Von Neumann regularity, split epicness and elementary cellular automata

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(Von Neumann) regular elements in monoids

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Let S be a monoid (set + associative product + identity element). We say $a \in S$ is *(Von Neumann) regular* if $\exists b \in S : aba = a \wedge b = bab$. We say b is a *generalized inverse* of a.

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For simplicity, let us concentrate on weak generalized inverses:

a **is regular iff the equation** $aba = a$ **has a solution**

Regular elements in monoids of functions

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A cellular automaton is regular iff you can pick preimages for all points (that have a preimage) by another cellular automaton.

Question of Castillo-Ramirez & Gadouleau

Castillo-Ramirez & Gadouleau study regular elements in cellular automata monoids (over general groups). They raise the following question:

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Which elementary cellular automata are regular (in the monoid $\mathrm{CA}(\{0,1\}^{\mathbb{Z}})$)?

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Recall that *elementary cellular automata* are maps $f : \{0,1\}^{\mathbb{Z}} \to \{0,1\}^{\mathbb{Z}}$ such that for some function $f_{\mathrm{loc}} : \{0,1\}^3 \mapsto \{0,1\}$ we have $\forall x \in \{0,1\}^{\mathbb{Z}} : f(x)_i = f_{\mathrm{loc}}(x_{[i-1,i+1]})$.

First cases of non-regularity

Lemma

If $f \in \mathrm{CA}(A^{\mathbb{Z}})$ is regular, then every p-periodic point $y \in f(A^{\mathbb{Z}})$ has a p-periodic f-preimage.

Proof.

A cellular automaton cannot increase the period, so the cellular automaton g giving the preimage must give one with the same period. □

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If $f \in \mathrm{CA}(A^{\mathbb{Z}})$ is regular, then so are all CA equivalent to it, i.e. obtained by flipping left and right, and/or pre- and/or postcomposing with a symbol permutation.

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Theorem (Castillo-Ramirez & Gadouleau, 2020)

The equivalence classes of the following ECA are not regular: 18, 22, 24, 25, 26, 30, 36, 37, 38, 45, 46, 54, 60, 62, 73, 90, 105, 122 and 126.

Proof.

Check the lemma above up to period 3. □

First cases of regularity

Theorem (Castillo-Ramirez & Gadouleau, 2020)

The equivalence classes of the following ECA are regular: 0, 2, 4, 5, 10, 11, 12, 13, 14, 15, 29, 35, 43, 51, 76, 128, 192 and 200.

Proof.

These have weak generalized inverses among ECA. □

Remaining cases

Question

Which of the ECA 6, 7, 9, 23, 27, 28, 33, 41, 57, 58 and 77 are regular?

Remaining cases

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Which of the ECA 6, 7, 9, 23, 27, 28, 33, 41, 57, 58 and 77 are regular?

Regularity is semi-decidable, by simply exhibiting a weak generalized inverse. How to semi-decide the non-regular cases?

The category of sofic shifts

Definition

A *category* consists of a class of *objects* $\mathcal{C}$ and a class of *morphisms* $f : X \to Y$ between objects $X, Y \in \mathcal{C}$, an *identity morphism* 1_X for each object $X \in \mathcal{C}$, and an associative composition rule that yields a morphism $g \circ f : X \to Z$ when given morphisms $f : X \to Y$ and $g : Y \to Z$.

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Definition

We say $X \subset A^{\mathbb{Z}}$ is a *sofic shift* if it is defined by a regular language L of forbidden words, i.e. $X = \{x \in A^{\mathbb{Z}} \mid \forall w \in L : \forall i \in \mathbb{Z} : x_{[i,i+|w|-1]} \neq w\}$.

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Definition

In the *category of sofic shifts* the objects are all sofic shifts, over all (finite) alphabets. Morphisms are the shift-commuting continuous functions (equivalently maps defined by local rules, equivalently restrictions of cellular automata).

Split epicness

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A morphism $f : X \to Y$ is *split epic* if and only if there exists a morphism $g : Y \to X$ such that $f \circ g = 1_X$. We say such g is a *section* of f, or *splits* f.

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Lemma

A cellular automaton $f : A^{\mathbb{Z}} \to A^{\mathbb{Z}}$ is regular in $\mathrm{CA}(A^{\mathbb{Z}})$ if and only if its codomain restriction $f : A^{\mathbb{Z}} \to f(A^{\mathbb{Z}})$ is split epic.

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Proof.

As previously observed, regularity means precisely that some cellular automaton can pick preimages for all points having a preimage. That's exactly what the definition says. □

Known results about split epicness

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Corollary

For each of the remaining ECA 6, 7, 9, 23, 27, 28, 33, 41, 57, 58 and 77, there exists a proof of either regularity or non-regularity!

What's the algorithm?

Theorem (Salo & Törmä, 2015)

If $X, Y \subset A^{\mathbb{Z}}$ are sofic shifts, then $f : X \to Y$ is split epic if and only if it admits a section with radius $3 + 9K^2 + K^{|A|^{2K+1}}$.

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We have $K = R(3, 3) = 6$, so the bound on radius is $327 + 6^{2^{13}}$. So we have

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candidates to consider. Problem: That would take hours!

Another approach based on consistent preimages

Say $X \subset A^{\mathbb{Z}}$ is *SFT* if it is defined by a finite family of forbidden words. Say $x \in A^{\mathbb{Z}}$ is *p-periodic* if p is the least shift-period of x. An *eventually periodic point* is one whose left and right tails are eventually periodic. Two points $x, y \in A^{\mathbb{Z}}$ are *right asymptotic* if $\exists n : \forall i \geq n : x_i = y_i$; *left asymptotic* defined symmetrically.

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If X is an SFT, then it is decidable whether a CA $f : X \to X$ is regular.

Proof.

Semidecidability: If it is regular, find a section.

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Semidecidability: If it is regular, find a section.

Co-semidecidability: If it is not regular, show that for some N, consistent preimages cannot be picked for $f : X \to f(X)$. There are finitely many choices of preimages for $\leq N$-periodic points, and for each choice checking non-consistency is an exercise in automata theory. □

What to do in practice: non-regularity

Lemma

The ECA 9 is not regular.

Proof.

The local rule of this ECA f maps $000 \mapsto 1, 011 \mapsto 1$, others to 0. Suppose f has a section g. [rest on whiteboard] □

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The ECA 9, 27, 28, 41 and 58 are not regular.

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A similar proof works in all cases: it is always enough to consider the g-images of points of period 1 (sometimes there are two choices to consider). □

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For all but ECA 9 and 28, the image is proper sofic, which in itself implies non-regularity, giving an alternative proof in these cases.

What to do in practice: regularity

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The ECA 7 is regular.

Proof.

The local rule maps $000 \mapsto 1, 001 \mapsto 1, 010 \mapsto 1$. Image is the SFT with the unique forbidden pattern 1001. Let's guess it has an inverse with radius 2, and deduce its local rule by looking at periodic points. [whiteboard] □

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Theorem

The ECA 6, 7, 23, 33, 57 and 77 are regular.

Proof.

For most of these, doing the above (by computer, for a suitable choice of radius) gives most (or all) values of a local rule. Guess the few remaining values. □

Weak inverse of ECA 6

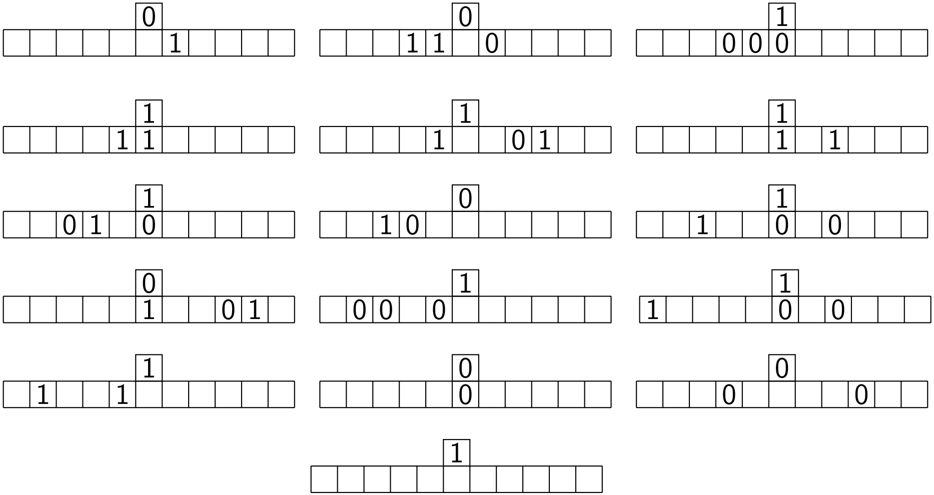

Figure: A weak generalized inverse of ECA 6. The rules are applied row by row, and on each row from left to right. An empty box denotes a wildcard symbol, and the first rule to apply is used. The rightmost coordinate is not actually read by any rule.

Weak inverse of ECA 7

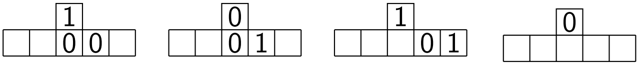

Figure: A weak generalized inverse of ECA 7. ECA 35 composed with σ.

Weak inverse of ECA 23

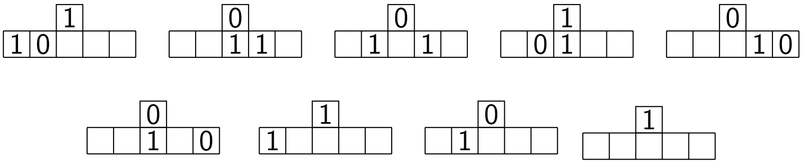

Figure: A weak generalized inverse of ECA 23.

Weak inverse of ECA 33

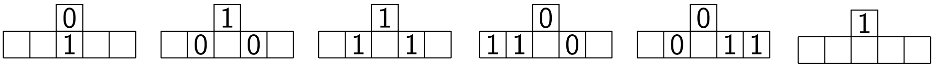

Figure: A weak generalized inverse of ECA 33.

Weak inverse of ECA 57

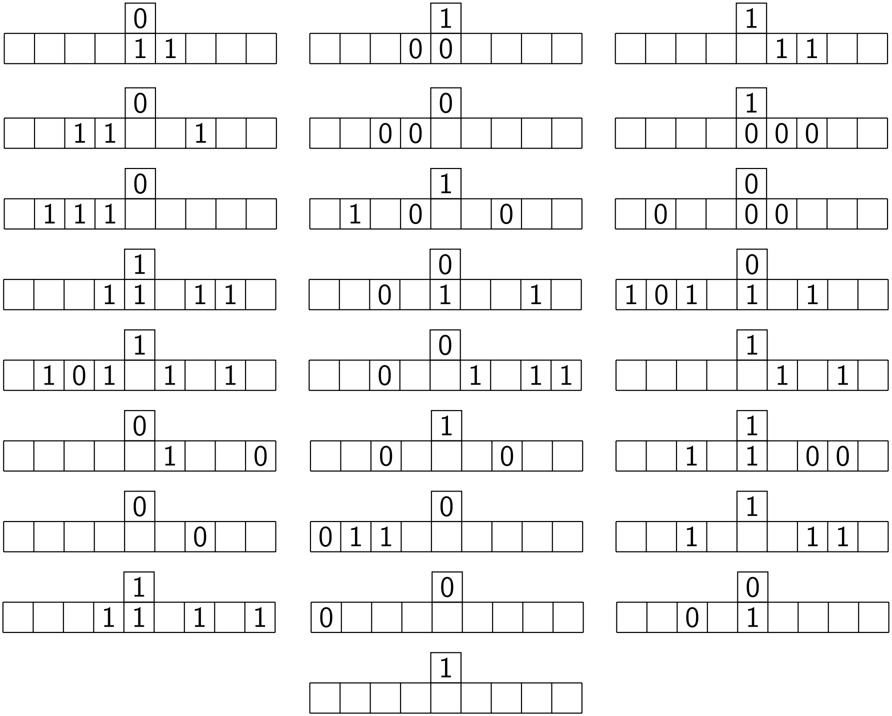

Figure: A weak generalized inverse of ECA 57.

Right inverse of ECA 77

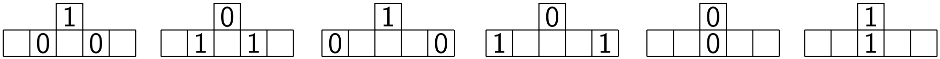

Figure: A weak generalized inverse of ECA 77.

Questions (one of them is probably easy!)

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Is there a "high-level" reason why regularity is decidable?

For injectivity and surjectivity, a high-level reason is you can program them in a suitable logic.

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For given n, r what is the probability that a randomly chosen n state radius-r cellular automaton is regular?

NB. If $n \geq 2$ and $n + r$ grows, the proportion of surjective CA tends to 0.

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Can we improve on the theoretical complexity bound on regularity?

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Question

Is there an undecidable equation over $\mathrm{CA}(\{0,1\}^{\mathbb{Z}})$ not involving inverses?

The End

Thank you for listening!