Isomorphic Boolean networks and dense interaction graphs

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A **Boolean network** (BN) with n components is a function

$$\begin{aligned} f : \{0,1\}^n &\to \{0,1\}^n \\ x = (x_1, \ldots, x_n) &\mapsto f(x) = (f_1(x), \ldots, f_n(x)). \end{aligned}$$

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Global transition function

Locale transition functions

$f_i : \{0,1\}^n \to \{0,1\}$

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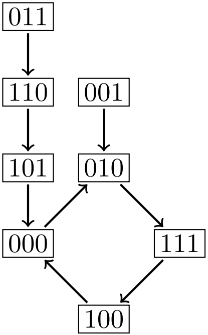
x	$f(x)$
000	010
001	010
010	111
011	110
100	000
101	000
110	101
111	100

$f_1(x) = x_2$
$f_2(x) = \overline{x_1}$
$f_3(x) = x_2 \wedge \overline{x_3}$



Dynamics $\Gamma(f)$

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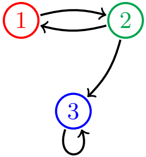

Interaction graph

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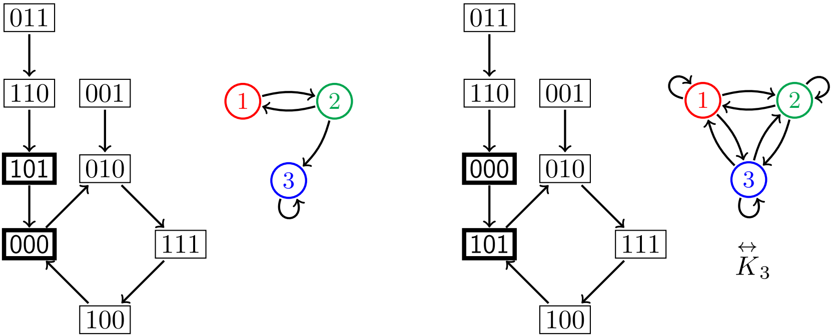
Example

If f^n is a constant function, what can be said on the interaction graph?

Two BNs are **isomorphic** if their dynamics are isomorphic.



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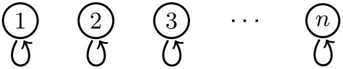

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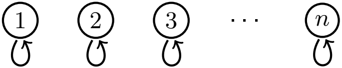

Question: Are there other f such that $|\mathcal{G}(f)| = 1$?

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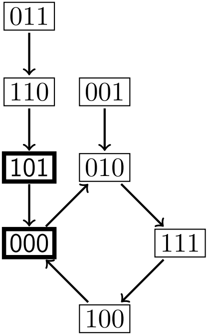

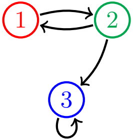

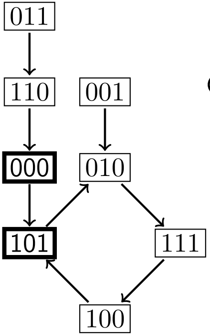

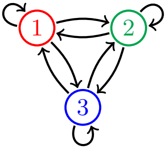

$\overleftrightarrow{K}_3$

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If $n \geq 5$, then some digraph in $\mathcal{G}(f)$ is not $\overset{\leftrightarrow}{K}_n$.

Corollary: If $n \geq 5$ we have $|\mathcal{G}(f)| = 1$ iff f is a constant or the identity.

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For $n \geq 1$ there is f such that any digraph in $\mathcal{G}(f)$ has at least $n^2/9$ arcs.

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Proof (sketch). We have $\overleftrightarrow{K}_n \in \mathcal{G}(f)$ whenever:

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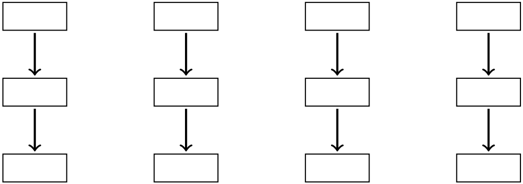

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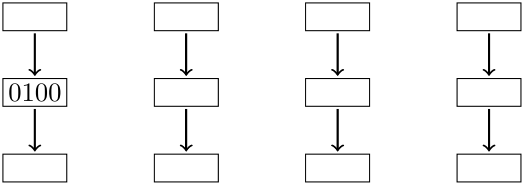

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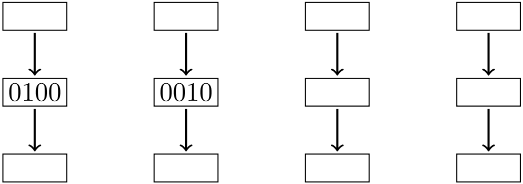

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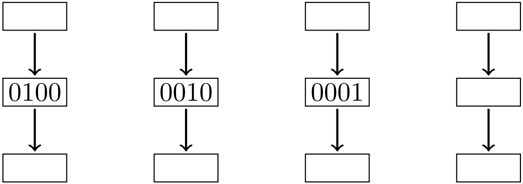

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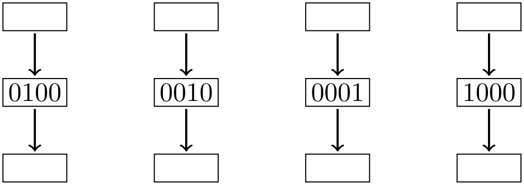

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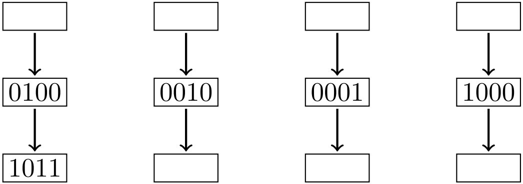

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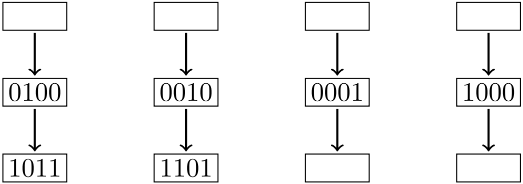

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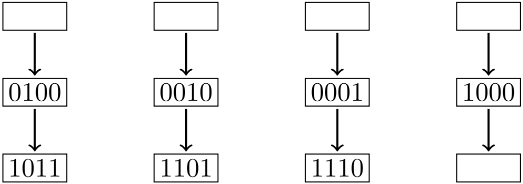

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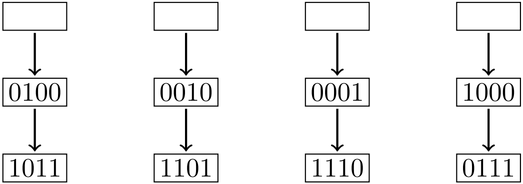

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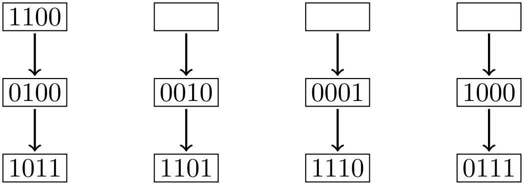

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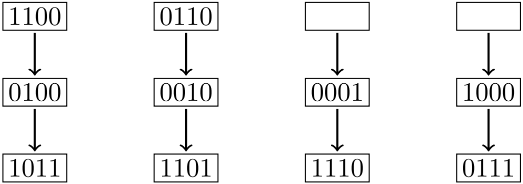

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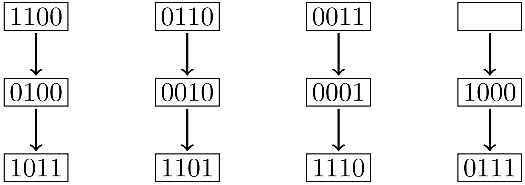

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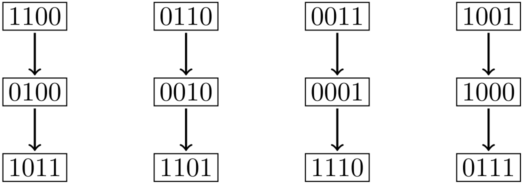

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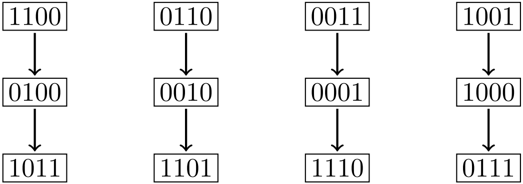

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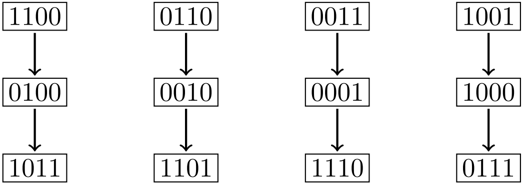

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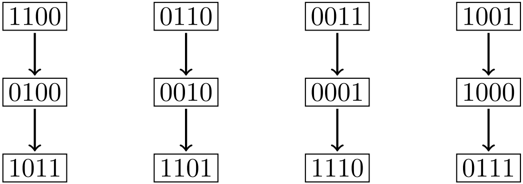

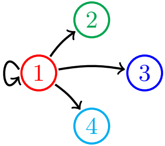

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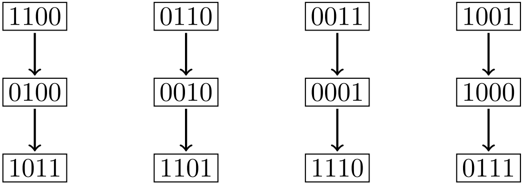

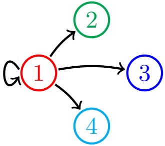

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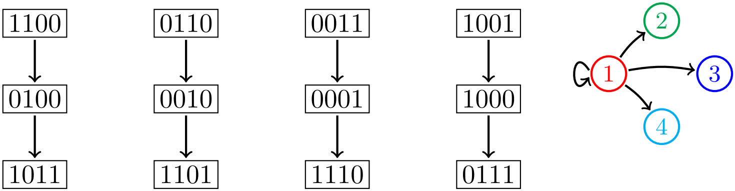

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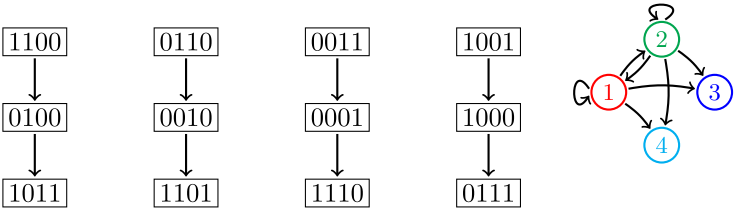

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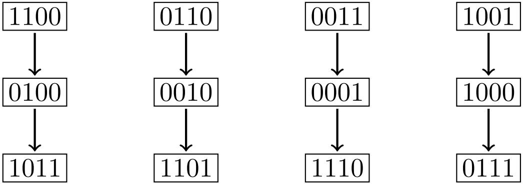

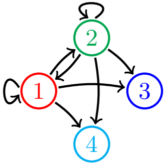

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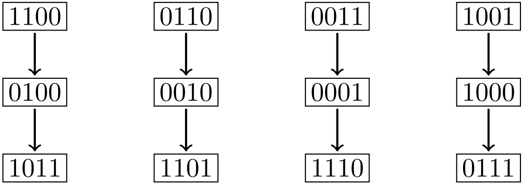

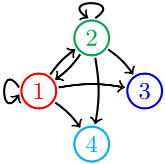

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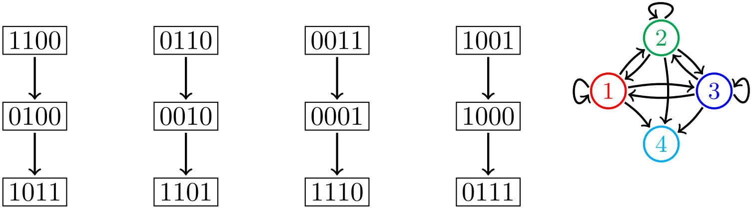

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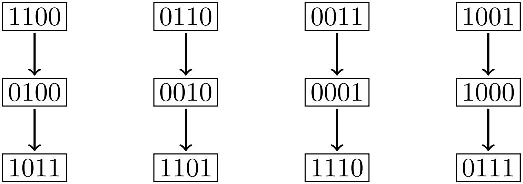

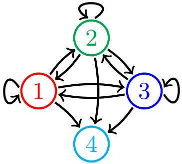

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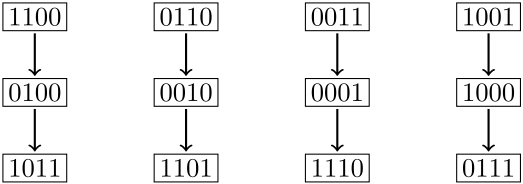

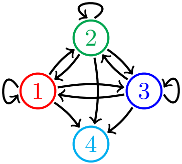

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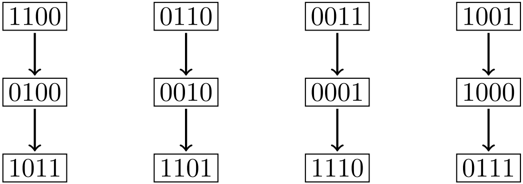

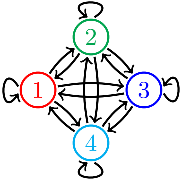

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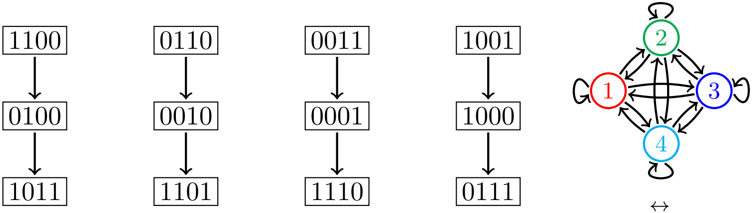

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