

DU Loa a la Seine: carnet de voyage. Form the Loa to the Seine: journey log. Eric Goles











Jorge Beauchef (* 1787, Velay, Francia -10 de junio de 1840, Santiago de Chile)





Beauchef

Los Estudiantes Rítmicos

Topics:

- 1) Neural or Threshold Networks: dynamics and energy
- 2) Regulation Networks: dynamics and Robustness.
- 3) Ants models and its complexity.
- 4) Prediction and Complexity
- 5) Social Science: Schelling Segregation, Sakoda's model and polarization
- 6) Periodical organisms: cicadas, bamboo

NEURAL AND THRESHOLD NETWORKS

$$x'_{ij} = 1$$
 iff $x_{i-1,j} + x_{i+1,j} + x_{i,j-1} + x_{i,j+1} \ge 2$



We consider a 4x4 lattice with periodic conditions, nearest interactions, states 0 or 1, and the local majority function: If the number of ones is bigger or equal to the number of zeros then the site takes the value 1.

Neural networks

$$X_{i} = S(\sum_{j=1}^{n} W_{ij}X_{j} - b_{i})$$
 for $1 \le i \le n$

$$W = (w_{ij})$$
 The weight matrix
 $b = (b_i)$ The threshold vector

$$s(u) = 1$$
 iff $u \ge 0$
0 otherwise

For arbitrary matrices W previous model may accept, iterated in parallel or block-sequentially, long period cycles and long transients ... But when W is symmetric the network admits short periods and an energy: (E.G and J.Olivos,

Discrete Mathematics, 1980, Discrete Applied Maths, 1981; E.G., SIAM J of Computing, 1982; E:G, F. Fogelman, Discrete Applied Maths(1985))

$$E(x(t)) = -\sum_{i=1}^{n} x_i(t) \sum_{j=1}^{n} w_{ij} x_j(t-1) + \sum_{i=1}^{n} b_i(x_i(t) + x_i(t-1))$$

Further, if diag (W) ≥ 0, any sequential update admits the energy (E.G., F. Fogelman, G. Weisbuch, Disc. Applied Maths. 1982)

$$E(x) = -\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}w_{ij}x_{i}x_{j} + \sum_{i=1}^{n}b_{i}x_{i}$$

For arbitrary matrices W previous model may accept, Iterated in parallel or sequentially, long period cycles and transients

But when W is symmetric the network converges to fixed point or two periodic cycles (parallel update),

And, if diag(W)≥0 to fixed point (sequential update).

E.G., J. Olivos, Periodic behaviour of generalized threshold functions, Discrete mathematics, vol 30, pp 187-189, 1980 E.G., Fixed Point behavior of threshold functions on a finite set, SIAM Journal on Alg. And Discrete Methods, vol 3(4), pp 2554-2558, 1982.

The most general dynamical result:

Consider the block-sequential scheme $s = \{I_1, ..., I_p\}$

The symmetrical threshold network *T*=(W, b, *s*)

Let $W(I_{k})$ the sub-matrix associated to the k-th block

If for every $k \in \{1,...,p\}$ $W(I_k)$ is non-negative-definite The network converges to fixed points

E. G., F. Fogelman-Soulie, D. Pellegrin, Decreasing energy functions as a tool

For studying threshold networks, Discrete Applied Mathematics, vol 12, pp261-277, 1985.

We will suppose now that every matrix is the incidence matrix of an undirected graph G=(V,E), so their entries belong to the set {0,1} W=W(G)= (w_{ii}) eventually with loops $(w_{ii} = 1)$

Consider the quantity:

$$\alpha(G) = -n - k + 2m - 4p$$

n = |V|, m = |E|, (without loops) K = the number of loops,

P = the minimum number of edges to remove such that the sub-graph is bipartite.

 Contents lists available at ScienceDirect

 Neural Networks

 journal homepage: www.elsevier.com/locate/neunet

 Dynamics of neural networks over undirected graphs

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Theorem: attractors for every block-sequential update.

Consider the block-sequential scheme $s = \{I_1, ..., I_p\}$

The symmetrical threshold network T=(W, b, s)

Let $G(I_k)$ the graph associated to the k-th block $\forall k \in \{1,...,p\} \ \alpha(G') < 0 \ \forall G' \subseteq G(I_k) \Rightarrow$ fixed points

 $\exists k \in \{1,...,p\} \text{ and } G' \subseteq G(I_k) \text{ such that } \alpha(G') \ge 0 \Rightarrow \text{ cycles}$





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FAUT-IL ADOPTER L'AVIS DE SES VOISINS?

La modélisation de réseaux d'individus dont l'opinion se conforme à celle de la majorité de leurs relations montre des évolutions parfois inattendues où, par exemple, le réseau se met à osciller entre deux états contradictoires.





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Jean-Paul Delahaye a récemment publié: Les Mathématiciens se pilent au jeu, une sélection de ses chroniques parues dans Pour la Science (Belin, 2017). ans une communauté ayant une décision à prendre, par exemple quant à la construction d'un pont, un vote doit avoir lieu. Les électeurs sont conciliants: chacun a un avis.

OUI ou NON, mais il est prêt à y renoncer si, parmi ses connaissances, plus de la moitié des avis sont contraires au sien. Pendant plusieurs jours, après un tour initial, chacun des électeurs consulte toutes ses connaissances et change d'avis le lendemain sur la construction du pont pour s'ajuster à l'avis majoritaire de ses connaissances consultées la veille. En cas d'égalité entre les pour et les contre parmi ses voisins, l'électur garde le même avis pour le lendemain. Oue va-t-il se passer?

Ce problème en apparence élémentaire des «dynamiques majoritaires» agite les mathématiciens depuis bientôt 40 ans. Ils en ont découvert une multitude de propriétés inattendues. Le premier résultat important à son sujet a été démontré en 1980 par mon ami Eric Goles, alors doctorant à l'université de Grenoble et aujourd'hui à l'université Adolfo Ibáñez, à Santiago du Chili, et Jorge Olivos, de l'université du Chili: cette dynamique conduit soit à une stabilisation des opinions des électeurs, soit à une oscillation des opinions entre deux configurations C et C': la configuration C donne le lendemain C' et C' donne le lendemain C. Aucune autre situation (oscillation entre trois états ou plus, désordre prolongé ...) n'est possible.

La modélisation du problème se fait avec un graphe dont les nœuds sont les membres de la

communauté concernée, et dont les arêtes (non orientées) indiquent qui se connaît: on place une arête entre A et B si A et B se connaissent et donc s'influencent peut-être d'un jour à l'autre.

Le «théorème de la période 1 ou 2» de Goles et Olivos stipule que, quelle que soit la répartition initiale des avis sur le graphe, l'application de la dynamique majoritaire conduit, en un nombre fini d'étapes, à une stabilisation complète des avis, ou à une double configuration des avis, chacune produisant l'autre.

Comme on le voit dans l'encadré ci-contre, si l'on part d'une configuration où le OUI l'emporte, le système se stabilise dans certains cas comme on l'attend en adoptant une configuration où le OUI reste majoritaire. Parfois cependant, la stabilisation se produit en adoptant une configuration où le NON l'emporte. Une troisième possibilité est prévue par le théorème de Goles et Olivos : le système se met à osciller entre deux états E et F. Il se peut même (voir l'encadré 1, A3 et B3) que pour E le OUI soit majoritaire et que pour F le NON soit majoritaire.

L'application de la dynamique majoritaire n'est donc pas le meilleur moyen d'arriver à un accord unanime, ou même seulement à un choix satisfaisant. Cependant, ce type d'évolution d'un graphe est très naturel et se rencontre dans plusieurs domaines scientifiques: réseaux d'automates finis, systèmes de votes, immunologie, interactions de cellules, réseaux de neurones, reconnaissance de formes, thermodynamique, etc. Le livre d'Eric Goles et Servet Martínez, Neural and Automata Networks – Dynamical Behavior and Applications >

LE THÉORÈME DE LA PÉRIODE 1 OU 2

A dopter l'avis majoritaire (« oui » les nœuds d'un graphe définit un mode d'évolution des opinions. Selon le graphe et selon l'état des opinions de chaque nœud, cette évolution dure plus ou moins longtemps avant soit de se stabiliser, soit d'osciller entre deux configurations. Les dessins A1 et B1 illustrent le cas où la dynamique majoritaire conduit à un état stable avec la même majorité qu'au départ. Dans les dessins A2 et B2, le graphe d'opinions es stabilise dans une configuration dont la majorité est inverse de celle de départ. Les dessins A3 et B3 montrent la troisième possibilité : le graphe finit par osciller entre deux configurations. Le théorème de Goles et Olivos, ou théorème de la période 1 ou 2, indique que des cycles entre plus de deux états sont impossibles : une dynamique majoritaire sur un graphe conduit soit à une stabilisation, soit à un cycle binaire.



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Fig. 3 The fission yeast cell-cycle threshold Boolean network. Using the same configuration as [3], (color online) the green/solid edges represent positive weights (activations), the red/dashed edges represent negative weights (inhibitory). The yellow/solid loops represent self-degradation.



Fig. 7 State transition graph for *Yeast1* using the parallel updating scheme. (Color online) The twelve red circles represent the fixed point states, the three blue circles represent the states that belong to the limit cycle.

The total number of updates is 545835

The equivalence classes are 15350

For $s \in P(z)$ and Ste9(t)=Rum(t)=0 there are 3984 equivalence classes with a limit cycle (unique).

For $s \notin P(z)$ there are 868 classes with a limit cycle (unique) Such that Ste9=0 and 661 classes with Ste9 non constant.

So there exists 5513 classes with a cycle (period between 2 and 5)



Fig. 8 State transition graph for Yeast1 using the block-sequential updating mode: $s(Wee1/Mik1) = s(Cdc25) < s(Cdc2/Cdc13) = s(Cdc2/Cdc13^*) = s(Ste9) < s(Rum1) =$ s(Slp1) = s(PP) = s(Start) = s(SK). (Color online) The twelve red circles represent the fixed point states, the four blue circles represent the states that belong to the limit cycle.



Cell cycle of the budding yeast

Li, Long, Lu, Tang,(2004) The yeast cell-cycle network is robustly designed, PNAS,101, 4781-4786

Fig. 12 The budding yeast cell-cycle threshold Boolean network. Using the same configuration as [14], (color online) the green/solid edges represent positive weights (activations), the red/dashed edges represent negative weights (inhibitory). The yellow/solid loops represent self-degradation.

THEOREM: for any update YEAST2 has only fixed points

ANTS as Complex systems: could be intelligence an emergent property?



A. Gajardo, E.G., A Moreira, Complexity of Langton's Ant, Discrete Applied Mathematics. Vol. 117, Issue 1-3, pp. 41-50, (with A. Gajardo, A. Moreira). (2002)

A. Gajardo, E.G, E. Moreira, Generalized Langton's Ant: Dynamical Behavior and Complexity, STACS'2001 Lectures Notes in Comp. Sci. 2010, pp.259-70. (2001)

A.Gajardo, E.G, A. Moreira, Dynamical Behavior and Complexity of Langton's Ant, in Complexity, Vol.6, N°4, pp 46-51 (2001)

A. Gajardo, E.G., Dynamics of a class of ants in a one dimensional lattice in Theorical Computer Science, vol. 322, number 2, pp. 267-283 (2004).

M. Schimick, E.G, M. Markus, Tracks Emerging by Forcing Langton's Ant with binary sequences, in Complexity 11,27-32 (2006).

Planar ant model (Langton's ant)



Some associated models and its dynamics



FIGURE 2: Same as Fig. 1, but with a one-bit perturbation in the forcing period.

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Ant's dynamics



Logic gate



THEOREM: The ant is P-Complete and Turing Universal

Dynamics of automata networks and computational complexity

We will consider decision problems (YES or NO answer). The class **P**: problems which we can be decided in a serial computer in polynomial time.

The class NC: problems which can be decided in a parallel machine (say a PRAM) in poly-logarithmic time by using a polynomial number of processors.

Bootstrap Percolation

Given a finite non oriented graph G=(V,E) and an initial configuration of O's and 1's.Consider the strict majority function operating at each node.

What is the relationship between the graph and the proportion of 1's such that iterated in parallel every node will become



Decision problem PRE: given an initial configuration and a specific node at value 0. does there exist T>0 such that this node becomes 1?

Theorem (E.G, P. Montealegre) For graphs such that its maximun degree \geq 5, PRE is P-complete. If the maximun degree \leq 4, PRE belongs to NC

COMPLEXITY for the majority

We consider the similar decision problem PRE

Von Neumann neighborhood in 2D OPEN

conjecture (C. Moore): easy

+

Nearest neighborhood In 3D P-Complete

(C. Moore)

THEOREM: For planar graphs PRE for the majority vote is P-Complete

E.G, P. Montealegre.





The cross-over gadget



Social Science Modelling: Schelling Segregation, Sakoda's model and polarization



The Model of Segregation by Shelling

Thomas C. Schelling (1969 - 1972)

- Lattice one or two dimensional with periodic conditions
- State $x_k = \pm 1$
 - Neighborhood Moore (green and red arrows) and von Neumann (red arrows)



So Tolerance threshold $θ ∈ {1,...,𝒴}$

Happiness threshold

An individual is unhappy if there are more than θ individuals on the other state in its neighborhood

The update rule

At each step, one lists the unhappy individuals of both species, and then randomly (for instance) one exchanges two individuals of opposite value.



Polarization.

Recent empirical findings suggest that societies have become more polarized in various countries, i.e. the median voter of today represents a smaller fraction of society compared to two decades ago. What is driving this polarization? Activist-voter interactions play a major role in political opinion formation. We study a macroscopic opinion model in which activists target certain groups of individuals in order to inject their political ideas. Polarization emerges when small heterogeneities among competing activists cause them to target different groups in society



Polarization model. In this example, the political spectrum consists of N = 9 different states and is divided in three groups: group A, a neutral set of agents, and group B. A transition from one state to its nearest neighbors occurs with probability p. A political activist A₊ or B₊ can locally decrease the transition probabilities ($p_- < p$) or increase them ($p_+ > p$).

Lucas B[°]ottcher, Hans Gersbach, (ETH) E.G. P. Montealegre.UAI

Periodic Organisms

New work in progress EG, Ivan Slapnicar, Marcos Lardies nature International weekly journal of science nature news home news archive specials opinion features news blog natu comments on this Published online 23 July 2001 | Nature | doi:10.1038/news010726-3 story News Cicadas appear in their prime ories by keywords Irregular emergence may foil insects' predators. cicada periodic Erica Klarreich orime Bob Dylan immortalized the rare zycle appearance of periodic cicadas in his 1970 song Day of the Locusts. But he may not have realized that he was is article elsewhere honouring a mathematical event as well **Blogs linking to** as a biological one. this article Periodic cicadas emerge from their Add to Digg underground homes to mate every 13 or 17 years. Both of these numbers are Add to Facebook prime - they can only be divided by one. Add to Newsvine Evolution could have selected for appearance in prime cycles, a new model Add to Del.icio.us suggests¹. Add to Twitter Cicadas emerge after a The notion is simple: cicadas with prime prime time. cycles coincide with their predators and © SPL parasites less often. "The philosophy is

E.G, M. Markus, Prime number selection of cycles in a predator-prey model, Complexity, Vol. 6, Nº4, 33-38, (2001).

E.G, M. Markus, Cicadas showing up every prime number of years, The Math. Intelligencer, Vol.24, N°1, pp.30-32 (2002).

E.G, M. Markus, O Schulz, Prey population cycles are stable in an evolutionary model if and only if their periods are prime, Science Asia 28 (2002): 199 - 203

Periodic Organisms

Consider two organisms, C_1 and C_2 , with a year period life $c_1, c_2 \in \mathbb{N}, c_1, c_2 \geq 2$. 2. Consider the year interval $T = c_1 \cdot c_2$, and the emergence functions

$$\chi_i: [1,T] \to \{0,1\},\$$

such that $\chi_i(t) = 1$ if and only if the organism C_i is present in year t. Clearly, organism C_1 appears exactly c_2 years in the interval [1, T] and, respectively, C_2 appears exactly c_1 times.

Let us now define the (local) fitness functions associated to C_1 and C_2 as:

$$f_i: \{0,1\} \times \{0,1\} \to \{-1,0,1\}, \quad i = 1,2$$

such that $f_1(0,*) = f_2(*,0) = 0$, that is, if organism is not present in a year, its fitness is zero. if the organism is present, its fitness with respect to the other organism may be 1, 0 or -1 (good, neutral or bad).

For given periods c_1 and c_2 , the global (cumulative) fitness functions over the interval [1, T] are obtained by simply adding yearly fitnesses and dividing them by the number of years in which the organism appears in this interval:

$$F_1(c_1, c_2) = \frac{1}{c_2} \sum_{t=1}^T f_1(\chi_1(t), \chi_2(t)),$$

$$F_2(c_1, c_2) = \frac{1}{c_1} \sum_{t=1}^T f_2(\chi_1(t), \chi_2(t)).$$

$$\begin{array}{c} + \\ C_{1} \\ + \\ C_{2} \\ + \\ C_{1} \\ + \\ C_{2} \\ + \\ C_{1} \\ + \\ C_{2} \\ + \\ C_{1} \\ - \\ C_{2} \\ + \\ C_{2} \\ - \\ C_{2} \\ + \\ C_{1} \\ - \\ C_{2} \\$$

$$\begin{array}{c} & & + \\ C_1 & & + \\ & & + \\ \hline \\ C_1 & & + \\ \hline \\ C_1 & & C_2 \\ & + \\ \hline \\ C_1 & & C_1 \\ & + \\ \hline \\$$

$$v_{1} = (-1, 0, 1, 1)$$

$$v_{2} = (-1, 1, 1, 1)$$

$$v_{11} = (1, 0, -1, 0)$$

$$v_{12} = (0, 1, 1, -1)$$

$$v_{13} = (0, 1, 1, 0)$$

 $v_{14} = (1, 1, 1, 0)$

 $v_{10} = (-1, 1, 1, 0)$

 $v_7 = (1, 1, -1, 0)$

 $v_3 = (0, 1, 1, 1)$

 $v_4 = (0, 0, 1, 1)$

 $v_5 = (-1, -1, 1, 1)$

 $v_6 = (1, 1, -1, -1)$

 $v_8 = (1, 1, 1, -1)$

 $v_9 = (-1, 1, 1, -1)$





Here we illustrate the behavior of the representative 4-tuples

 $v_7, v_{10}, v_{11}, v_{14}, v_1, v_2, v_6, v_8, v_9,$

from Propositions 2-5. We display graphs for (c_1, c_2) where $c_1, c_2 = 2, 3, 4, \ldots, 40$, for two cases, [-p, p] = [-1, 1] and [-p, p] = [-4, 4].



Les Grenoblois



Francois Robert, The boss



The brilliant all terrain friend



Jacques Demongeot The friend who I owe so much



The wise partner, the chief, the minister



J. L Lion The academician.





Maurice Nivatdon Marco SchützembergerMembership Number:
Membership type:the academician
and the artist.Sectioof the submarine tab.
Elected:Main Country of Residence: FRANCE



Dominique Perrin, the Chilean-French partner; the Malraux chilien.



the Polytechnicien du corps de Mines The philosopher and the friend



Steve Wolfram the office partner at the Artificial Intelligence lab. MIT



Stuart Kauffman, ethe one with crazy idea. The unreserved friend who gift me Boolean Networks

Maurice Tchuente , The partner The friend The minister.





Michel Cosnard, the partner



Yann Lecun partner from la Montée sainte Genovieve (Turing Price 2018)



Francoise the friend from Paris

Henri Atlan, The philosopher and Biophysicist of disorder *Entre el Cristal y el Humo*







































































































And from the Loa to the Mapocho river then Hanoi. Tomorrow Shanghai, Bristol, Marseille and again the Seine and Paris, and Antofagasta and Santiago and go ahead, again and again, because the journey never ends.