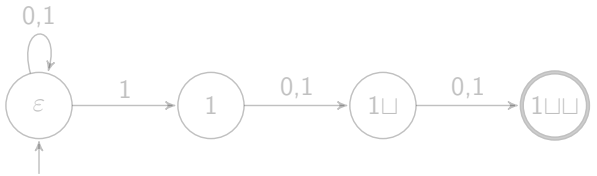
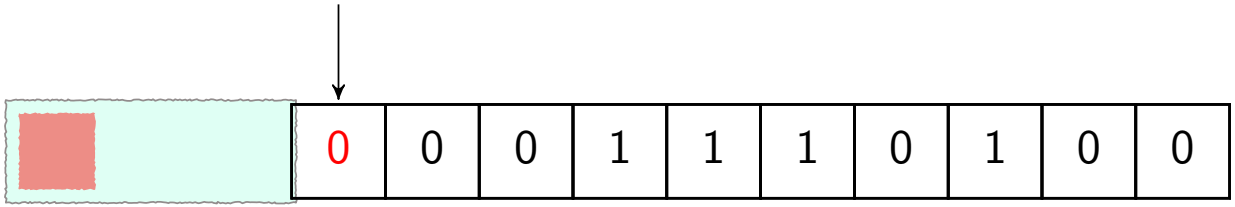
DFA NFA

$$\{\, w \mid \text{the } 3^{rd}\text{-last symbol of } w \text{ is } 1 \,\}$$







DFA NFA

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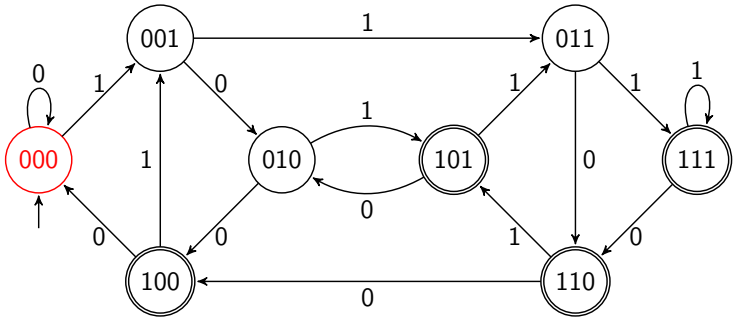

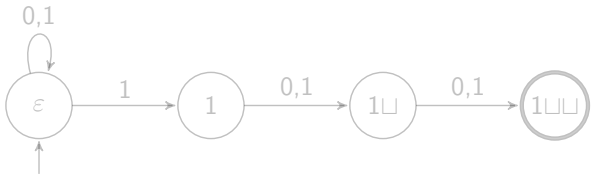

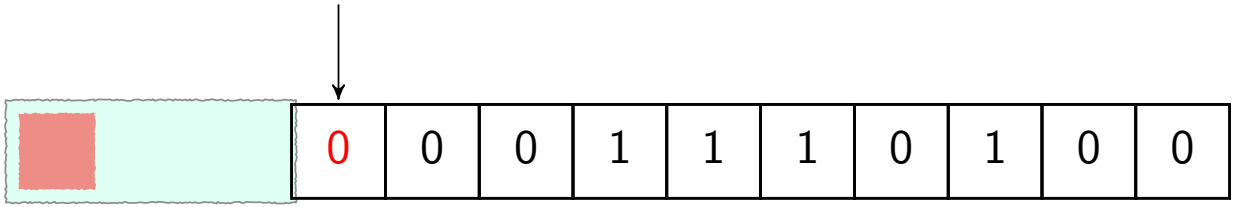

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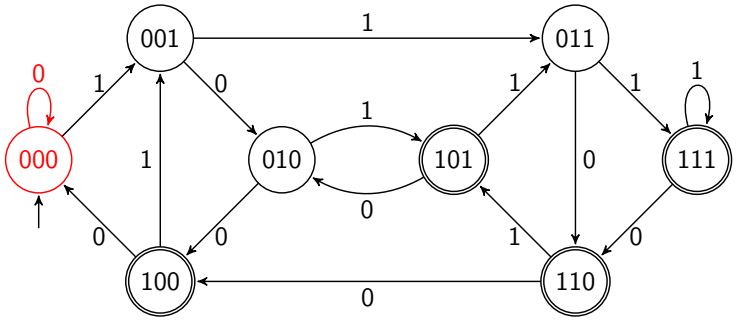

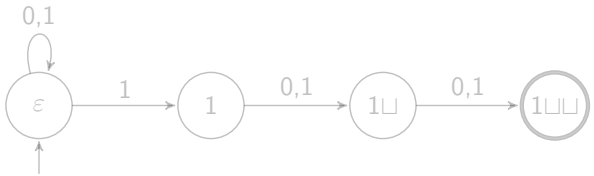

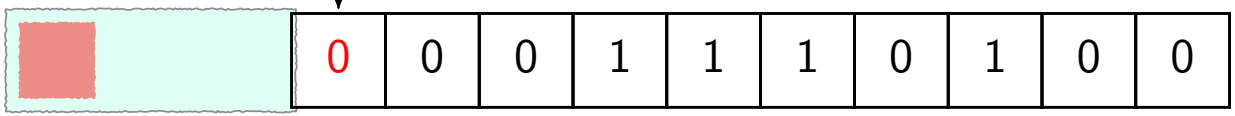

DFA NFA

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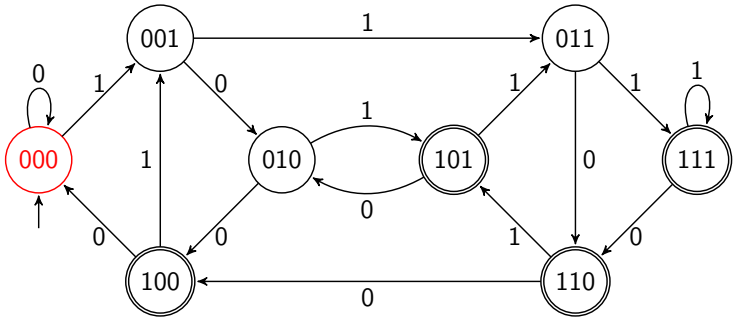

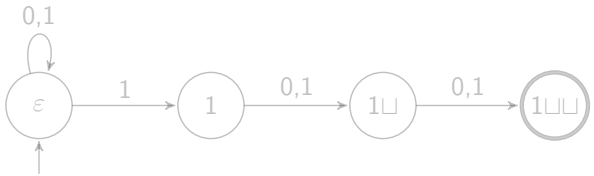

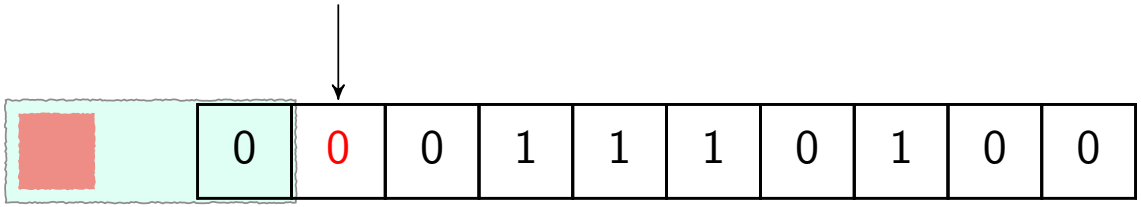

DFA NFA

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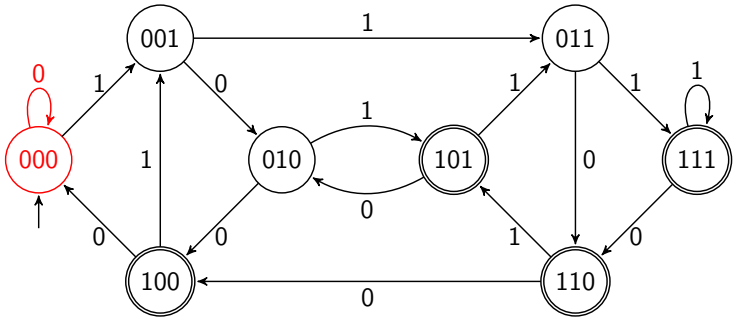

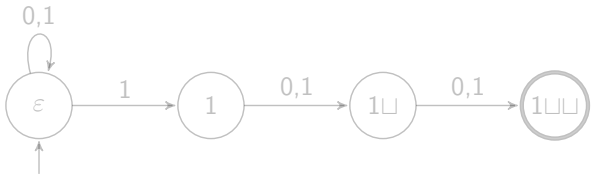

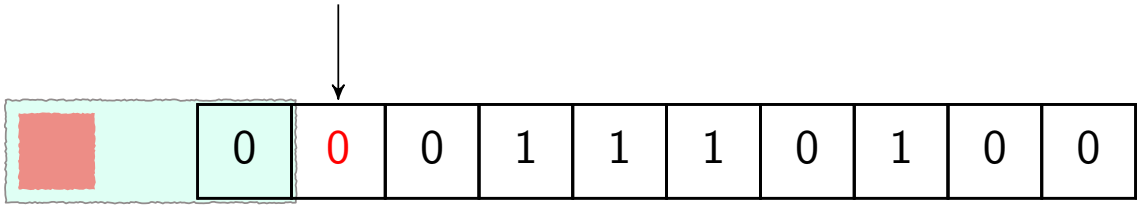

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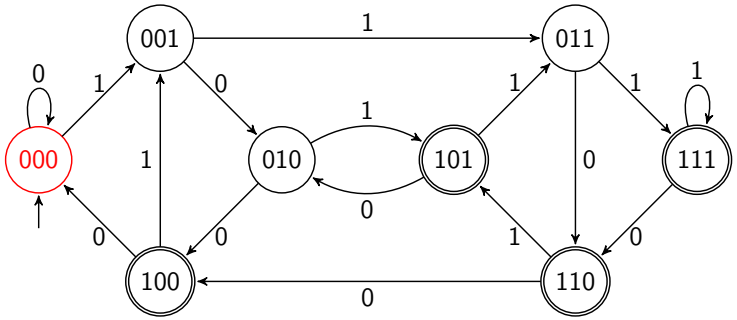

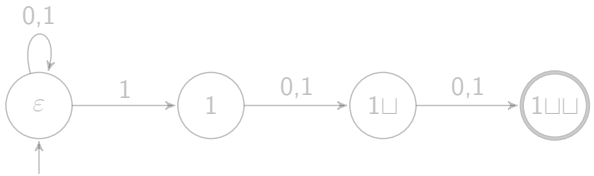

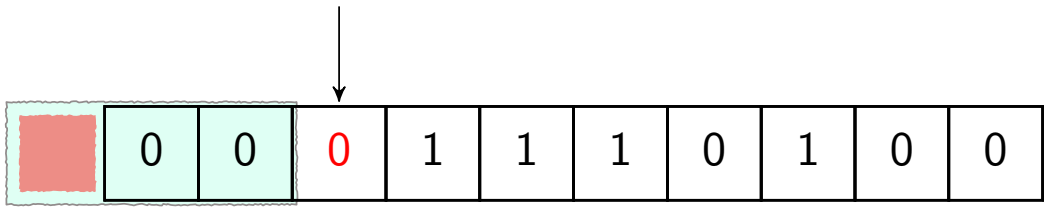

DFA NFA

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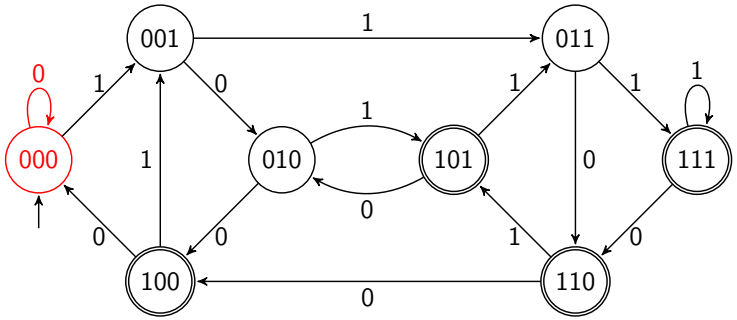

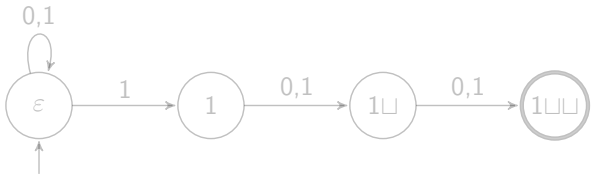

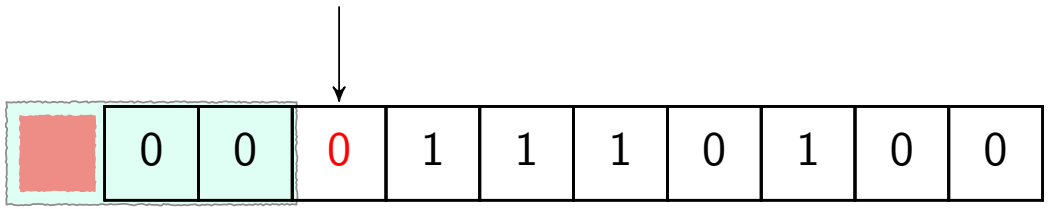

DFA NFA

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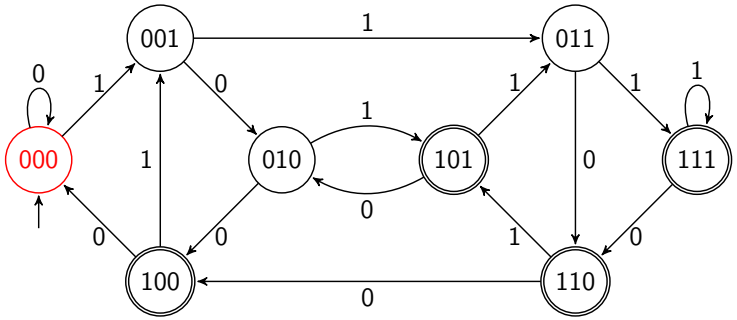
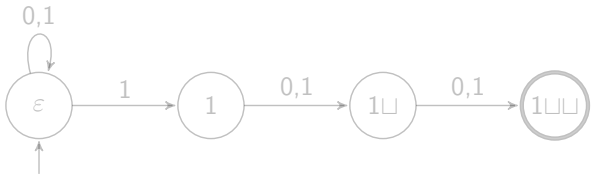
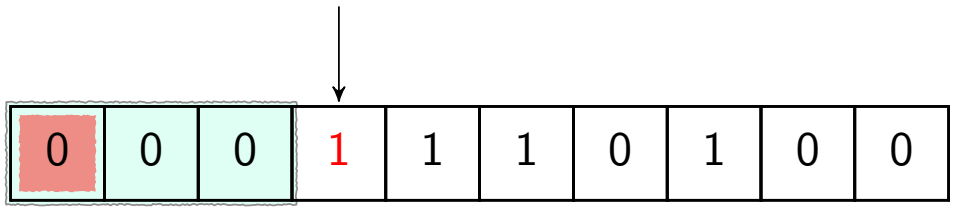
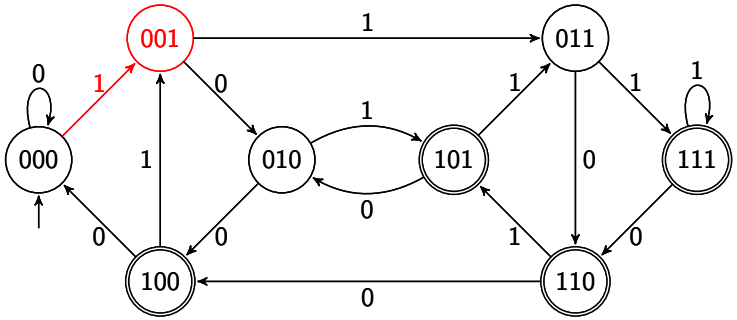


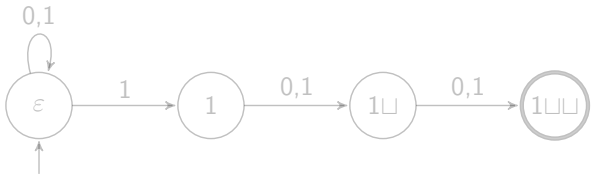


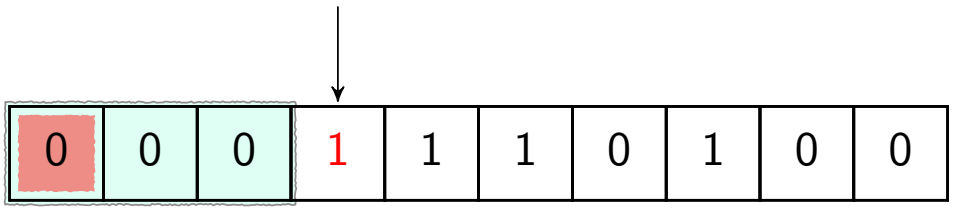


DFA NFA

$\{ w \mid \text{the } 3^{rd}\text{-last symbol of } w \text{ is } 1 \}$







DFA NFA

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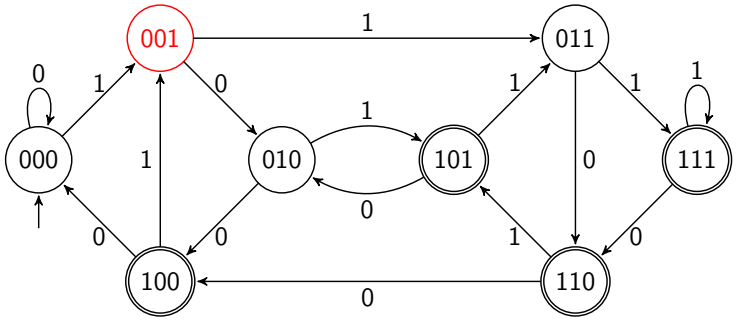

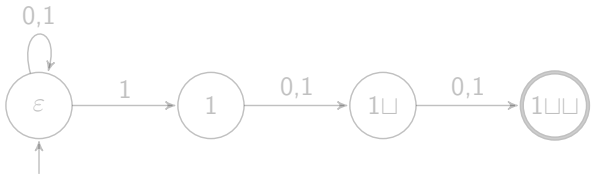

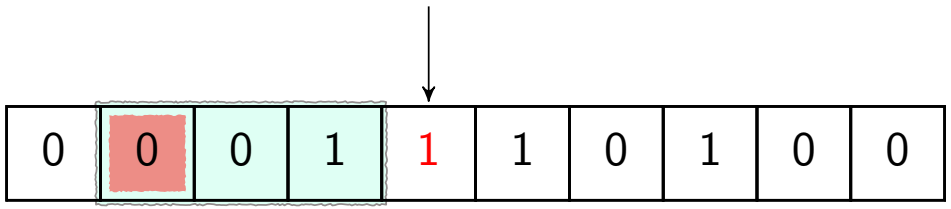

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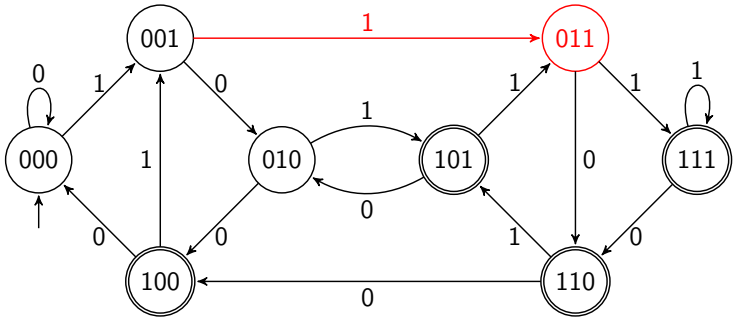

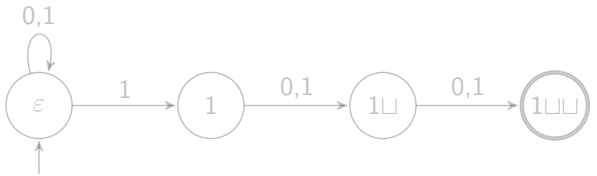

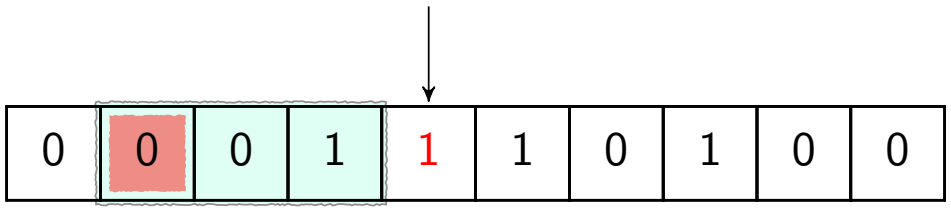

DFA NFA

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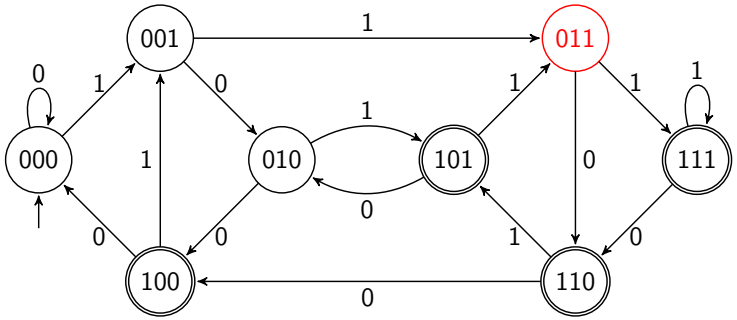

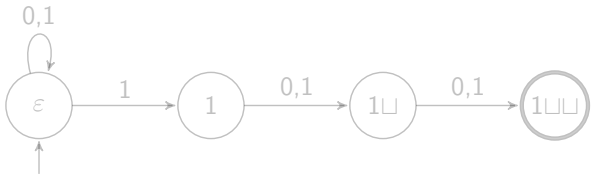

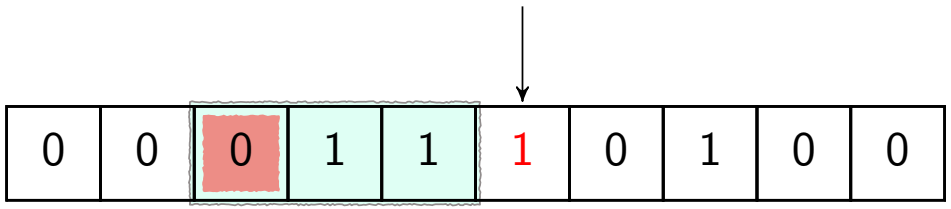

DFA NFA

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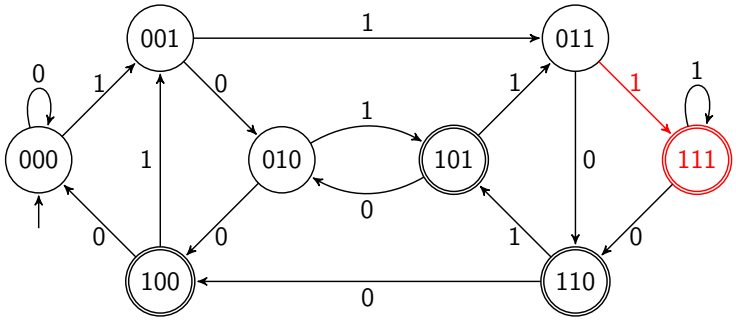

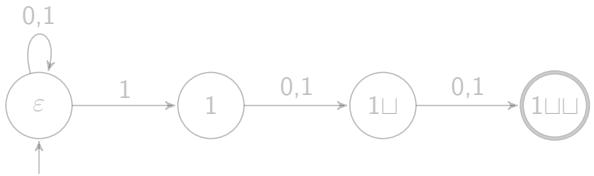

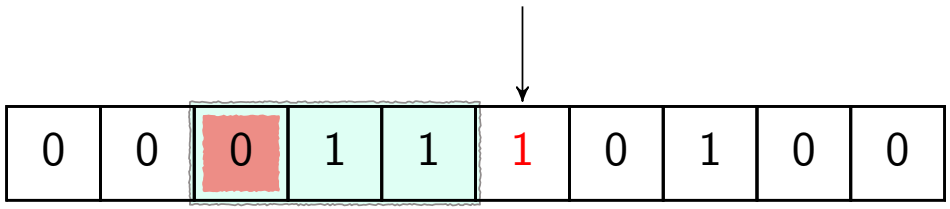

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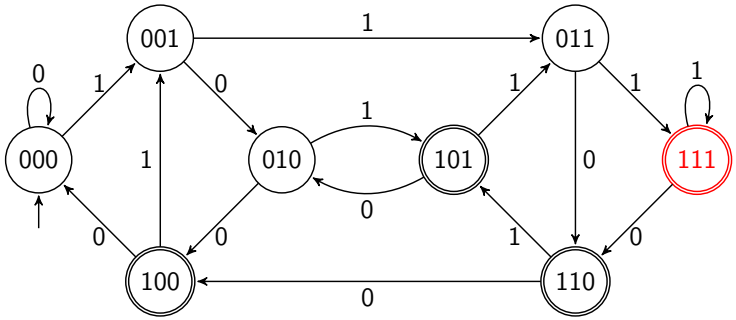

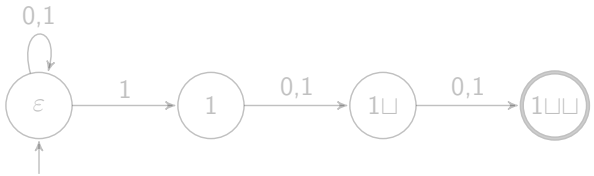

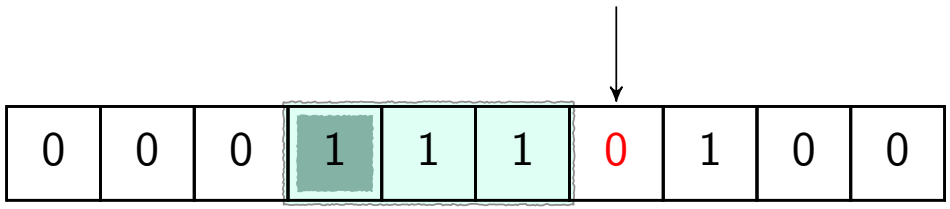

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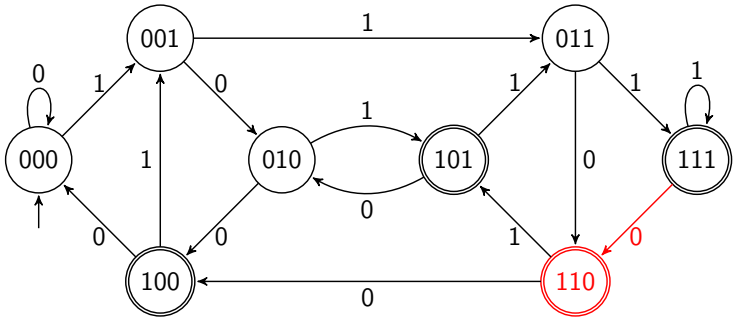

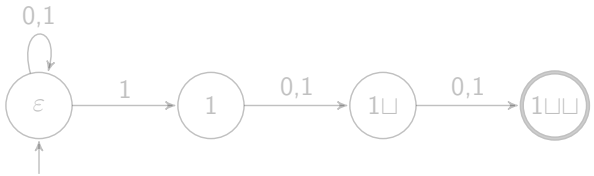

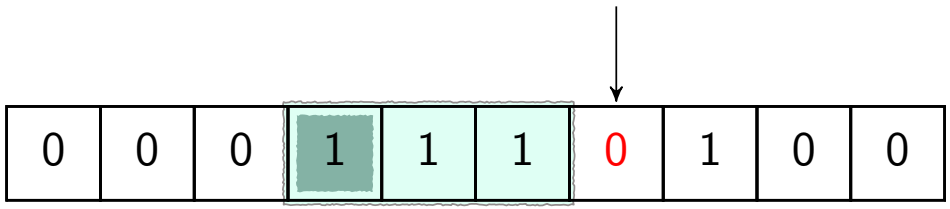

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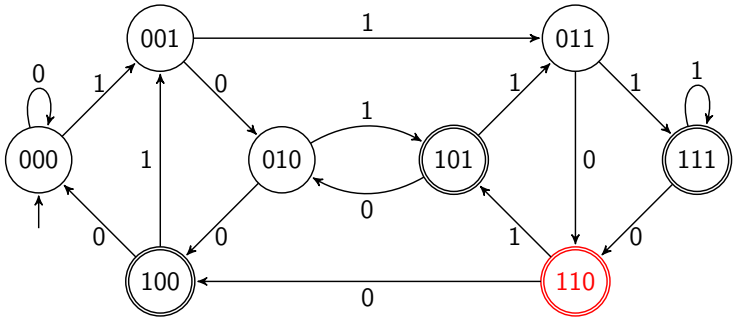

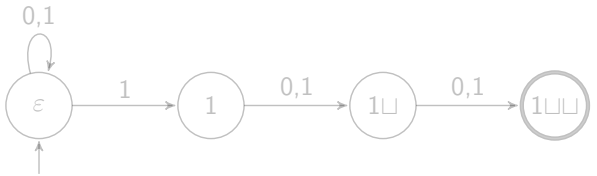

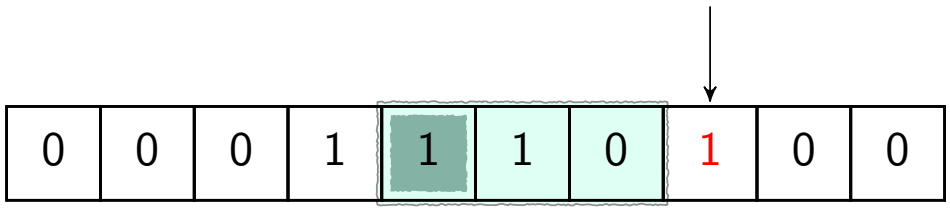

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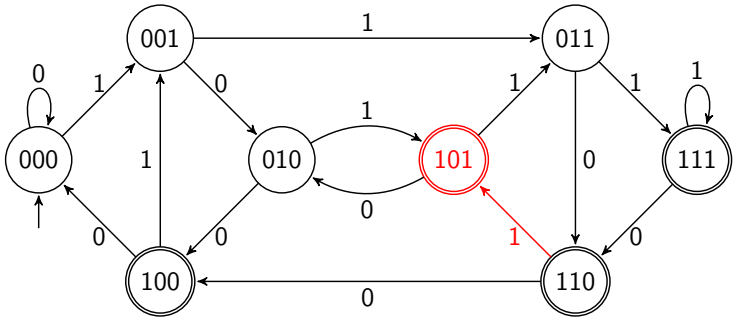
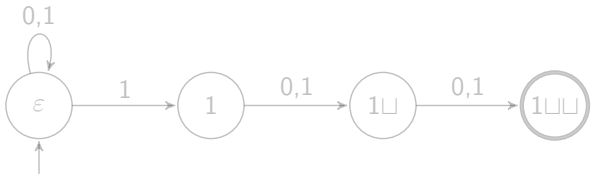
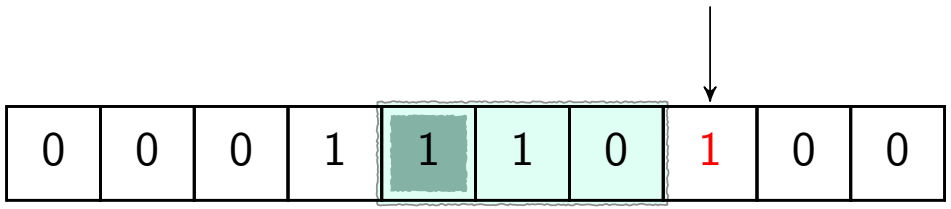
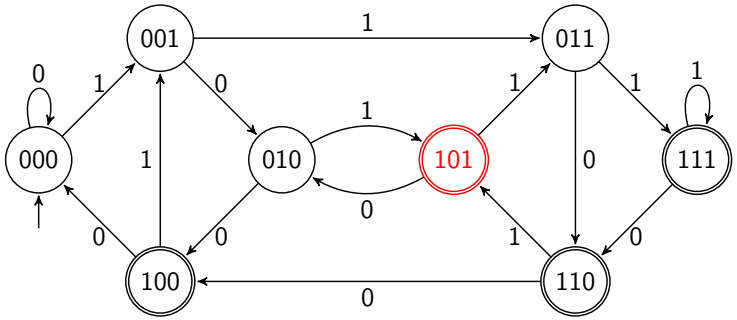


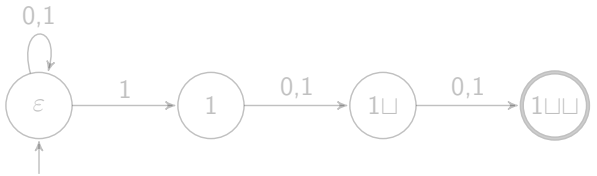


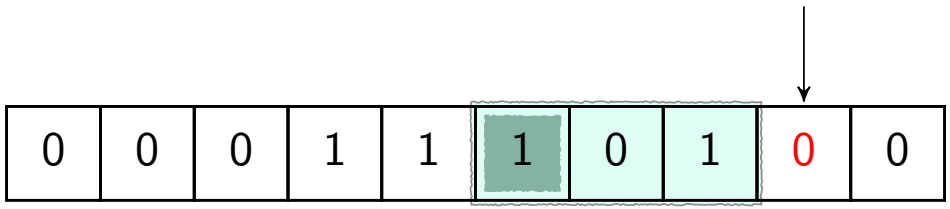


DFA NFA

$$\{\, w \mid \text{the } 3^{rd}\text{-last symbol of } w \text{ is } 1 \,\}$$







DFA NFA

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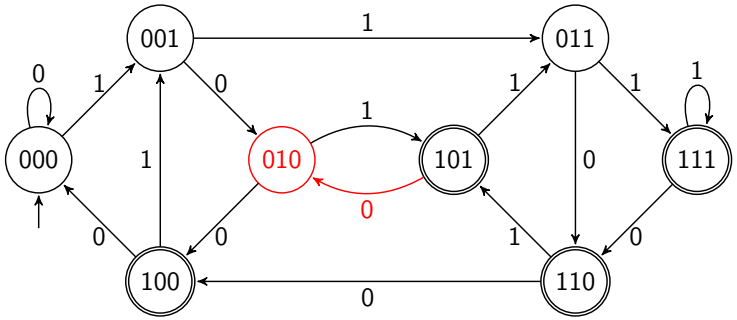

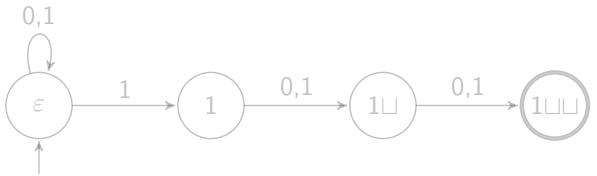

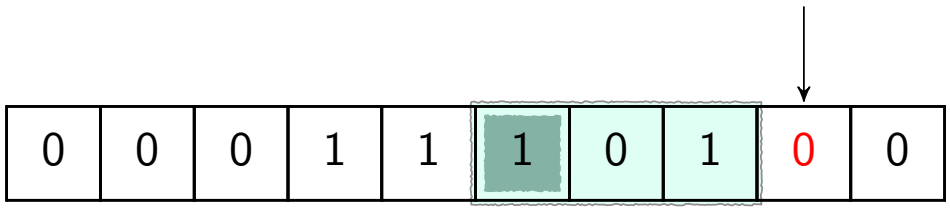

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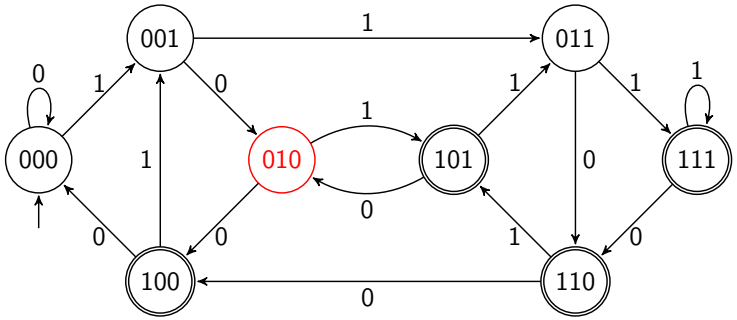

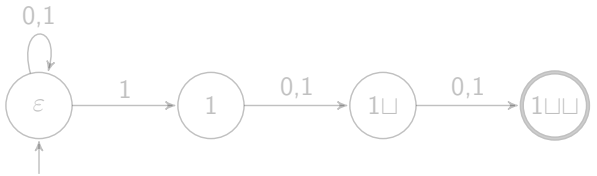

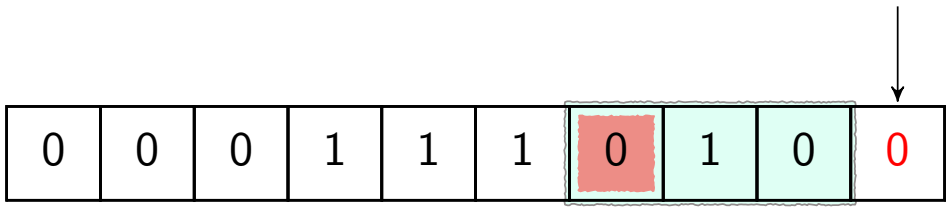

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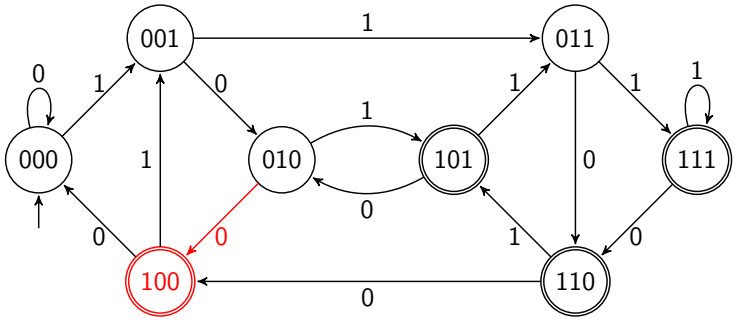

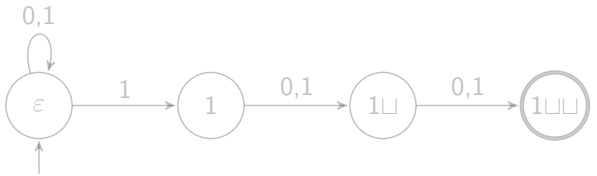

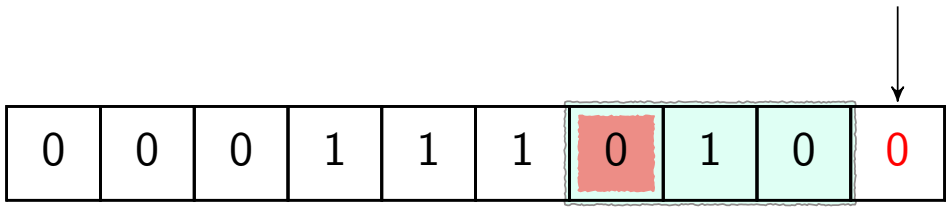

DFA NFA

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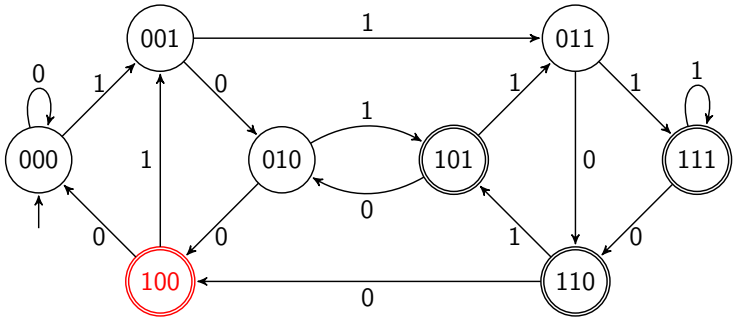

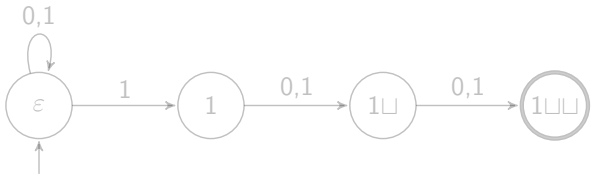

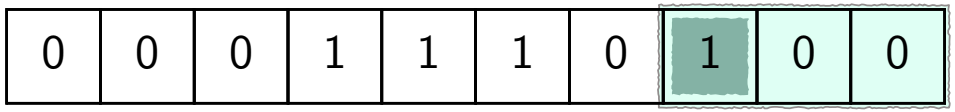

DFA NFA

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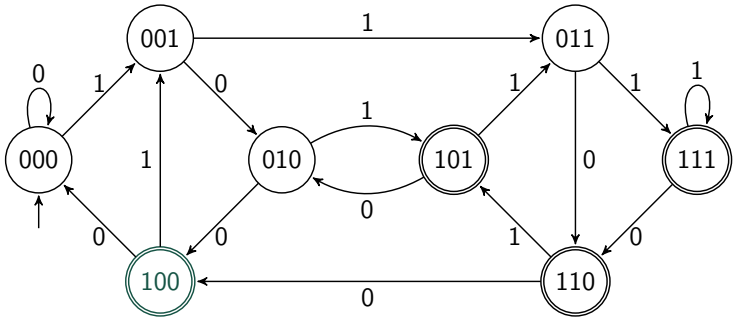
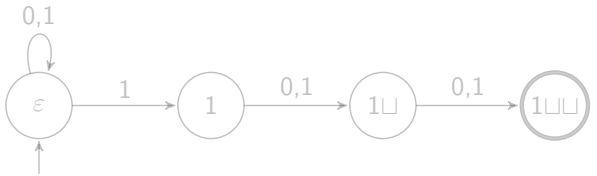
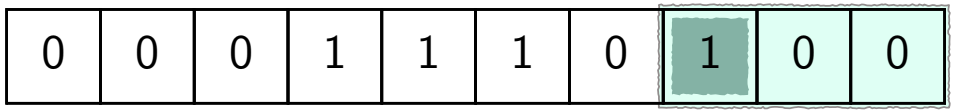
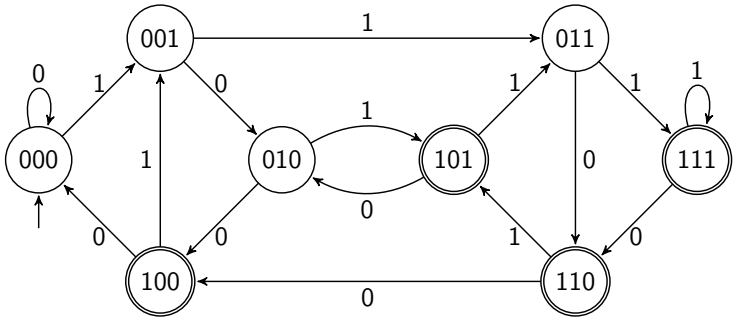


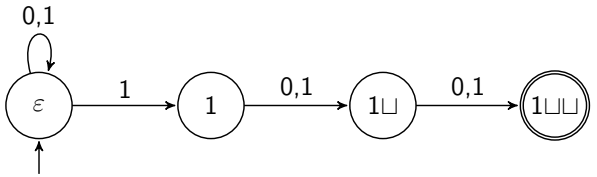




DFA NFA

$$\left\{ w \mid \text{the } 3^{rd}\text{-last symbol of } w \text{ is } 1 \right\}$$





DFA NFA

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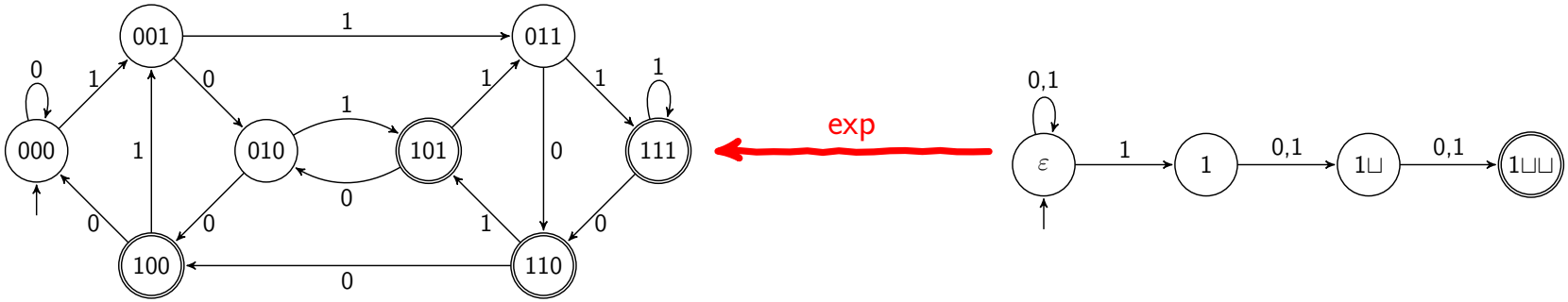

DFA NFA

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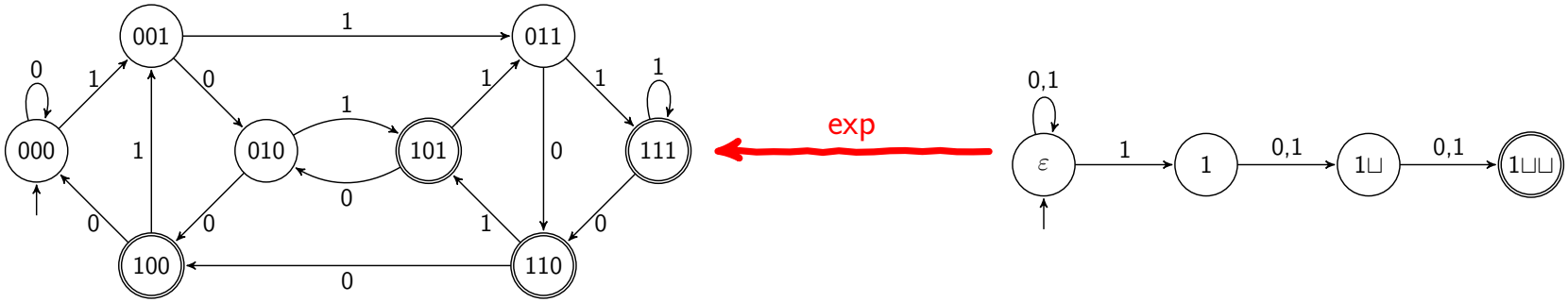
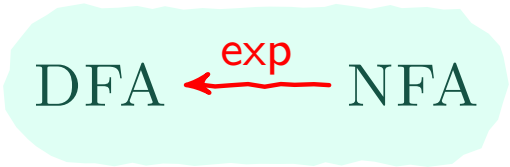
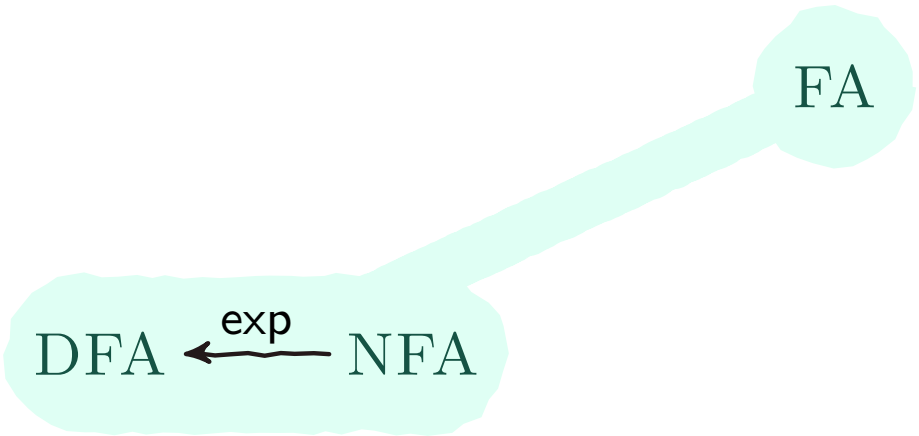
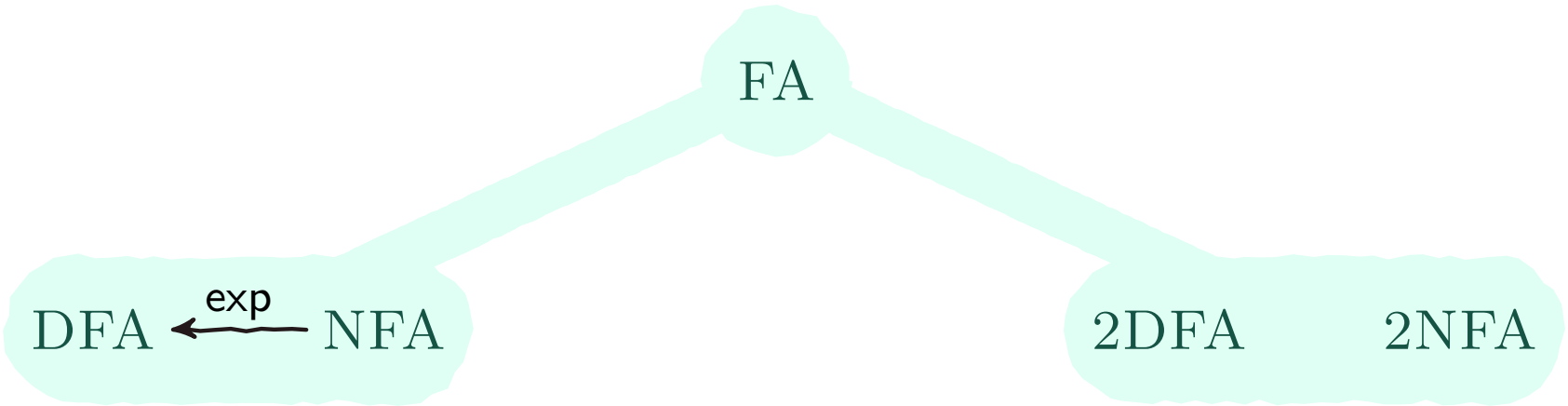




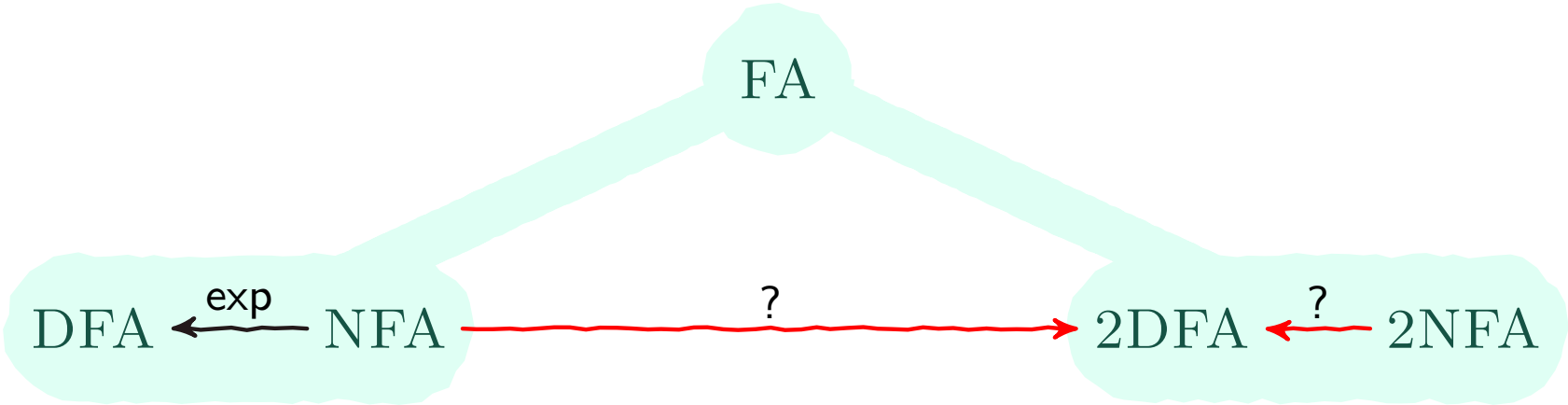
NSE h-PDA 1-LA 1tTM



NSE h-PDA 1-LA 1tTM



NSE h-PDA 1-LA 1tTM



NSE h-PDA 1-LA `1tTM`

FA

$$\{ a^{2^k} \mid k \geq 0 \text{ is a fixed integer} \}$$

NSE h-PDA 1-LA 1tTM

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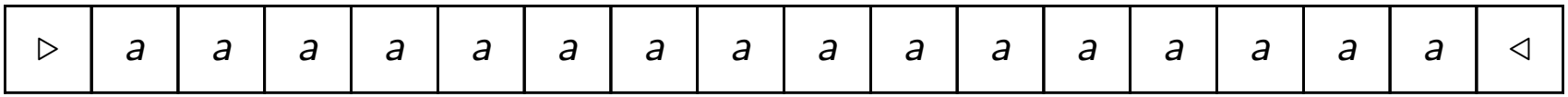

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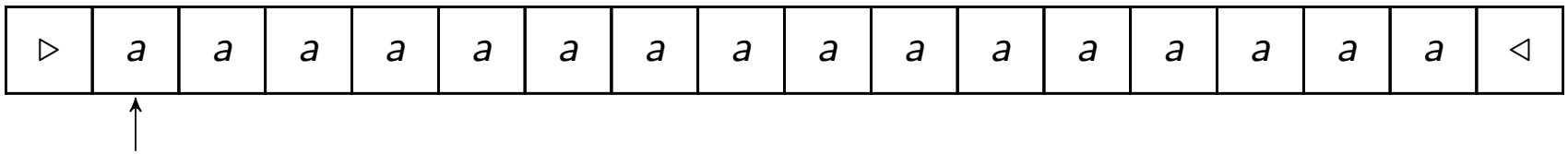
NSE h-PDA 1-LA 1tTM



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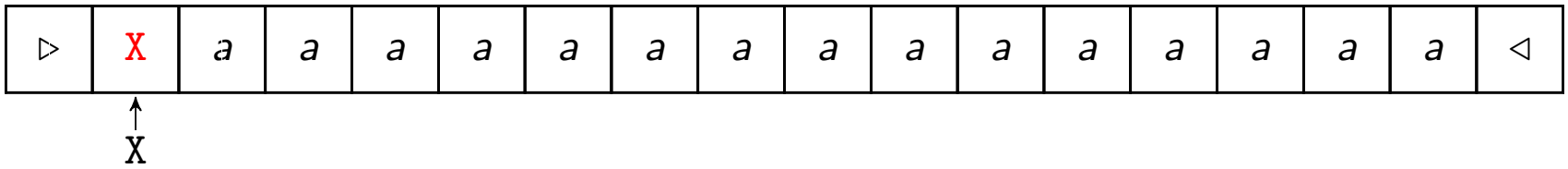
NSE h-PDA 1-LA 1tTM



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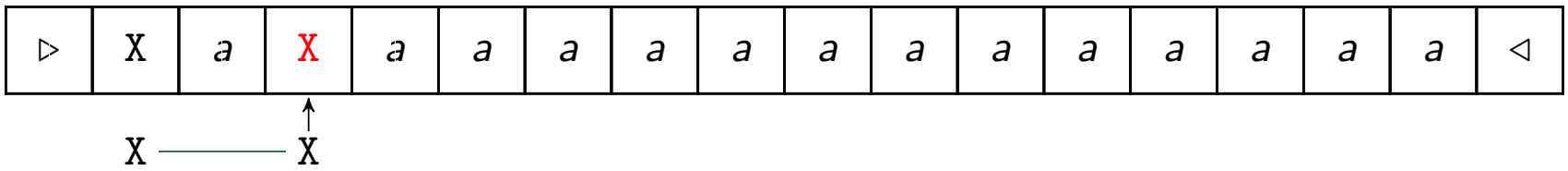
NSE h-PDA 1-LA 1tTM



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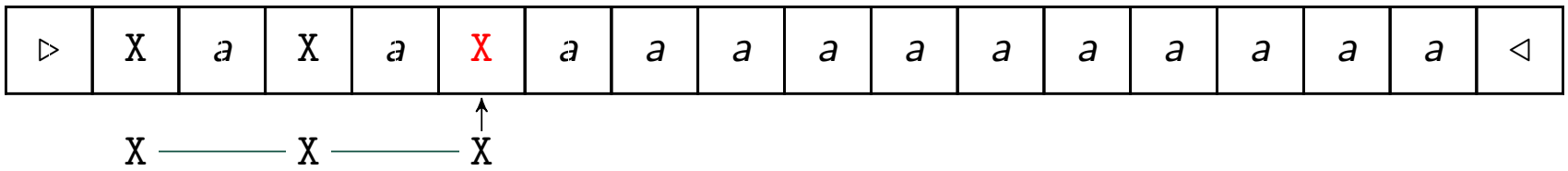
NSE h-PDA 1-LA 1tTM



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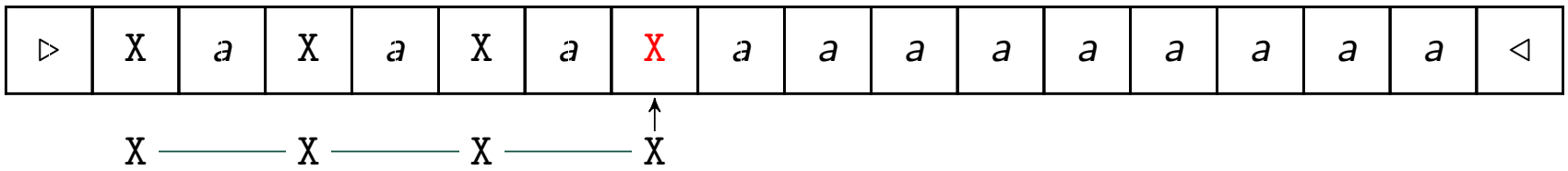
NSE h-PDA 1-LA 1tTM



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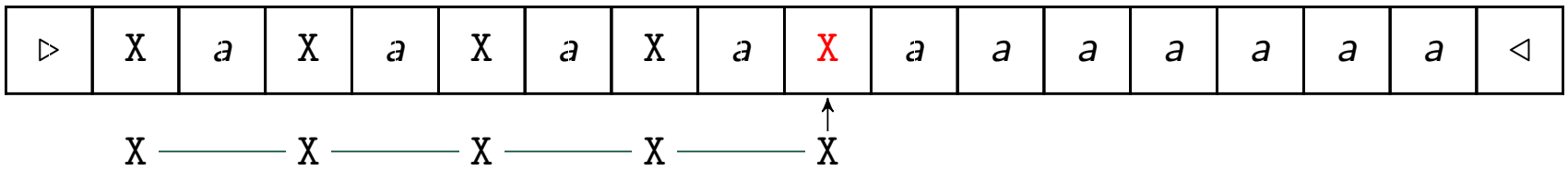
NSE h-PDA 1-LA 1tTM



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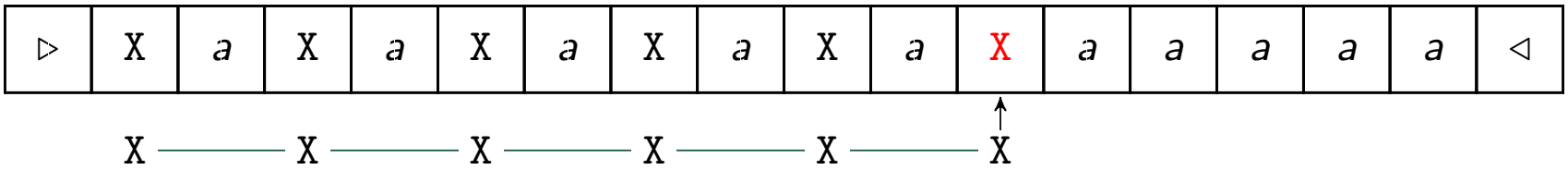
NSE h-PDA 1-LA 1tTM



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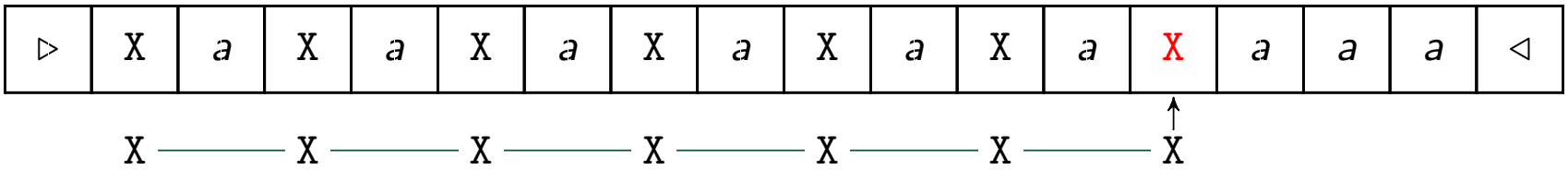

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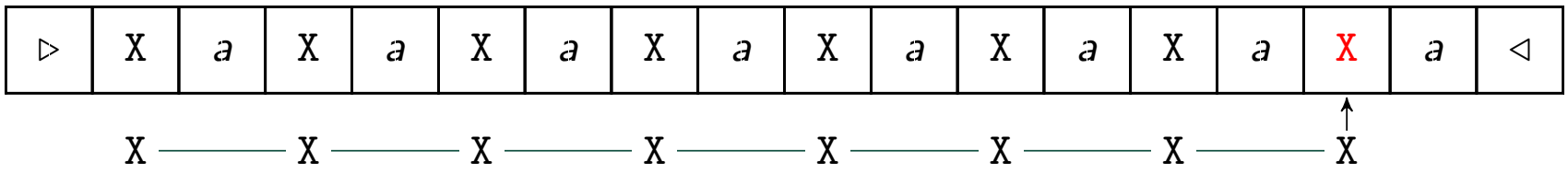
NSE h-PDA 1-LA 1tTM



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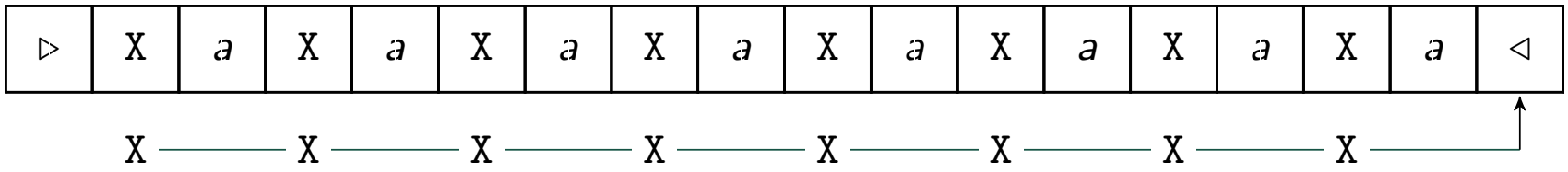
NSE h-PDA 1-LA 1tTM



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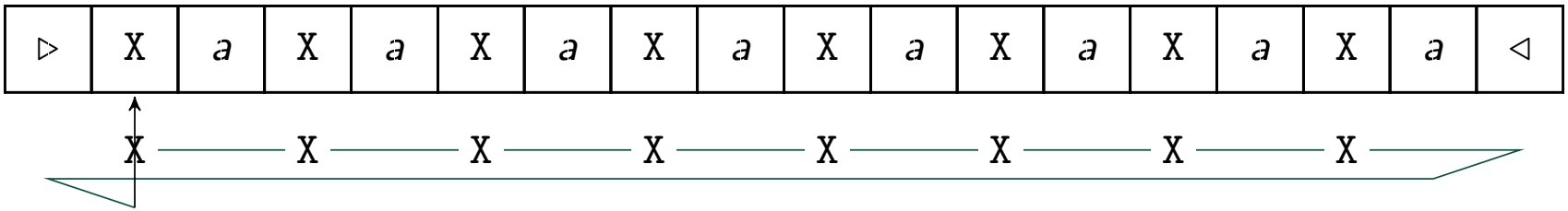
NSE h-PDA 1-LA 1tTM



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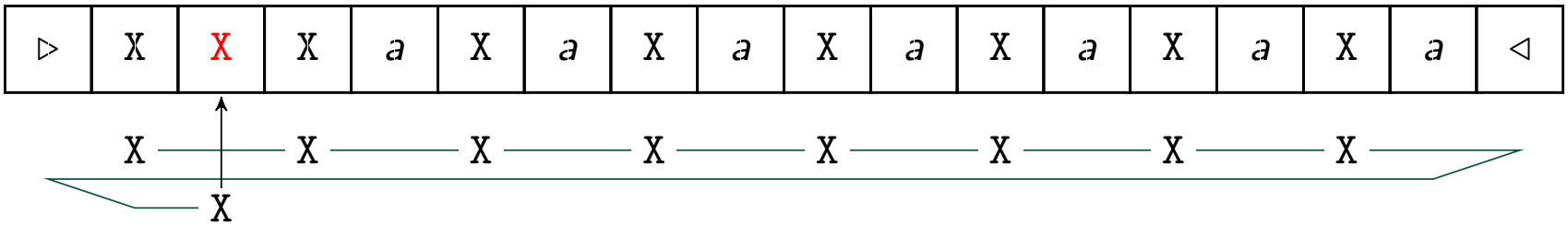
NSE h-PDA 1-LA 1tTM



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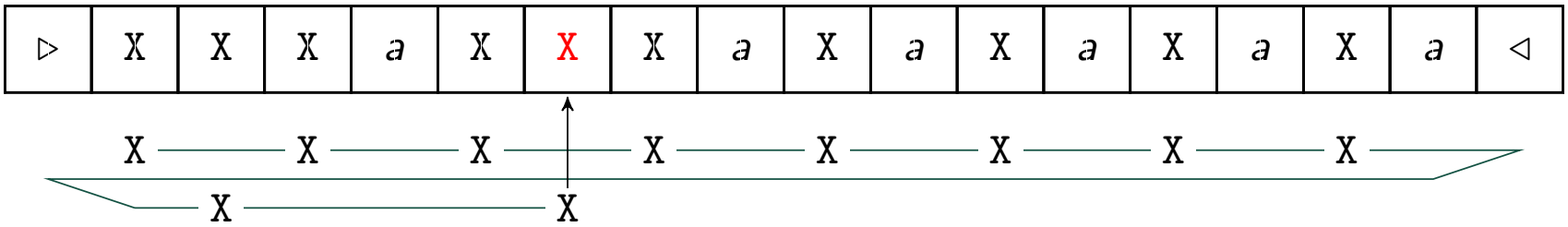

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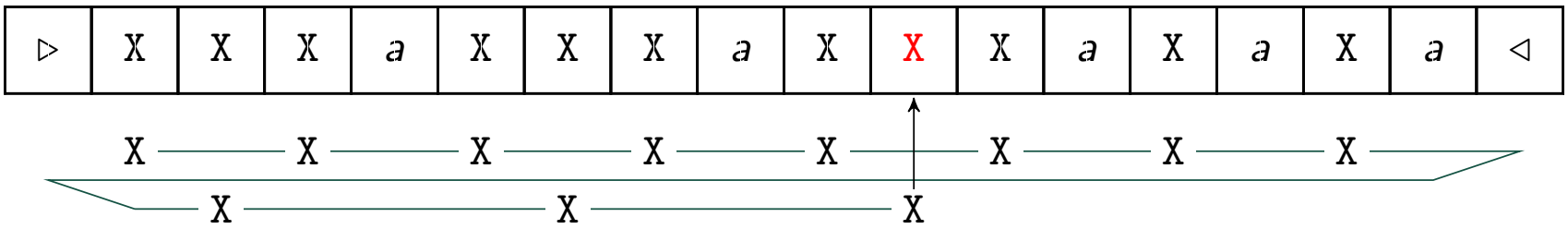

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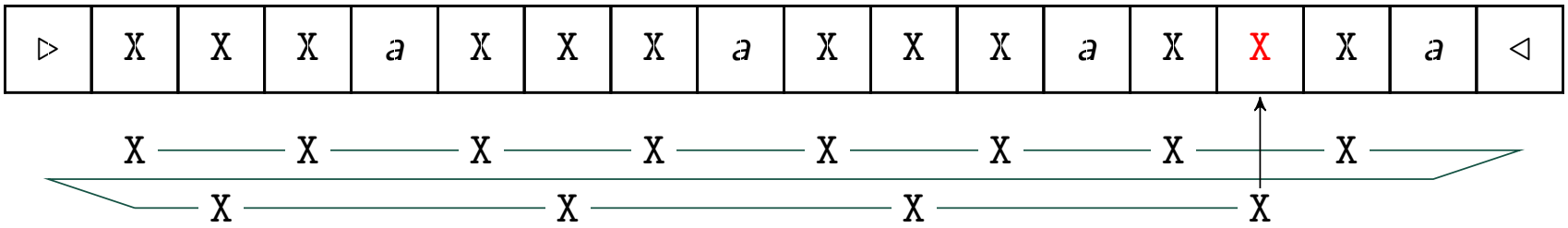

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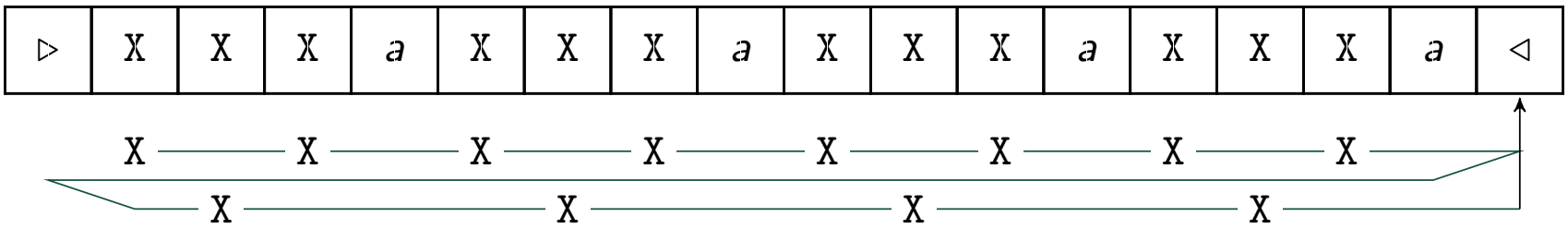

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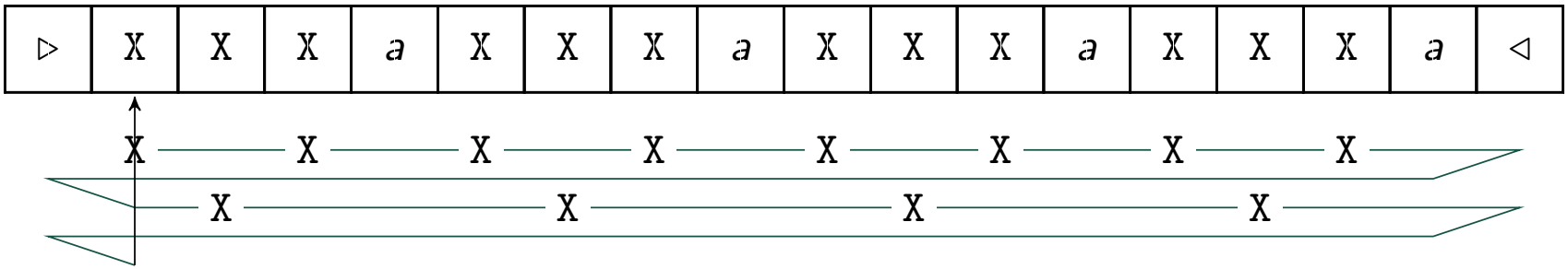

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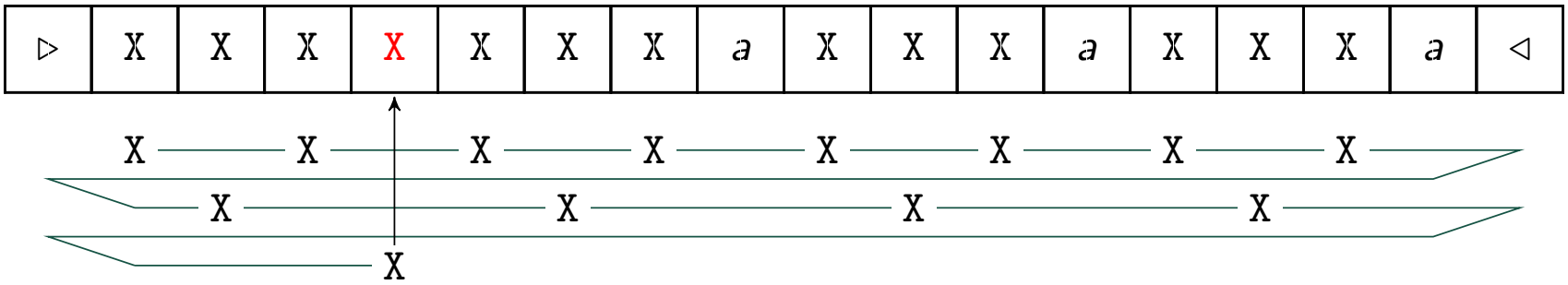

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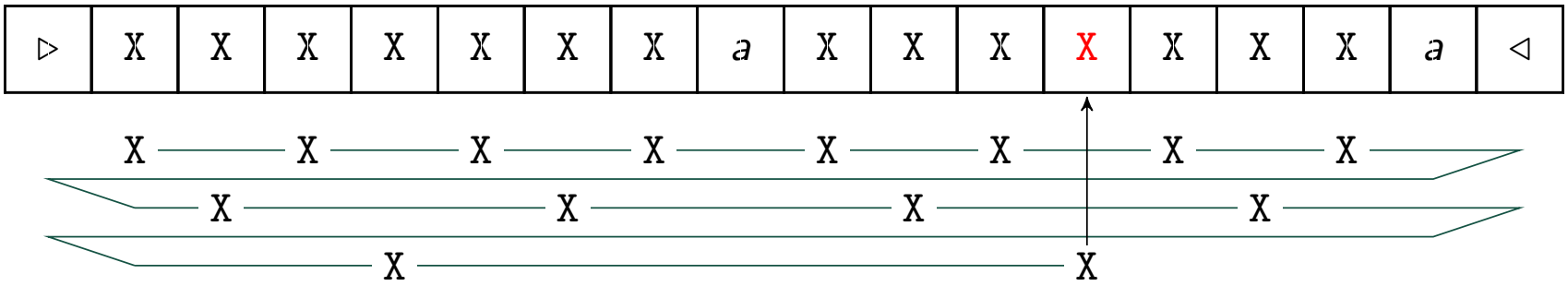

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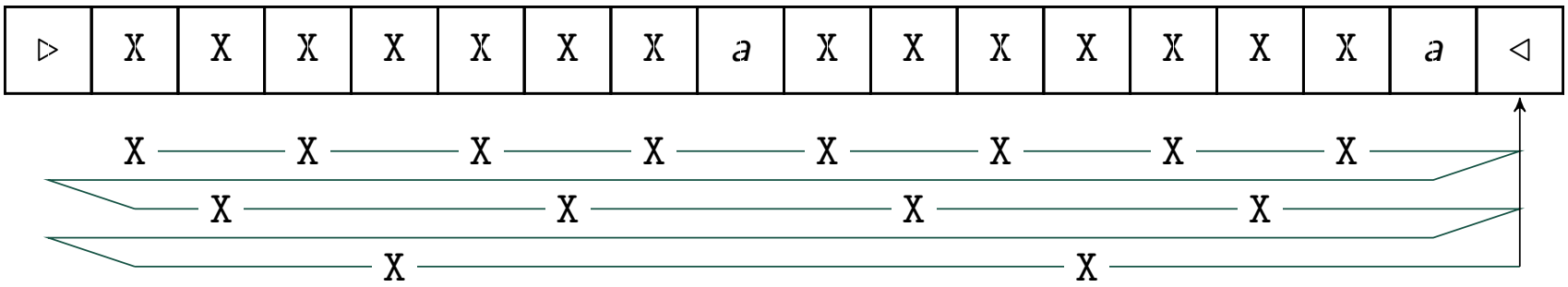

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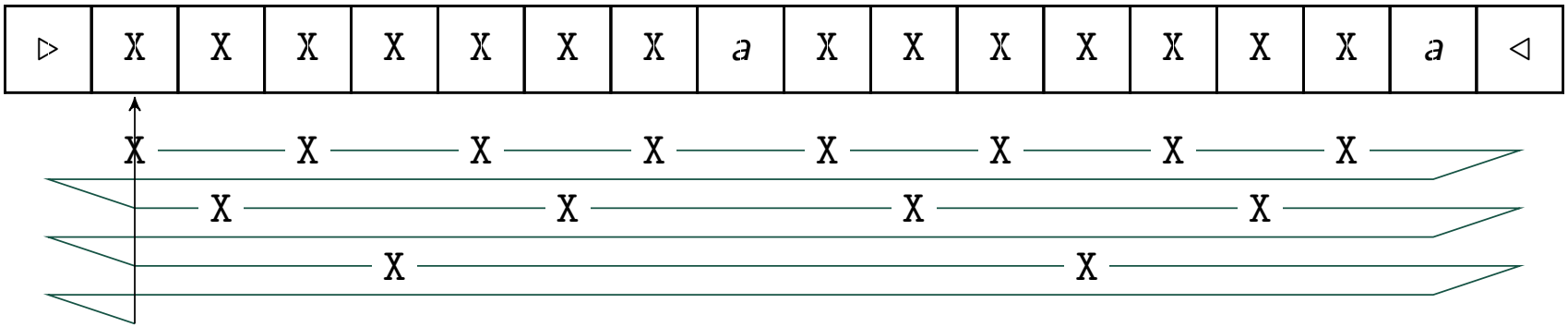
NSE h-PDA 1-LA 1tTM



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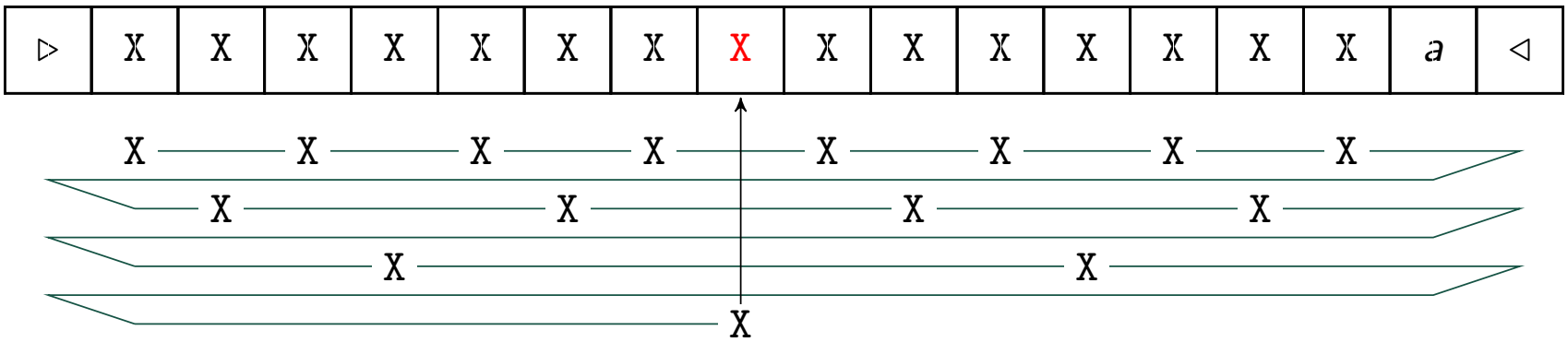

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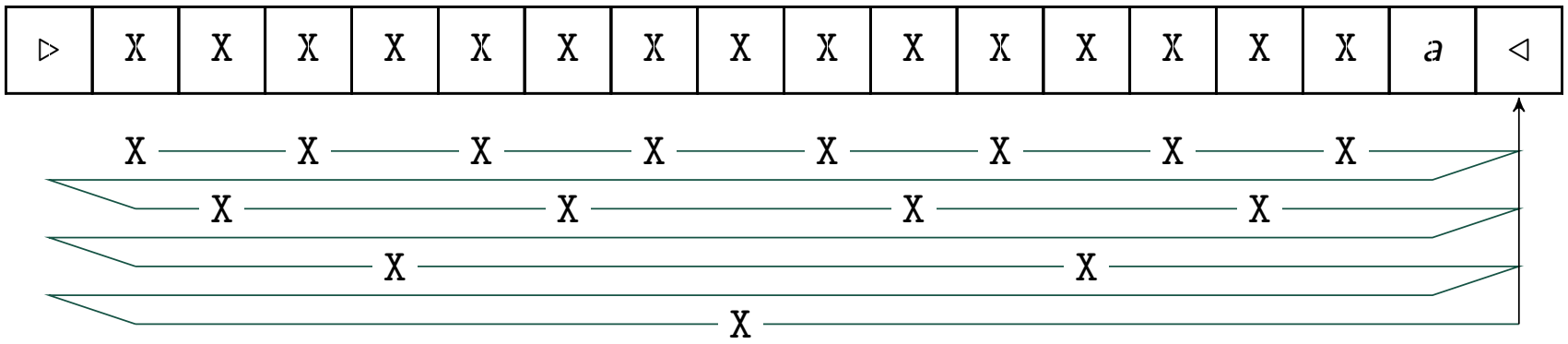

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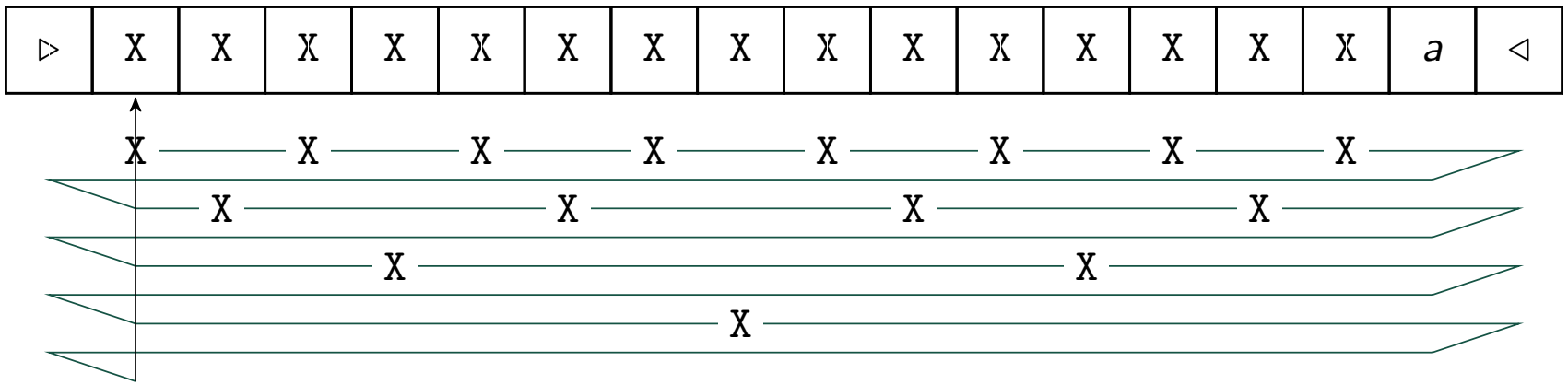
NSE h-PDA 1-LA 1tTM



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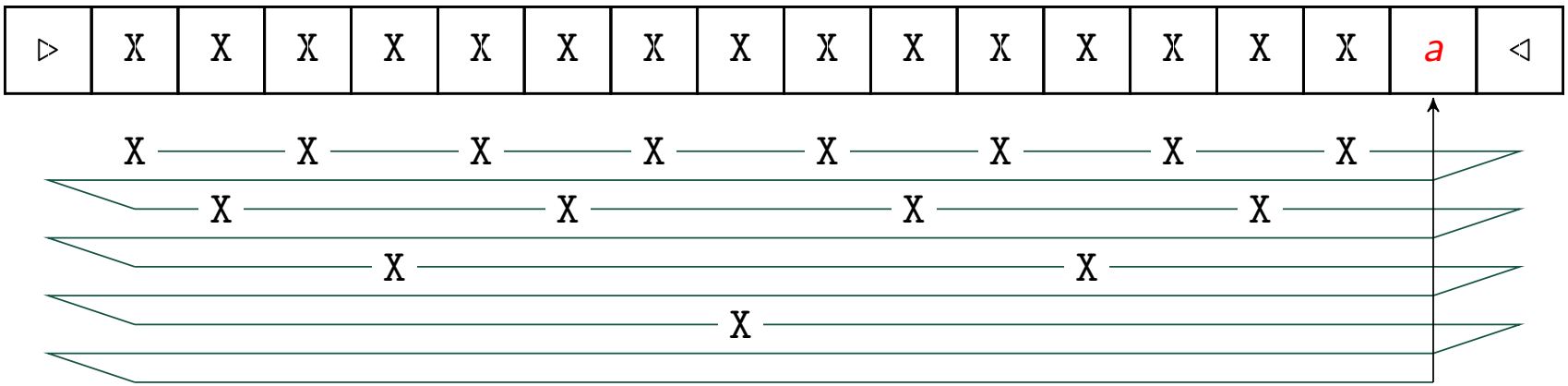

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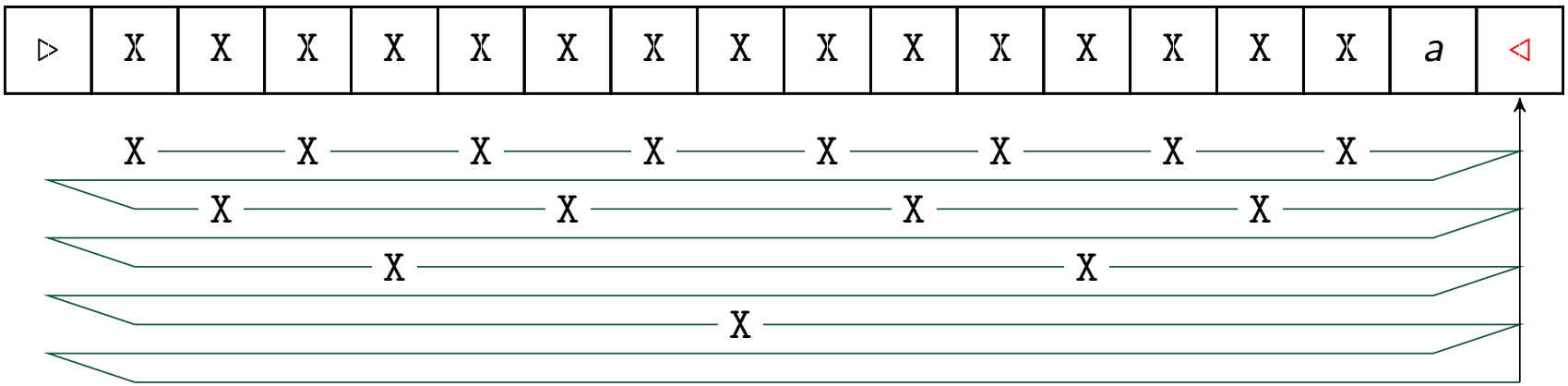

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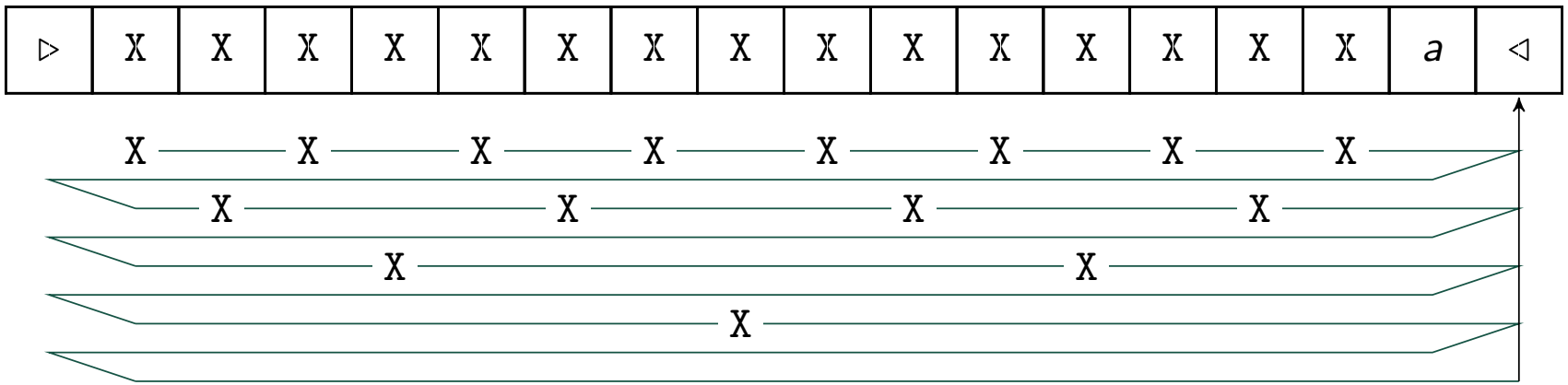

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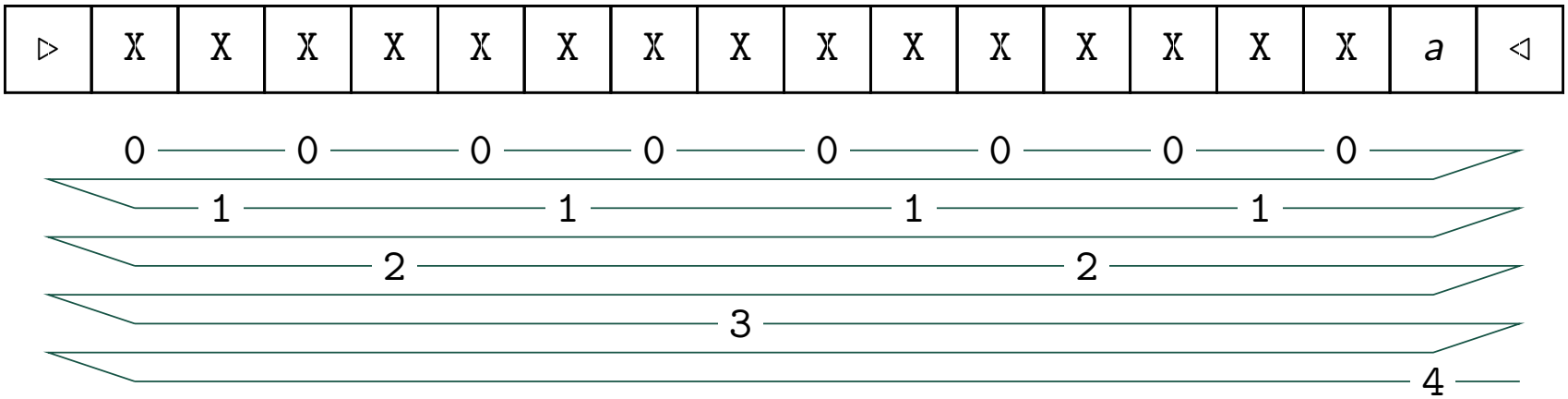

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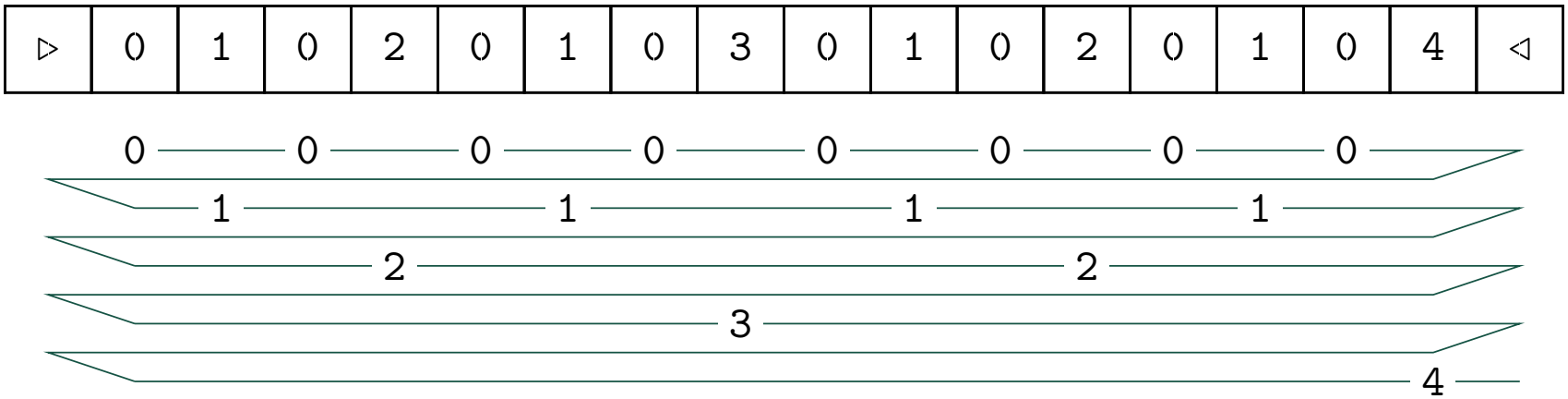

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NSE h-PDA 1-LA 1tTM

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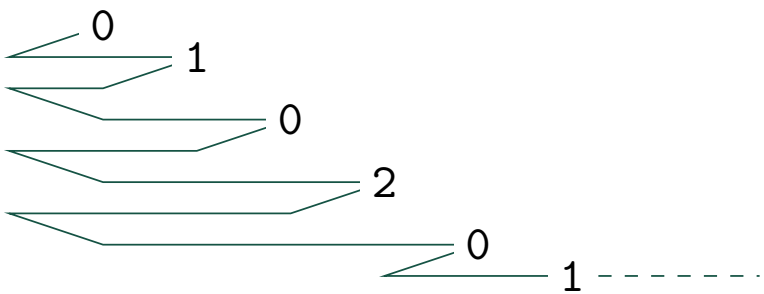

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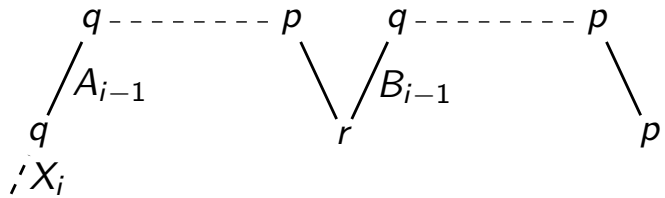

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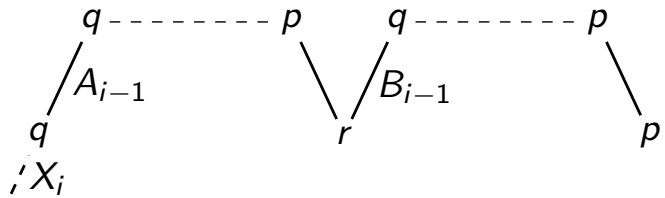

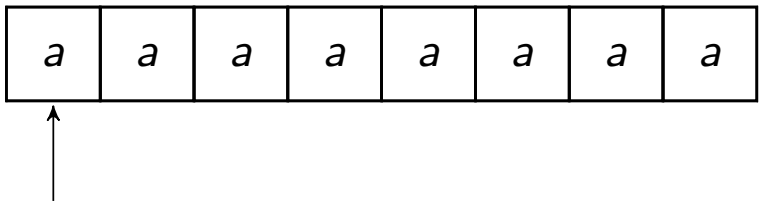

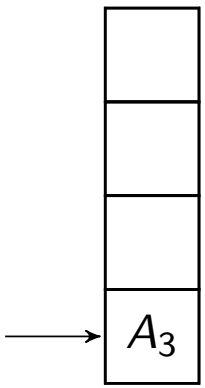

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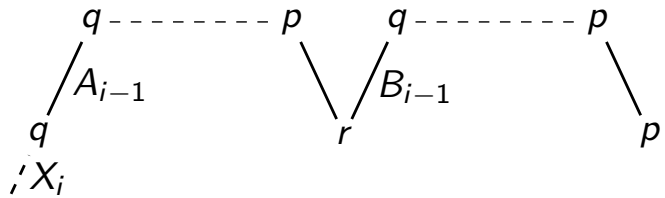

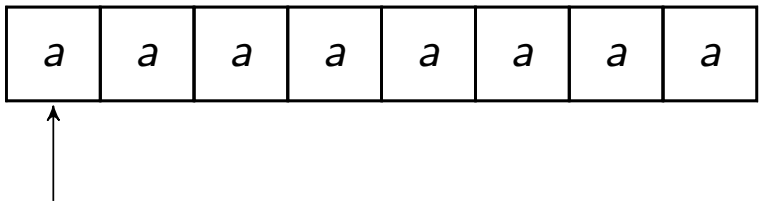

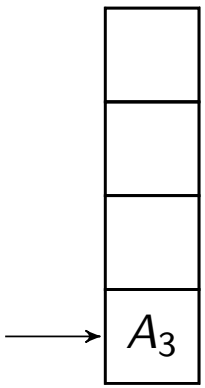

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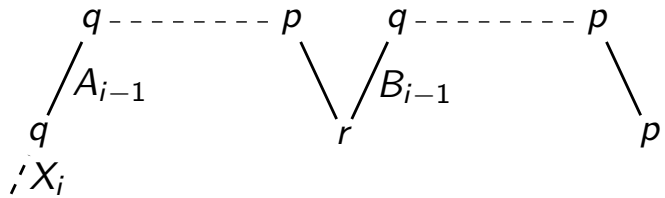

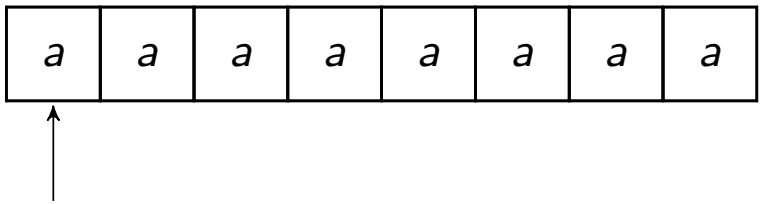

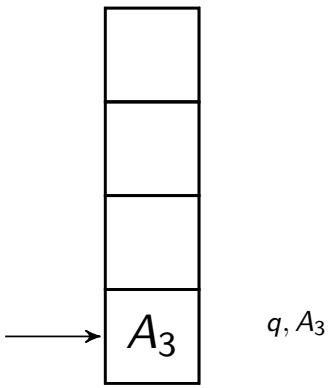

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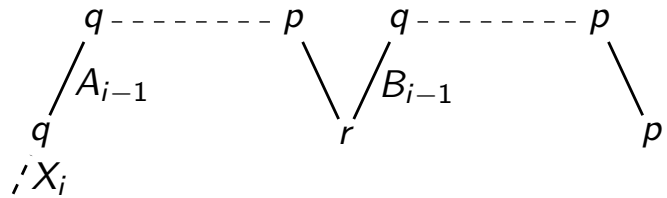

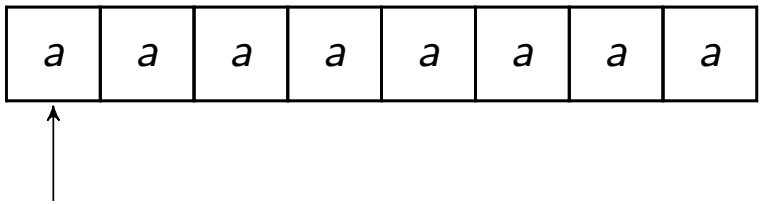

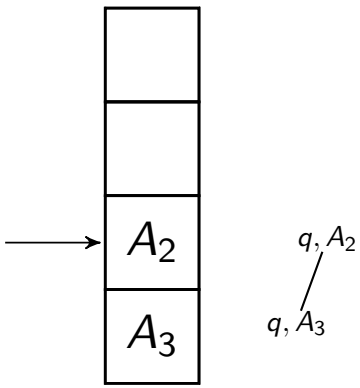

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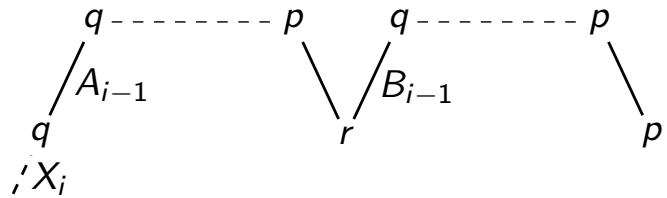

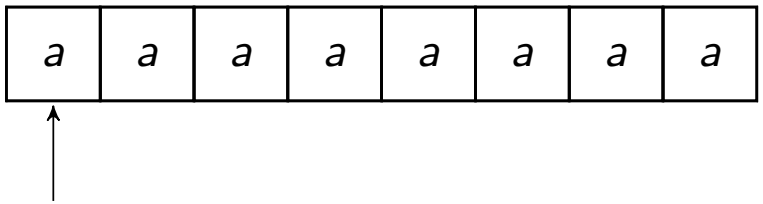

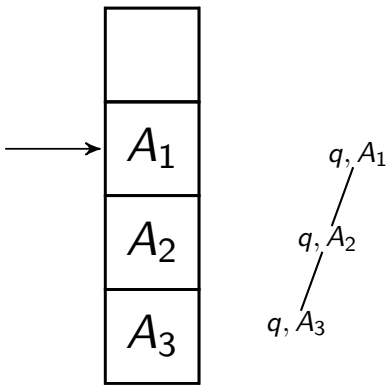

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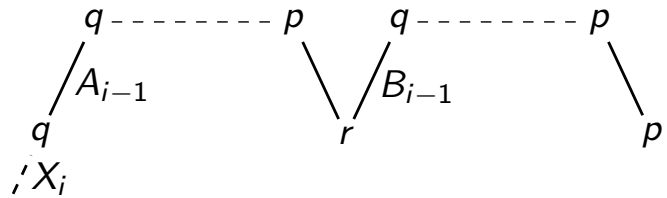

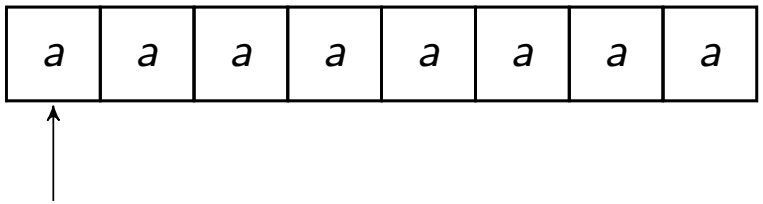

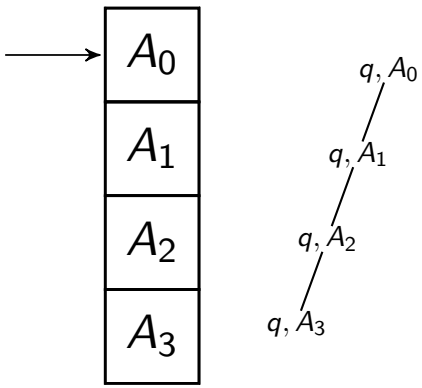

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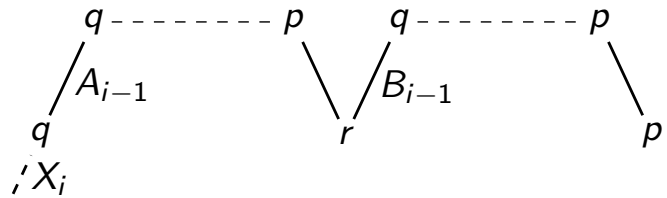

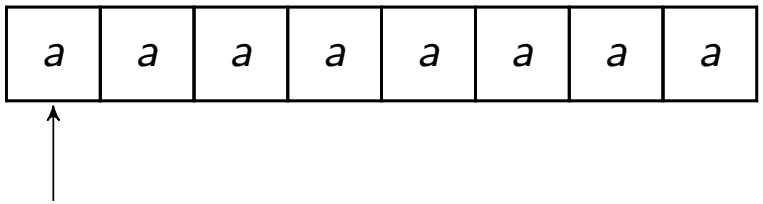

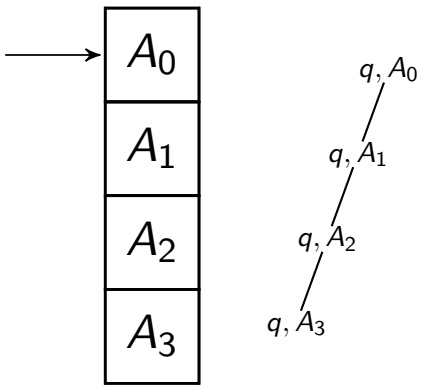

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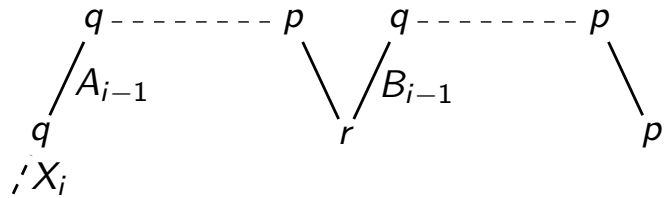

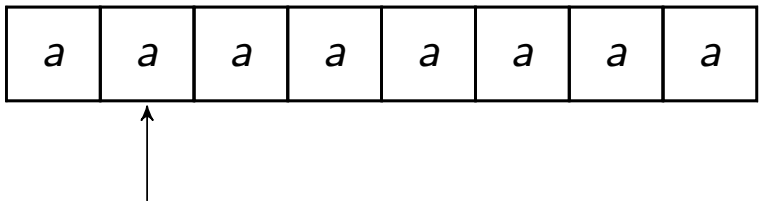

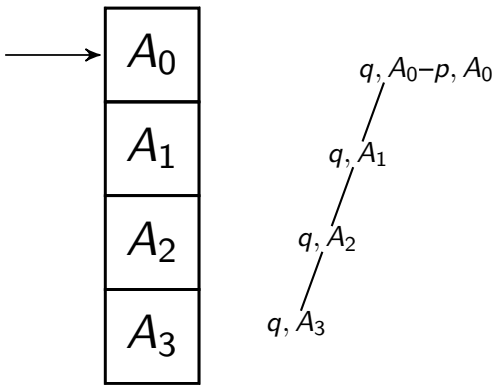

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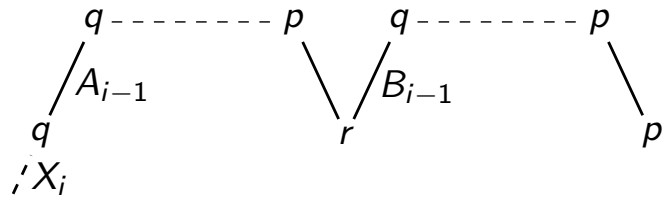

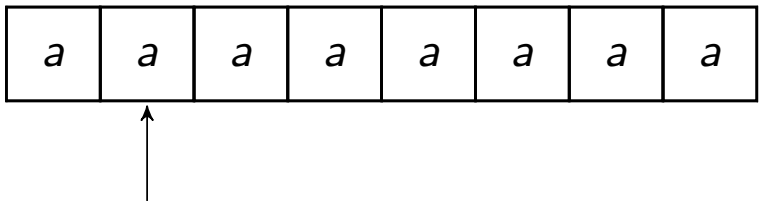

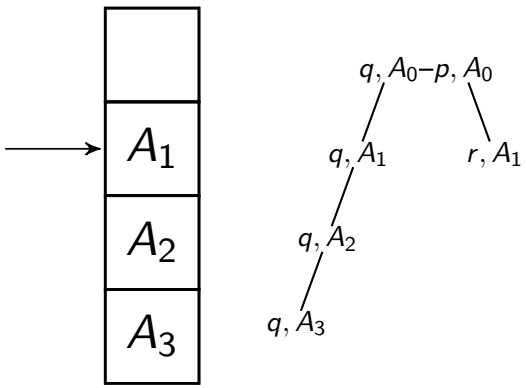

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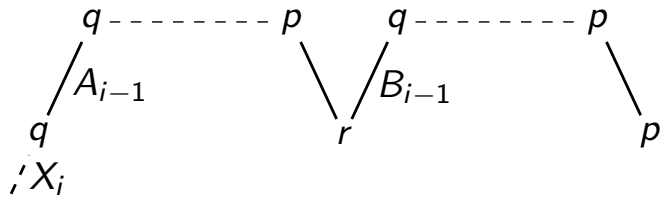

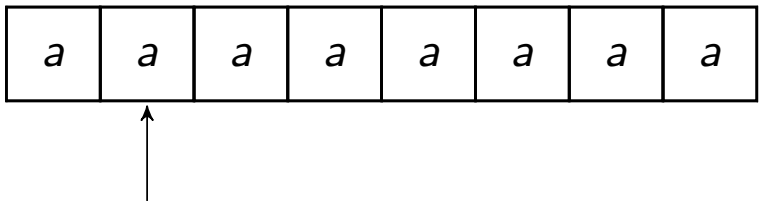

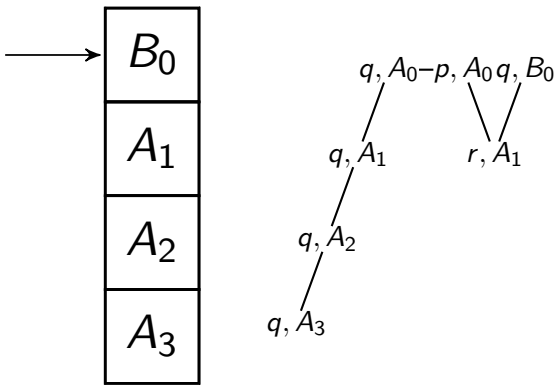

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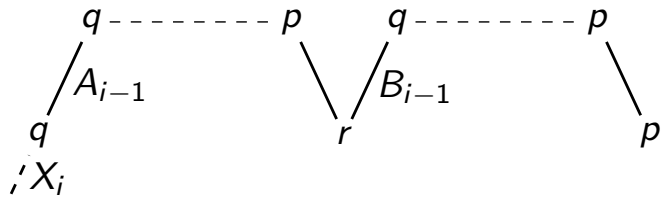

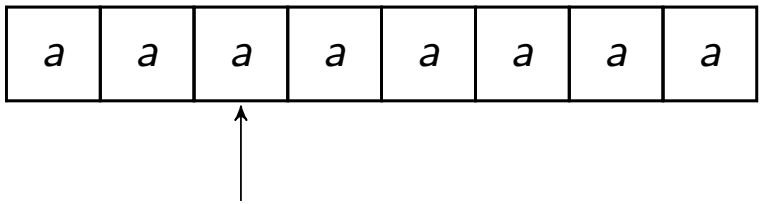

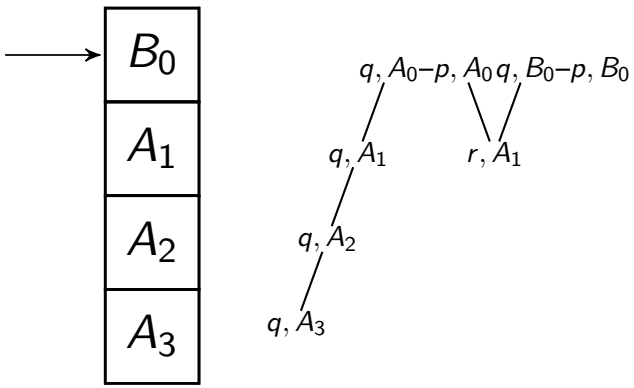

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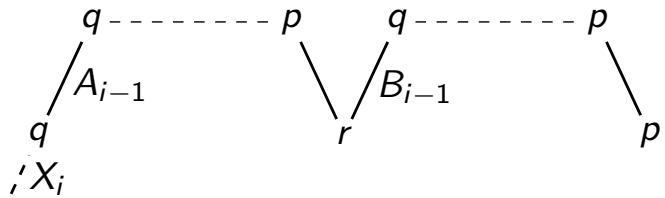

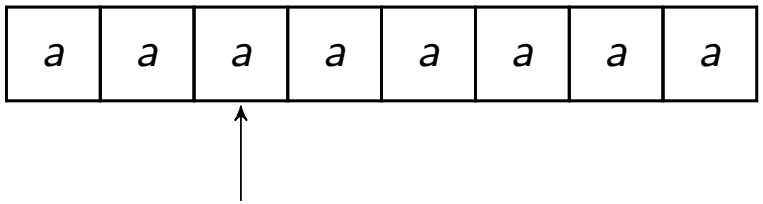

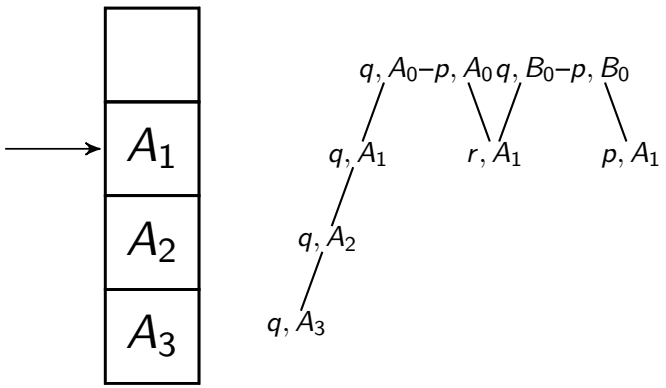

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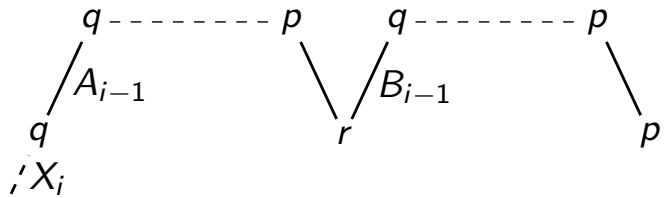

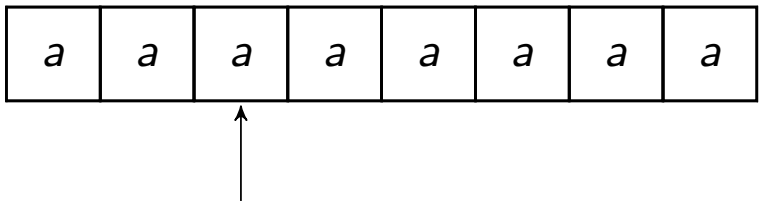

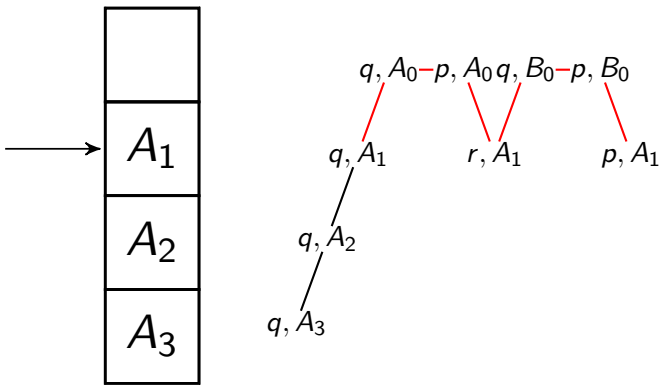

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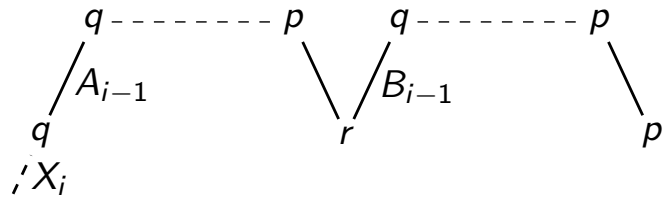

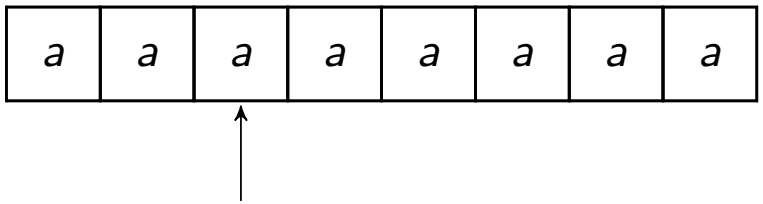

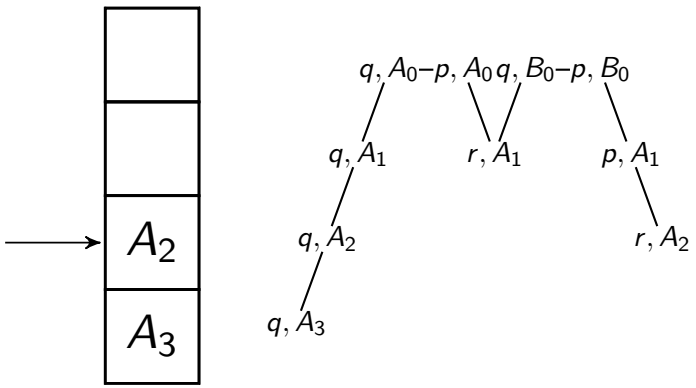

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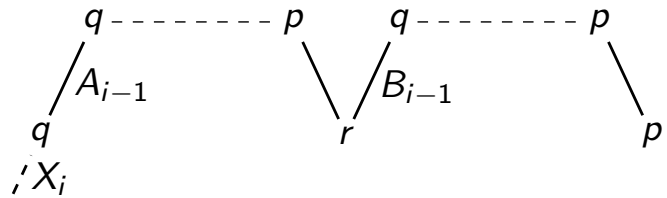

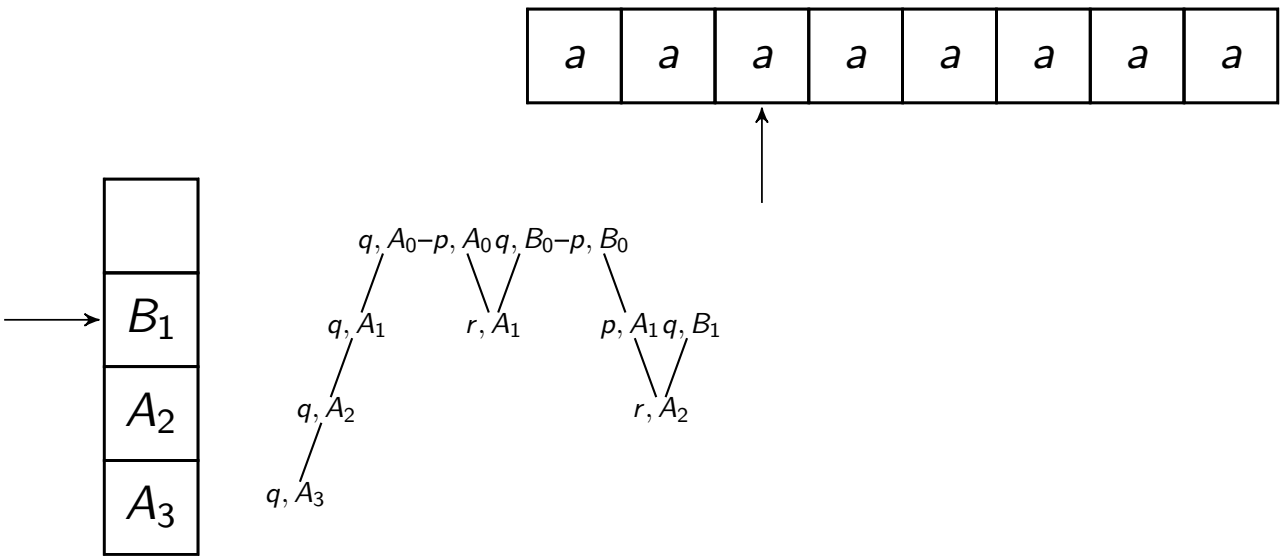

$$\{\, a^{2^k} \mid k \geq 0 \text{ is a fixed integer} \,\}$$

NSE h-PDA 1-LA 1tTM

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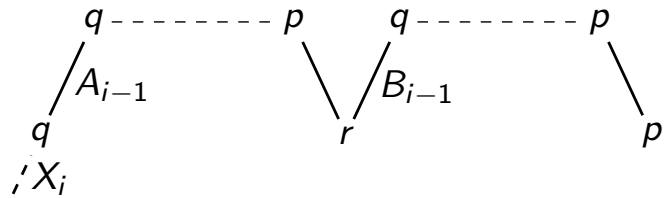

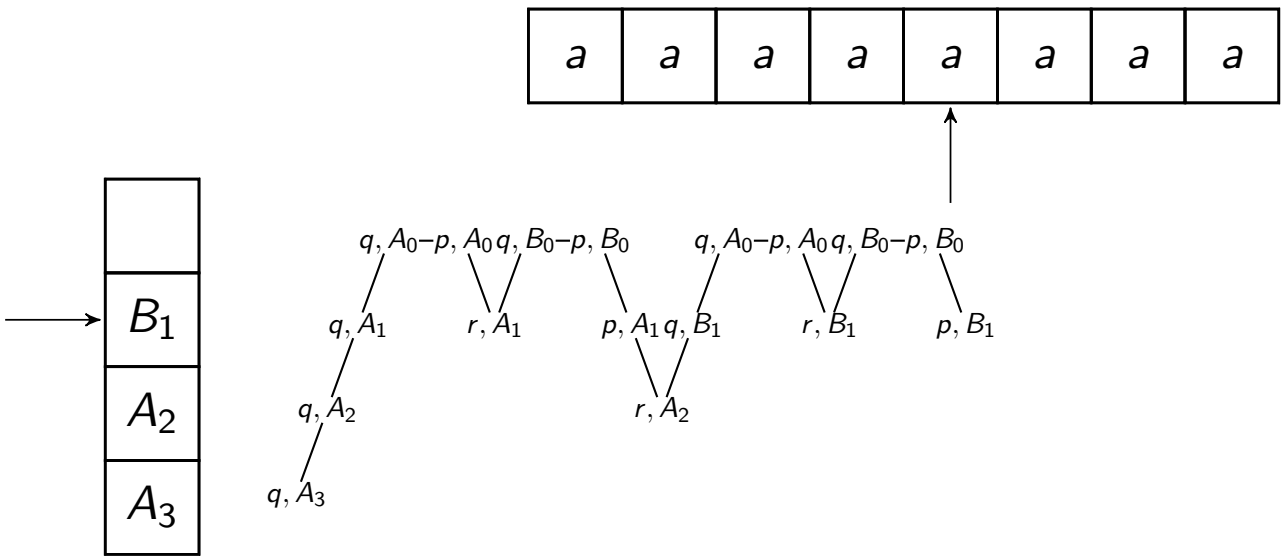

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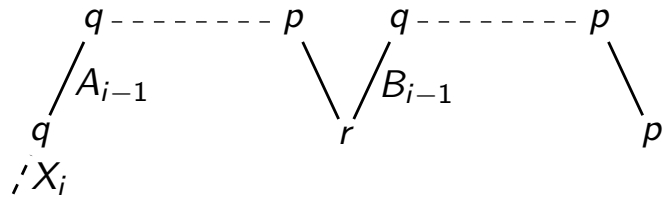

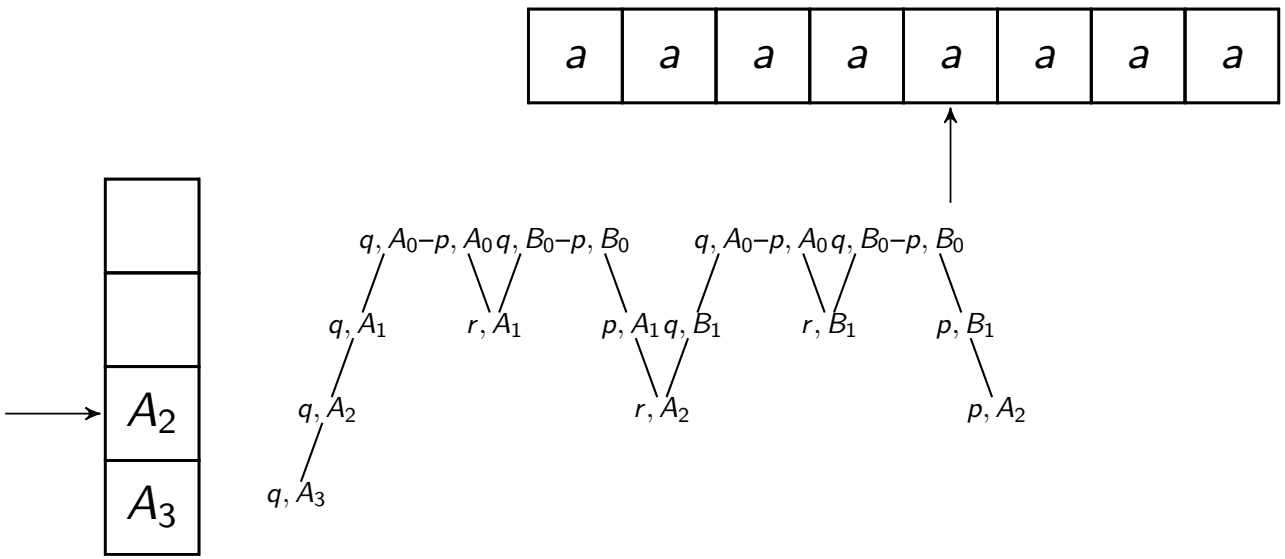

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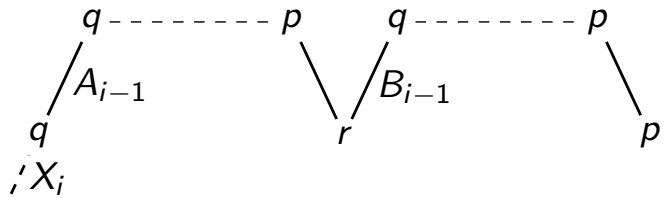

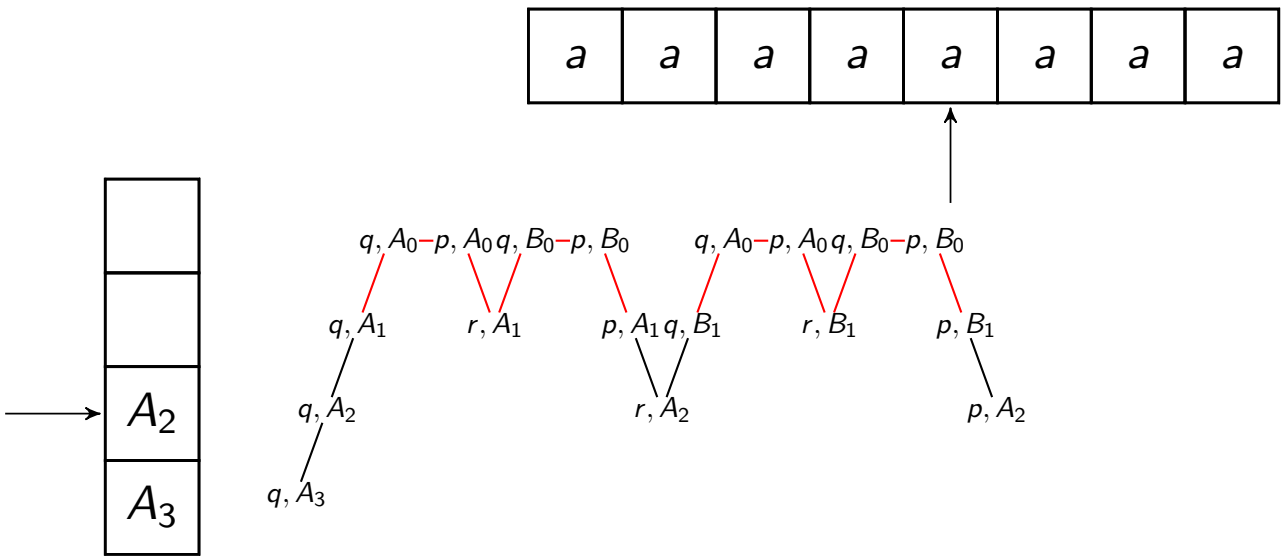

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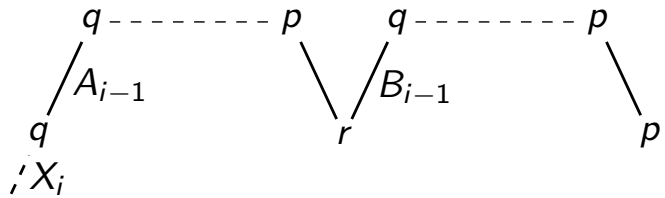

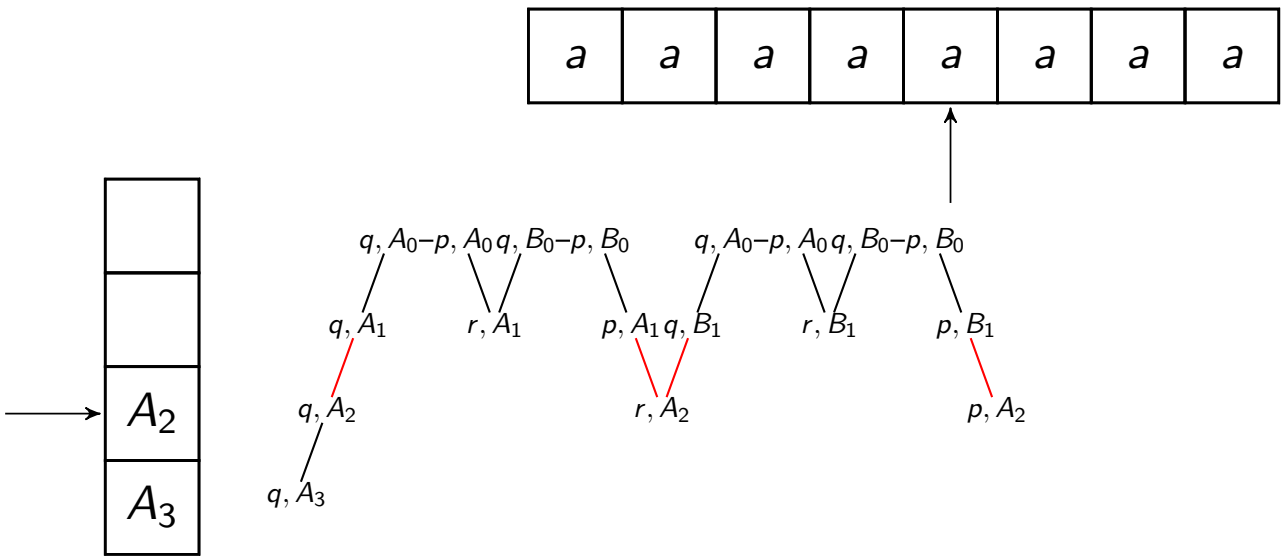

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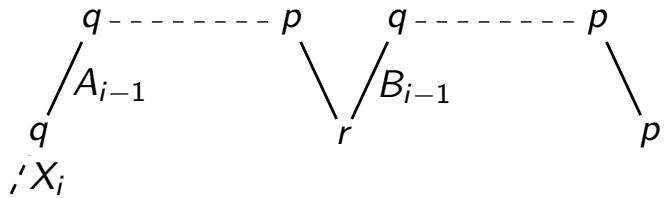

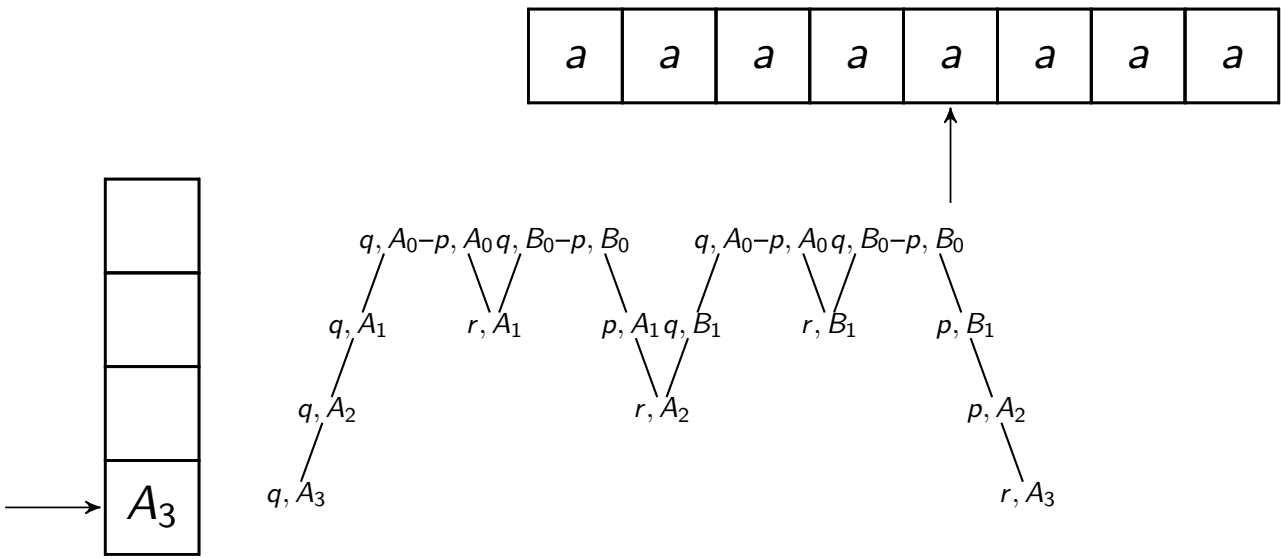

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NSE *h*-PDA 1-LA 1tTM

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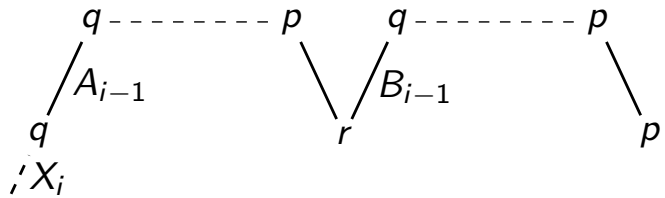

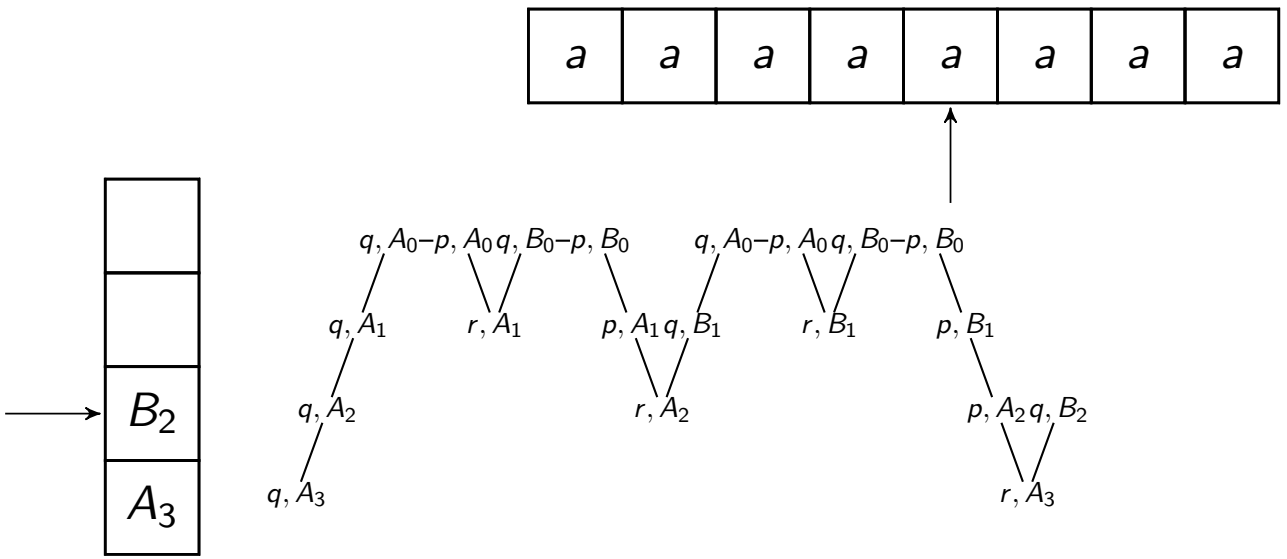

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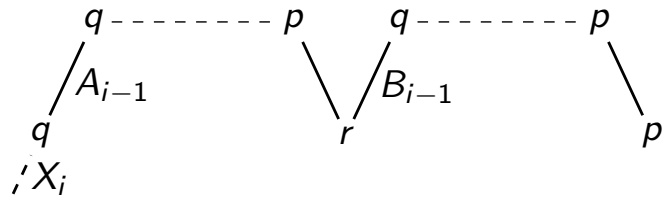

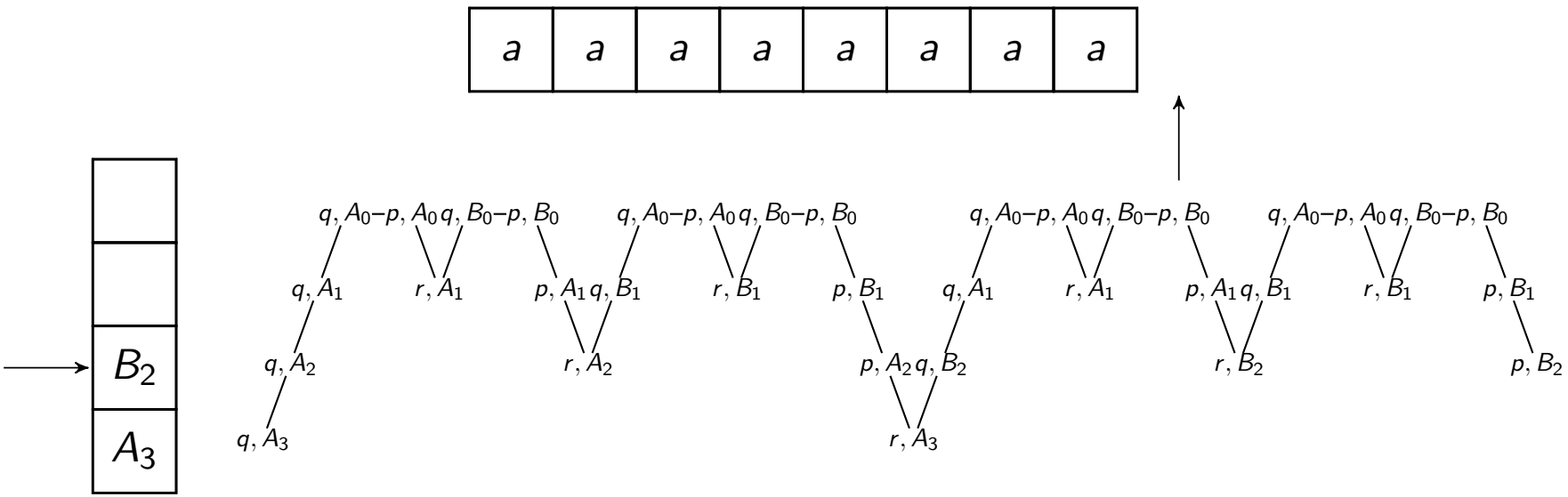

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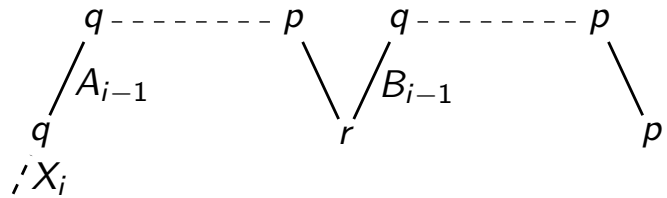

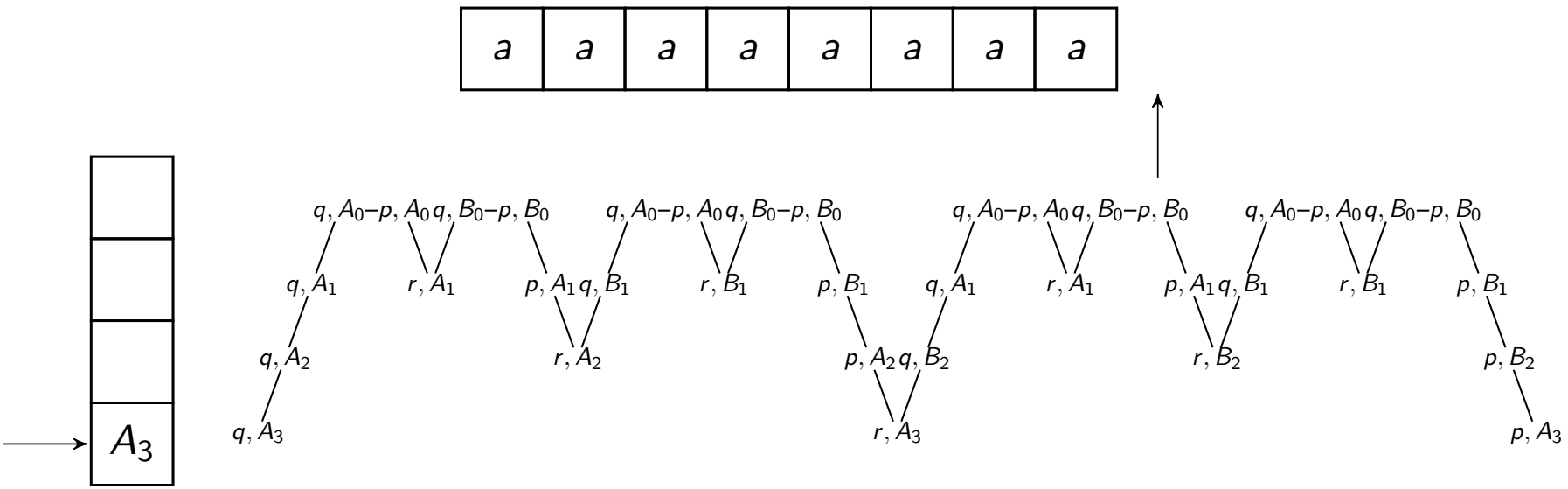

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NSE h-PDA 1-LA 1tTM

$$f(i) = \begin{cases} 1 & \text{if } i = 0 \\ 2f(i-1) & \text{otherwise} \end{cases}$$

$$A_0 \to a$$
$$A_i \to A_{i-1}A_{i-1}$$

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NSE h-PDA 1-LA 1tTM

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$$A_3$$

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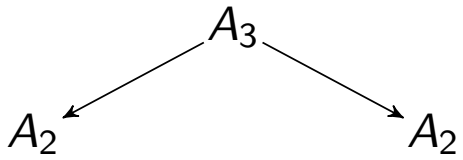

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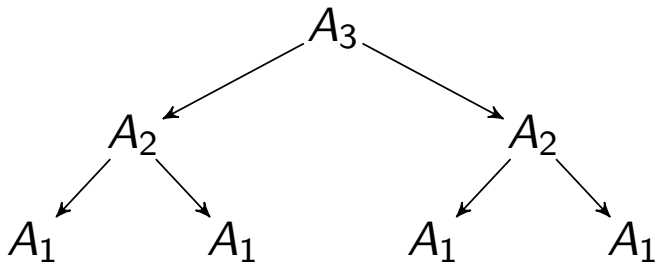

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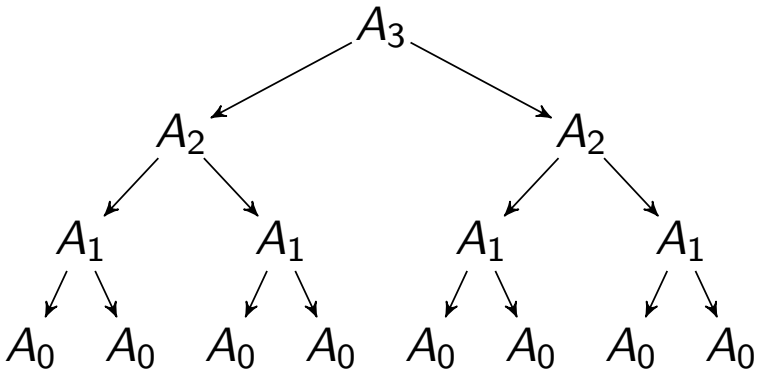

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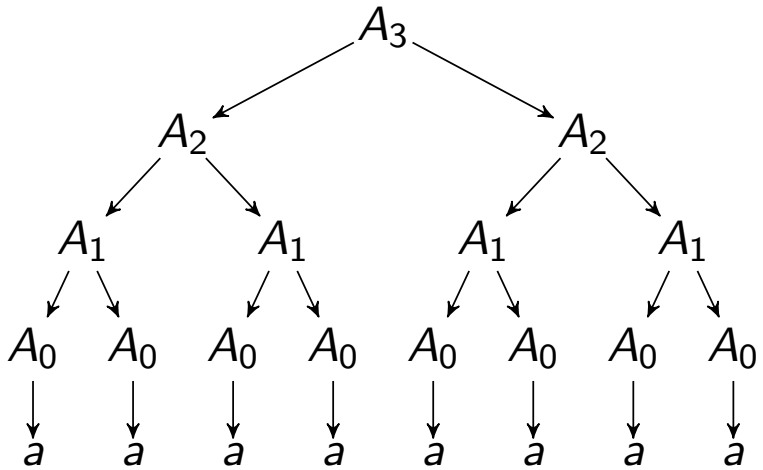

NSE h-PDA 1-LA `1tTM`

FA


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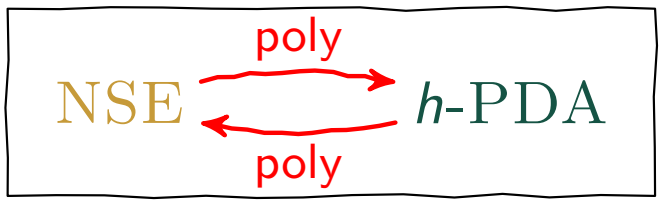

1-LA

ltTM

FA

Bruno Guillon, Giovanni Pighizzini, and Luca Prigioniero:
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International Journal of Foundations of Computer Science 31(8), pp. 1133–1157, 2020.
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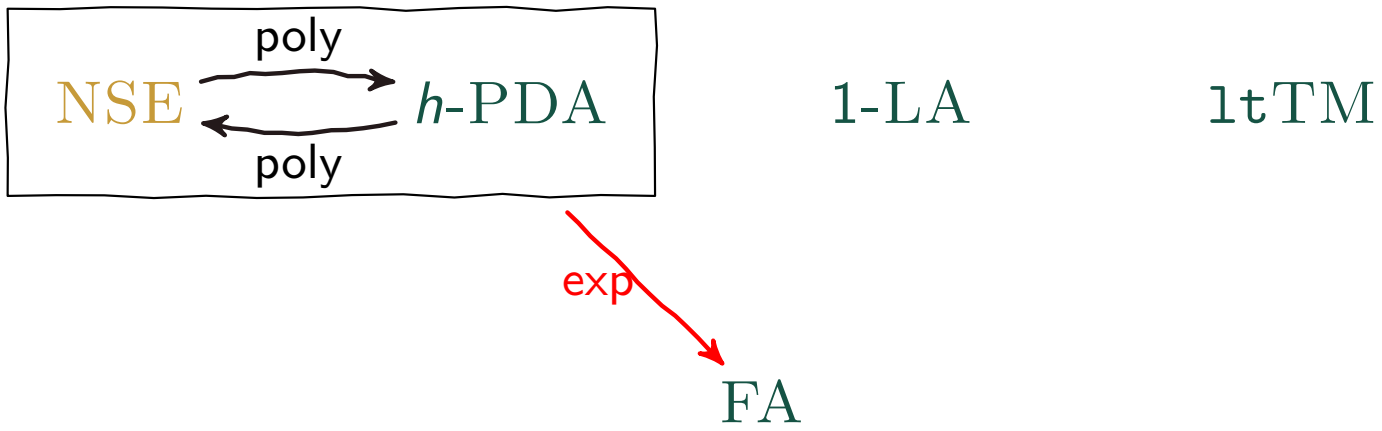

Giovanni Pighizzini and Luca Prigioniero:
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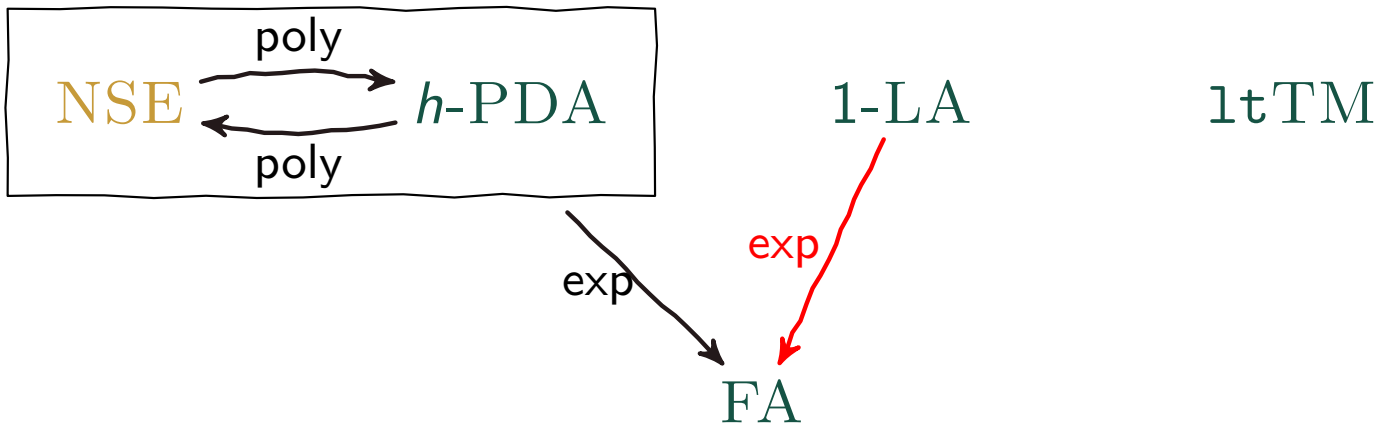
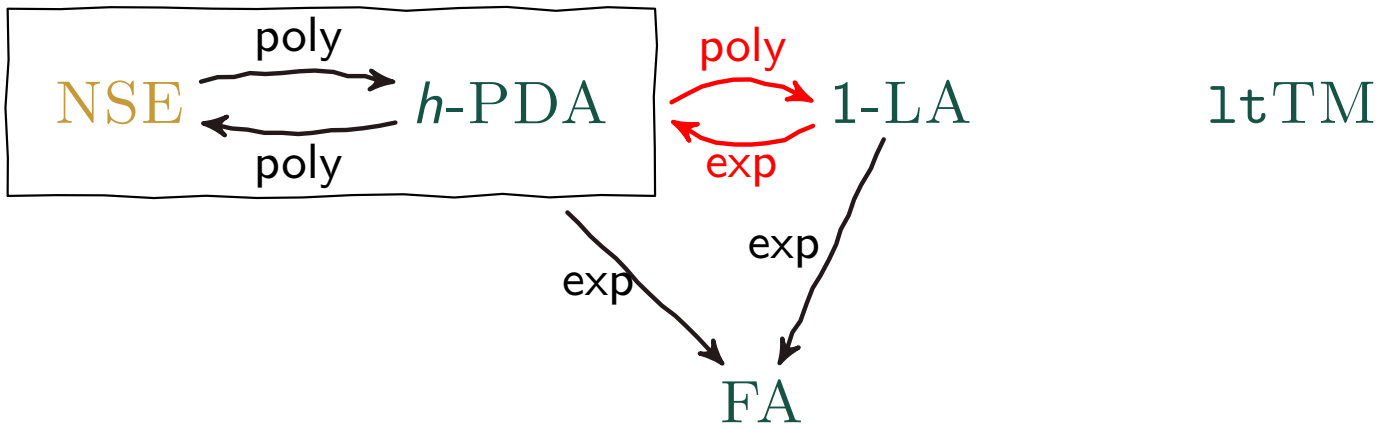
DOI: 10.3233/FI-2021-2036. URL: https://doi.org/10.3233/FI-2021-2036



Giovanni Pighizzini and Luca Prigioniero:
Limited Automata and Unary Languages.
Information and Computation 266, pp. 60–74, 2019.
DOI: 10.1016/j.ic.2019.01.002



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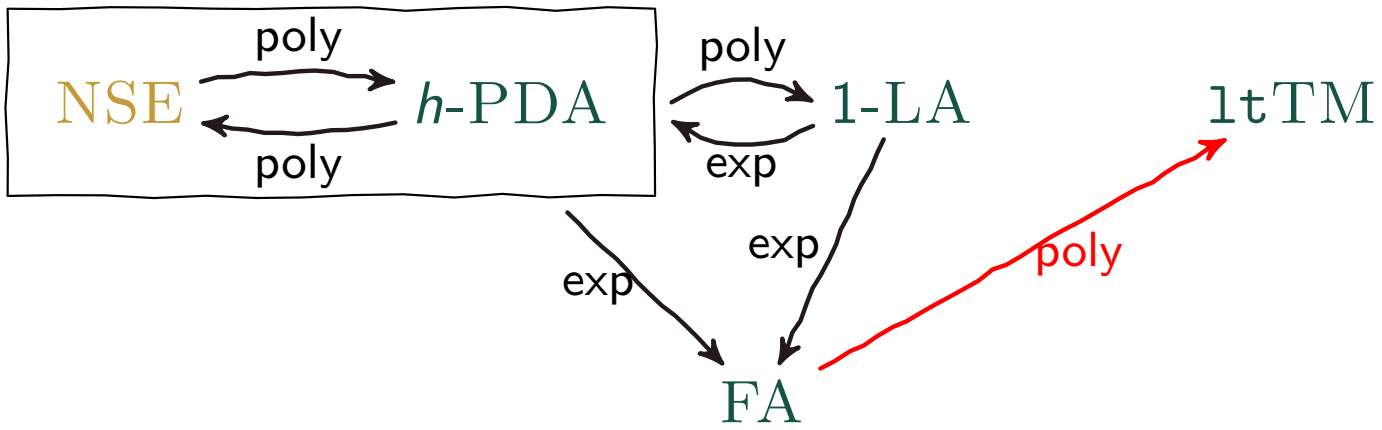
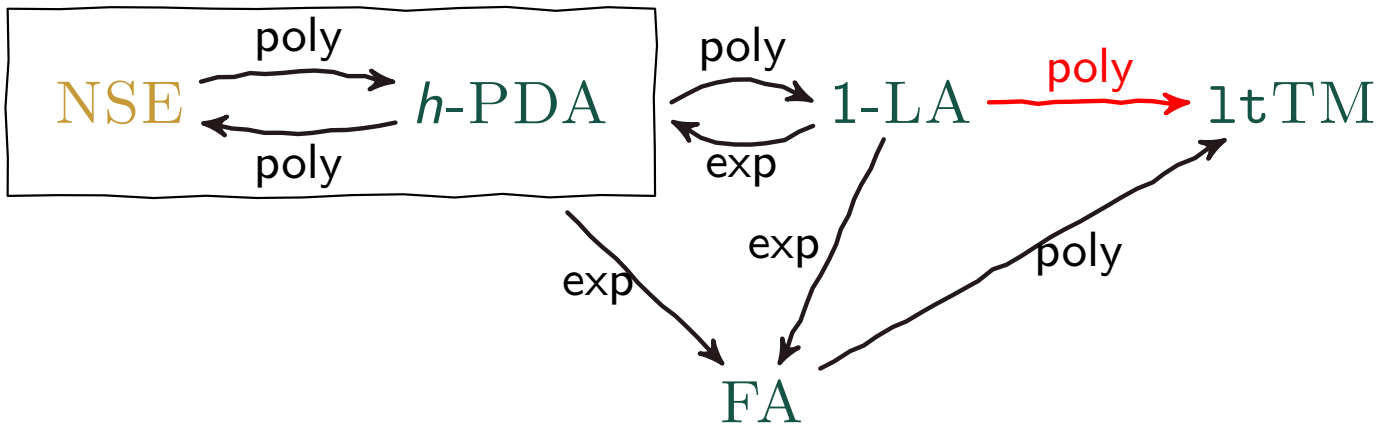


Bruno Guillon, Giovanni Pighizzini, Luca Prigioniero, and Daniel Průša:
Two-Way Automata and One-Tape Machines - Read Only Versus Linear Time.
22nd International Conference on Developments in Language Theory, DLT 2018.
Lecture Notes in Computer Science, vol. 11088, pp. 366–378. Springer, 2018



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Linear-time limited automata.
Theoretical Computer Science 798, pp. 95–108, 2019.
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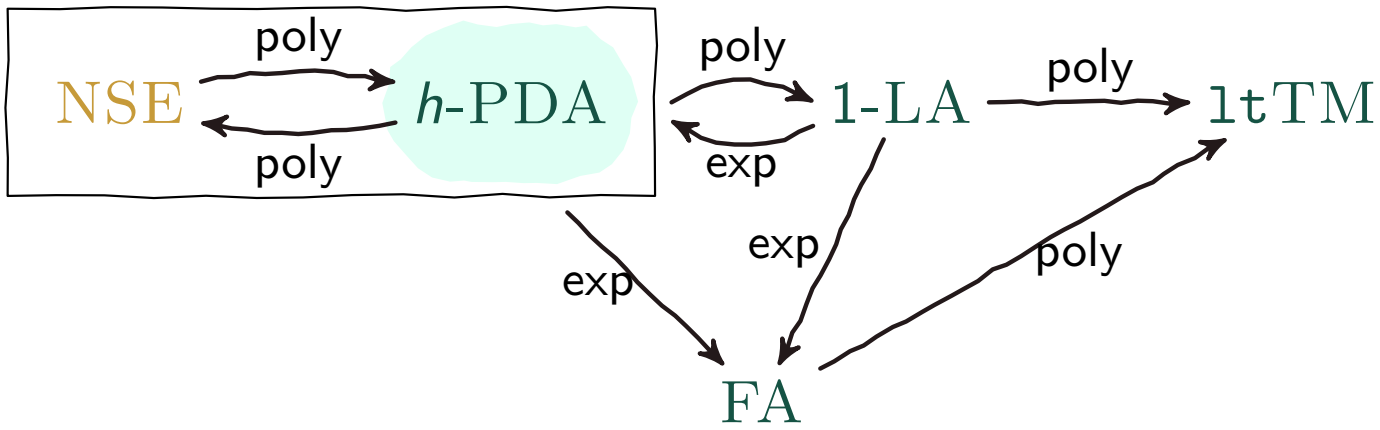
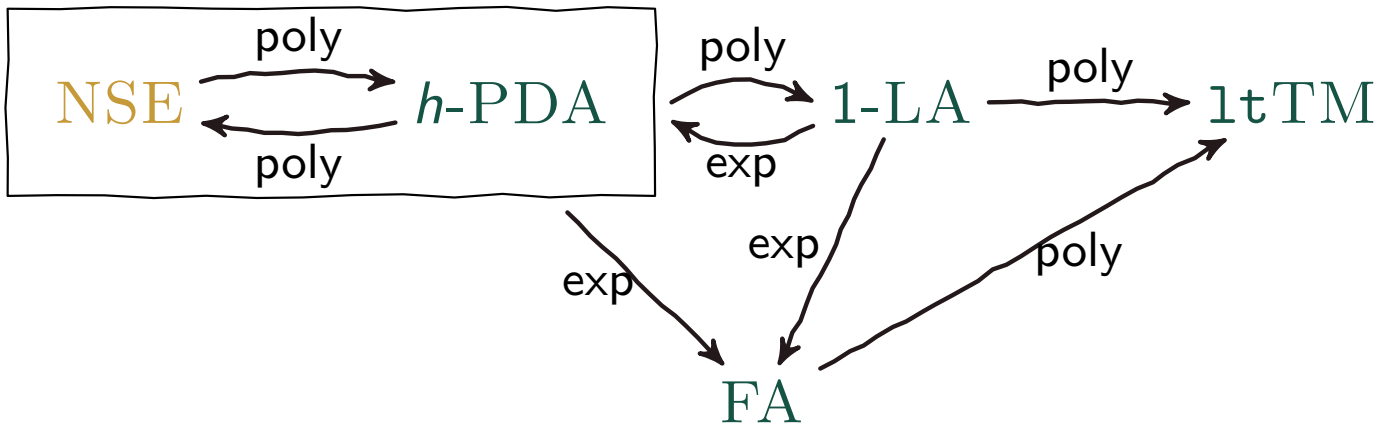


Giovanni Pighizzini and Luca Prigioniero:
Pushdown Automata and Constant Height: Decidability and Bounds.
21st International Conference on Descriptional Complexity of Formal Systems, DCFS 2019.
Lecture Notes in Computer Science, vol. 11612, pp. 260–271. Springer, 2019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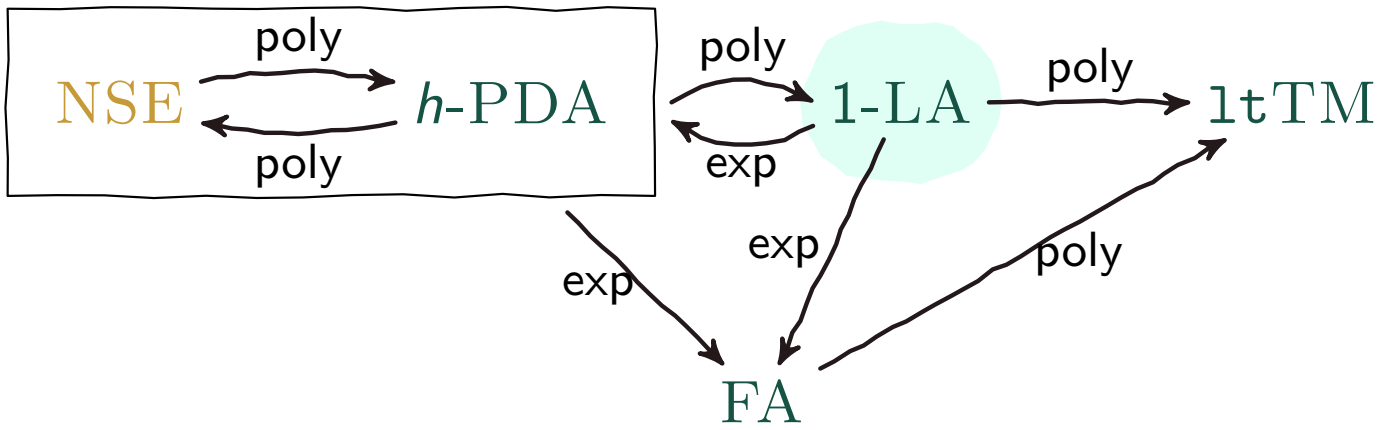
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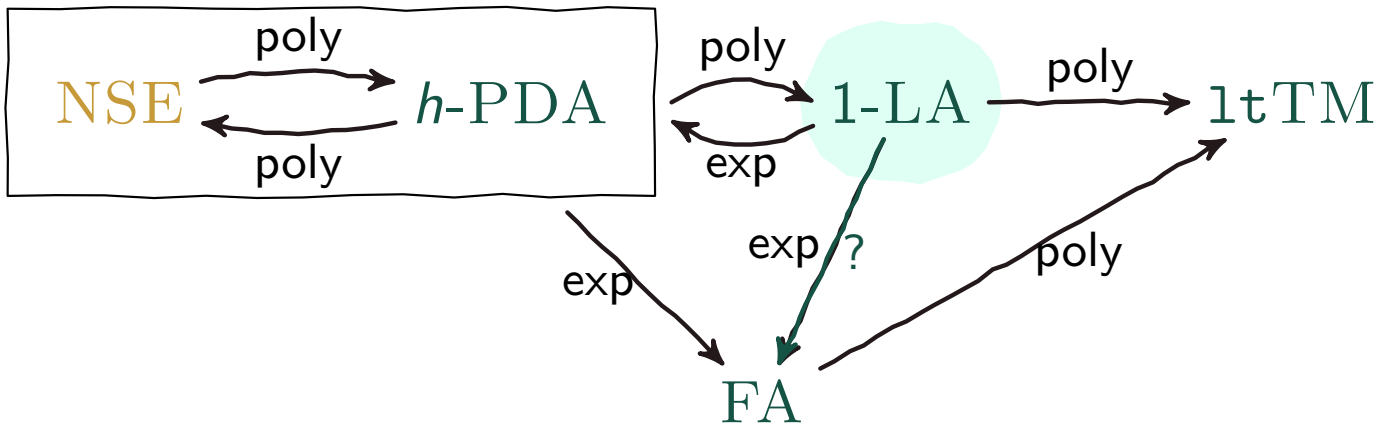
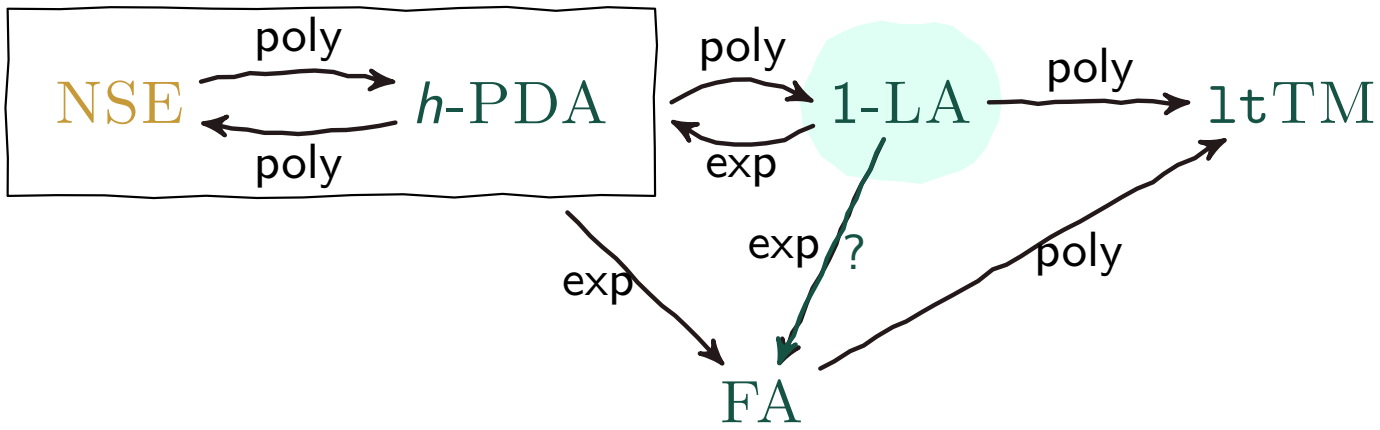
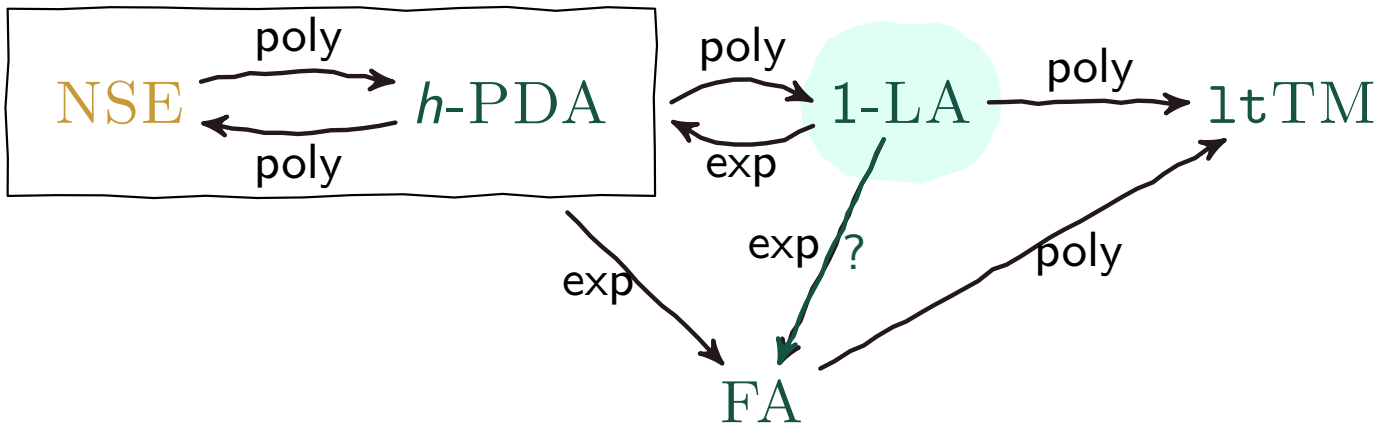


$$\text{unary 1-LA} \xrightarrow{?} \text{DFA}$$

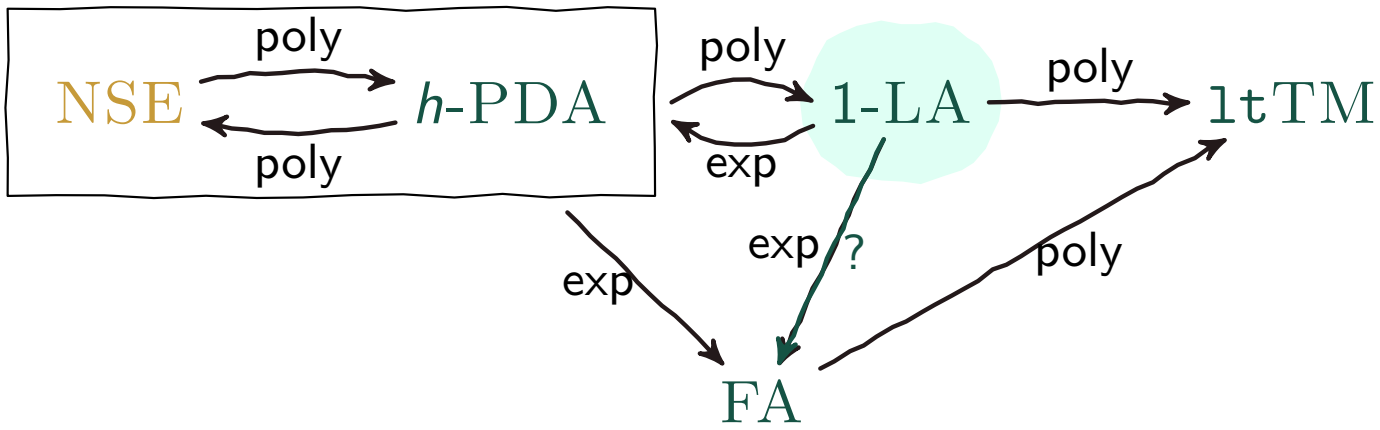


$$\text{unary 1-LA} \stackrel{?}{\to} \text{DFA}$$

$$\text{1-LA} \stackrel{?}{\to} \text{deterministic 1-LA}$$



$$\text{unary 1-LA} \stackrel{?}{\to} \text{DFA}$$
$$\text{1-LA} \stackrel{?}{\to} \text{deterministic 1-LA}$$
$$\text{1-LA} \stackrel{?}{\to} \text{2NFA}$$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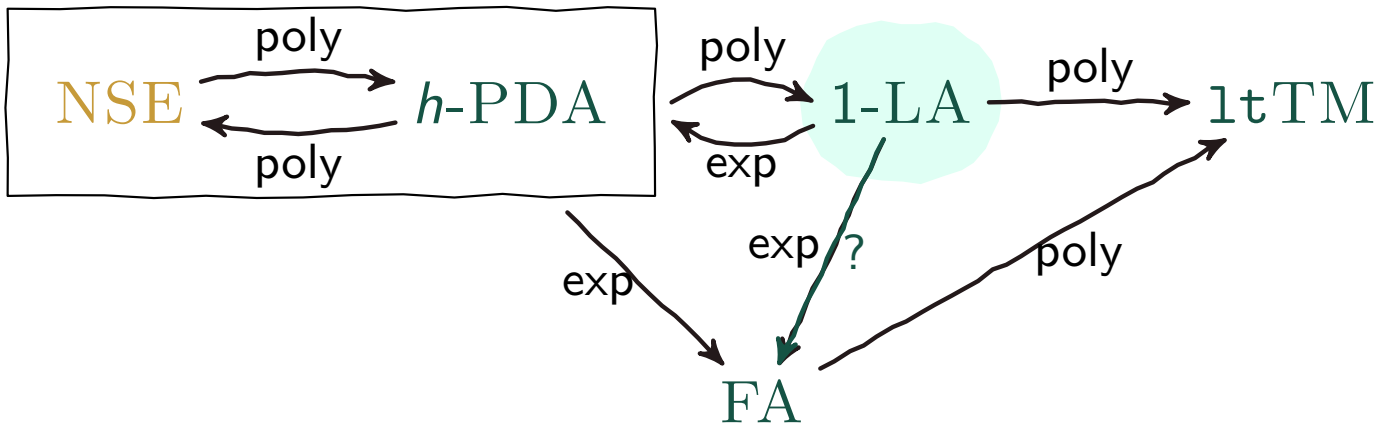

Questions?